Searching the proportional level of operating costs – specification of the minimum volume of production

Proporcionování vlastních nákladů na výrobu – stanovení minimálního objemu výroby

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Abstract: This article concerns the topic of various types of costs and refers to relations between them. It also solves relations between costs and the output in in-kind units, between costs and revenues (in financial units), between the unit-price and the sale profit. The article also describes two possibilities how to find the break-even point: one way is to compare the dynamics of revenues and costs, the second one is to compare actual and expected variable costs. The article also suggests the way how to find the break-even point using cost-revenue ratio indicators.

Key words: proportional costs, fixed costs, over-proportional costs, under-proportional costs, unit costs, break-even point, minimum profit, minimum profitability of production, variable costs, cost-revenue ratio, rate of profit

Abstrakt: Článek se zabývá různými druhy nákladů podniku a vazbami mezi nimi. Dále je zde nastíněn vztah těchto nákladů k výkonům podniku v naturálních jednotkách, jeho tržbám (vyjádřených v peněžních jednotkách), jednotkové ceně a zisku z prodeje. V další části článku jsou rozebírány dva způsoby stanovení bodu zvratu, a to na základě porovnání dynamiky tržeb a vlastních nákladů a porovnáním skutečných a předpokládaných variabilních nákladů. Na závěr je nastíněna možnost stanovení bodu zvratu pomocí ukazatelů nákladovosti.

Klíčová slova: proporcionální náklady, stálé náklady, nadproporcionální náklady, podproporcionální náklady, jednotkové náklady, bod zvratu, minimální zisk, minimální rentabilita výroby, variabilní náklady, nákladovost, výnosnost

INTRODUCTION

One of the important principles, when preparing production, is to find the correlation between operating costs and revenues to reach a profitable production. Relations between costs and incomes must be, among other, taken into consideration also when choosing an investment option and when making other strategic decisions. We can often see that the first step to be done in many companies is the technical preparation of production, which is later followed by changes of the economy of production. These types of strategy often do not lead to an economic solution and the whole preparation of production leads to uneconomic results.

The problems solved here are, by no means, the new ones, but we try to present a comprehensible approach and to show certain economic principles that are generally valid.

SEARCHING THE PROPORTIONAL LEVEL OF OPERATING COSTS

When preparing any type of production, the top-priority task is to reach the appropriate relationship between revenues, the volume of production and costs to gain a requisite profit. This process is called "searching the proportional level of operating costs."

The task of searching the proportional level of operating costs is the following:

a) To define the minimum volume of production which can ensure that the costs of the company will be covered by revenues and also an adequate profit will be gained.
b) To guess how the increase of the production will influence the amount of revenues, costs and what the company economic result will probably be.

Searching a proportional level of operating costs is an important component of the process of preparation of any type of production. It is of extreme importance especially when preparing a new investment. On principle, the first thing to be done is an economic calculation of production; this calculation shows the necessary investment and parameters and then the suitable investment should be chosen. The vice versa procedure – i.e. to choose an investment as the first step and to find an appropriate economic calculation as a second step, is usually incorrect.

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Improper investments often result in reaching their effectiveness at an inadequate high volume of production.

Inadequate volume of production usually results in the following negative effects:

1. The required volume of production is usually reached after a certain time and not immediately after the investments implemented. This fact causes protraction of the pay-off period.
2. If the efficiency of the investment is planned for an inadequately high volume of production, the effect from the production enlargement (leading to the raising profit) does not stay in the particular company but is used to reimburse the investment costs and flows over to other companies.
3. To reach a higher volume of production often requires higher costs. This fact is often omitted when the investments efficiency is assessed.

It is recommendable to use the method of searching the proportional level of operating costs not only prior to any new investment, but also prior to any important technological and organisational change in production, as well as in the case of changes of the external environment (e.g. market price changes, demands on salaries, etc.).

PROPORTIONAL, FIXED, OVER-PROPORTIONAL AND UNDER-PROPORTIONAL COSTS

It is vital to explain basic relations between operating costs, the volume of production and unit costs for the characteristic cost function types. It is necessary to introduce the terms of proportional, fixed, over-proportional and under-proportional costs.

Proportional costs

Proportional costs are those costs that are directly proportional to the volume of production. The cost function of proportional costs is given by a line which goes through the origin. The character of the proportional costs shows that the unit proportional costs are constant and are equal to the tangent of the right line of the proportional costs (Figure 1).

Development of proportional and proportional unit costs can be described by the following equations:

$$VN(p) = a \times Q$$

$$jN\,(p) = \frac{VN\,(p)}{Q} = \frac{a \times Q}{Q} = a$$

jN = proportional unit cost
$VN(p)$ = proportional costs
Q = volume of production in in-kind units

The above-mentioned equations indicate that the bigger the angle of the line of the proportional costs, the higher the unit costs.

Proportional costs assume constant unit costs. The increase of the volume of production does not influence their efficiency. Proportional costs development shows typical signs of an extensive increase of the volume of production. Increase of proportional costs is usually explained by material costs; these material costs are proportional to the volume of production.

Fixed costs

It is typical for the fixed costs that they remain constant within a certain range of production. The cost function of the fixed costs is given by a line. This line is parallel to the *x*-axis at the value of a particular unit fixed cost. Depending on the volume of production, these fixed costs change in jumps (Figure 2).

The cost function of fixed costs can be described by the following equations:

$$VN_{(S)} = SN$$

$$jN(S) = \frac{SN}{Q} \qquad \lim_{Q \to \infty} jN\,(S) = 0$$

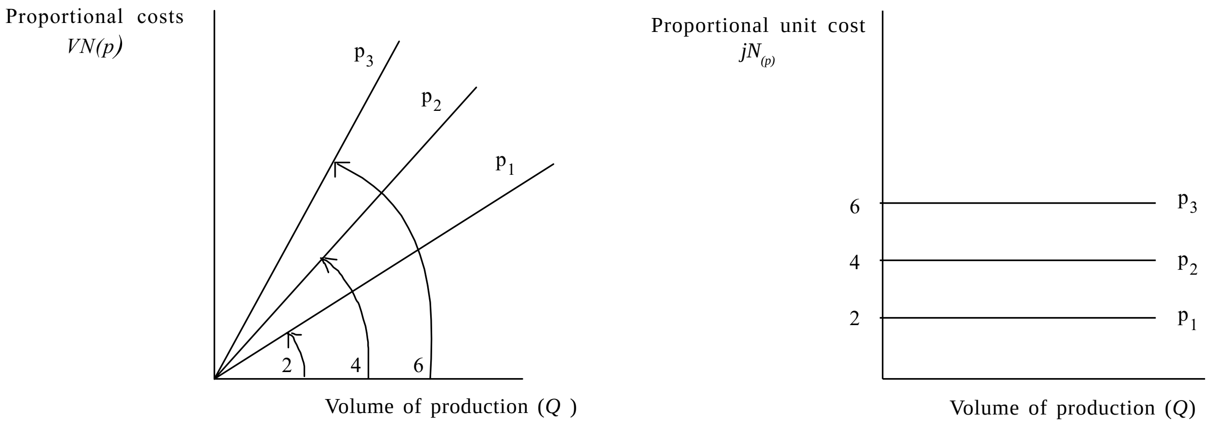


Figure 1. Development of the proportional costs

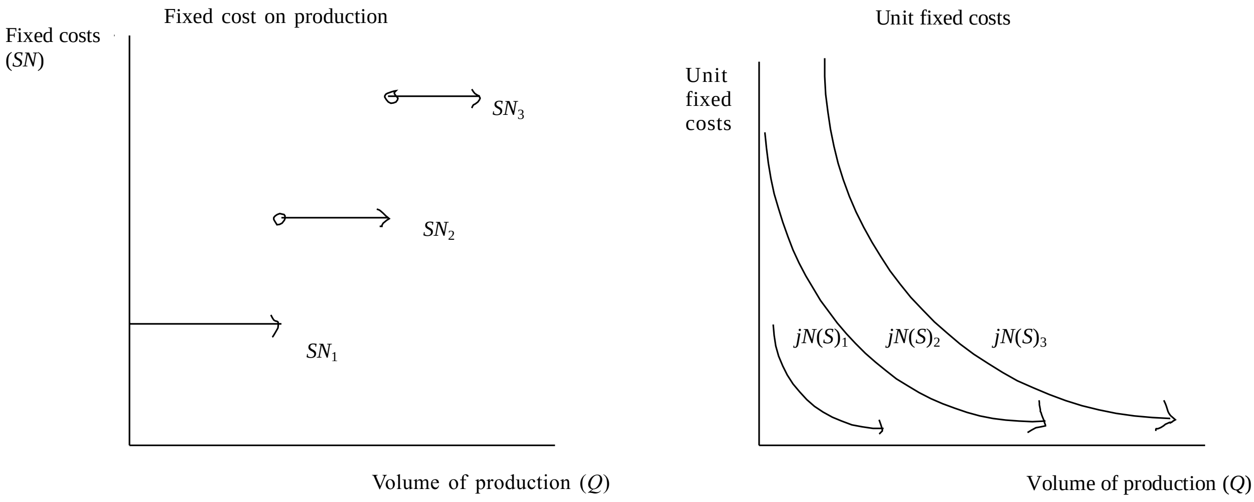
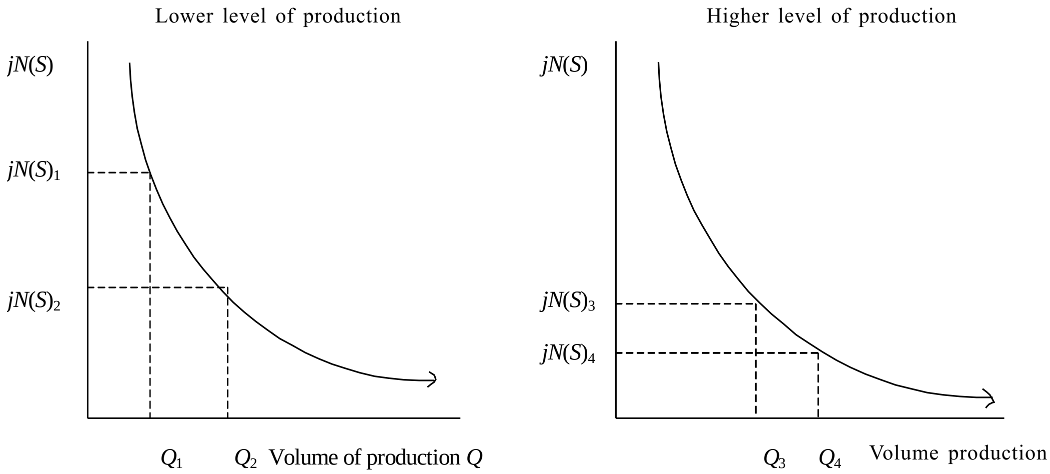


Figure 2. Development of fixed costs



for $(Q_2 - Q_1) = (Q_4 - Q_3)$, is valid that $(jN(S)_1 - jN(S)_2) > (jN(S)_3 - jN(S)_4)$

Figure 3. Various reductions of unit costs for various levels of production

$VN_{(S)}$ = costs
SN = fixed cost
$jN(S)$ = unit fixed cost
Q = volume of production in in-kind units

The following economic rules can be derived from the above mentioned equations:

1. Reduction of unit fixed costs, corresponding to constant increase of production, declines with the increasing production (Figure 3).

For the same increase of the volume of production, the unit fixed cost saving is higher at a lower level of production than at a higher one.

2. When fixed operating costs are rising, unit fixed costs are rising as well. For the constant increase of the volume of production, the unit fixed cost saving is higher at higher operating costs than at lower ones (Figure 4).

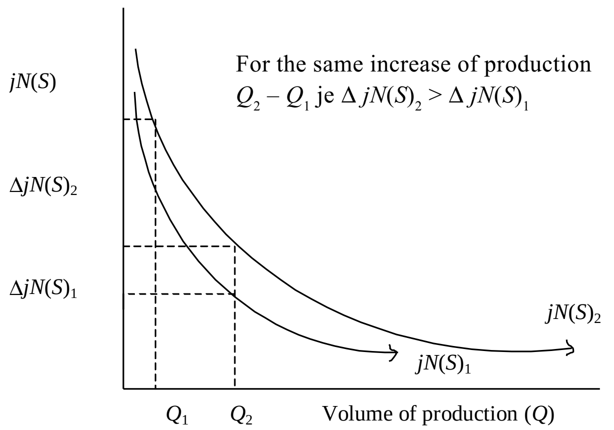


Figure 4. Changes of fixed unit costs resulting from different fixed operating costs

Manufacturing capacity utilisation of more expensive technologies should be paid higher attention than the cheaper ones utilisation. Better manufacturing capacity utilisation of more expensive technologies brings larger reduction of fixed unit costs and therefore also better relative saving of fixed costs.

Over-proportional costs

Over-proportional costs are created by the merger of two cost types – i.e. proportional costs and fixed costs.
The relations of over-proportional costs can be interpreted by the following equations:

1. Over-proportional operating costs

$$VN_{(np)} = SN + VN_{(p)}$$

$$VN_{(np)} = SN + jN_{(p)} \times Q$$

$VN_{(\mathrm{np})}$ = over-proportional costs
SN = fixed operating costs
$VN_{(p)}$ = proportional cost
$jN_{(p)}$ = proportional unit cost

2. Over-proportional unit costs

$$\frac{VN_{(np)}}{\mathrm{Q}} = \frac{SN}{Q} + \frac{VN_{(p)}}{Q}$$

$$jN_{(np)} = jN_{(S)} + jN_{(p)}$$

$$\lim_{Q\to\infty} jN_{(np)} = jN_{(p)}$$

$jN_{(np)}$ = over-proportional unit cost
$jN_{(S)}$ = fixed unit cost

For over-proportional costs, similar economic rules are valid as for proportional costs and the fixed costs (Figure 5).

Under-proportional costs

Under-proportional costs are used when preparing production; they are composed of negative costs and proportional costs (Figure 6).
The development of under-proportional costs can be described by the following equations:

1. Under-proportional operating costs

$$VN_{(pp)} = VN_{(p)} - SN$$

2. Under-proportional unit costs

$$jN_{(pp)} = \frac{VN_{(p)}}{\mathrm{Q}} - \frac{SN}{Q}$$

$$jN_{(pp)} = jN_{(p)} - jN_{(S)}$$

$$\lim_{Q\to\infty} jN_{(pp)} = jN_{(p)}$$

$VN_{(pp)}$ = under-proportional costs
$jN_{(pp)}$ = under-proportional unit costs

It is typical that the under-proportional unit costs grow up to the level of the proportional unit costs.

METHODS OF SEARCHING OPERATING COSTS PROPORTIONAL LEVEL

Searching the production parameters proportional level and searching the minimum volume of production can be done by using several methods. The following text describes the two basic methods:

1. Comparison of revenues and costs dynamism
2. Comparison of real and assumed variable costs

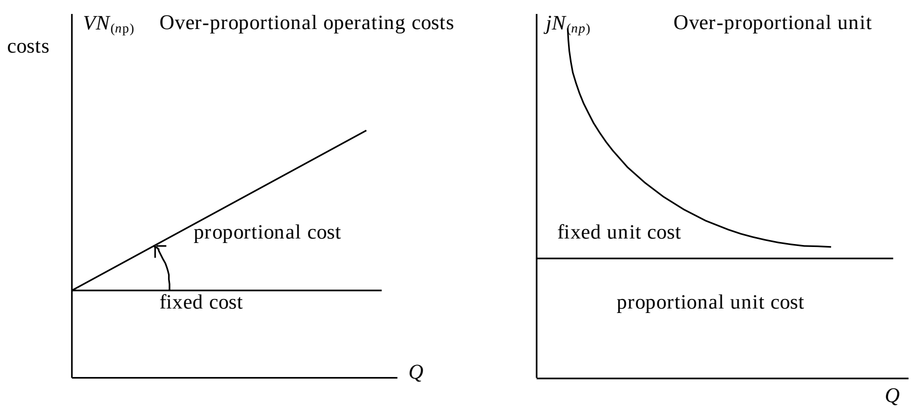


Figure 5. Development of over-proportional costs

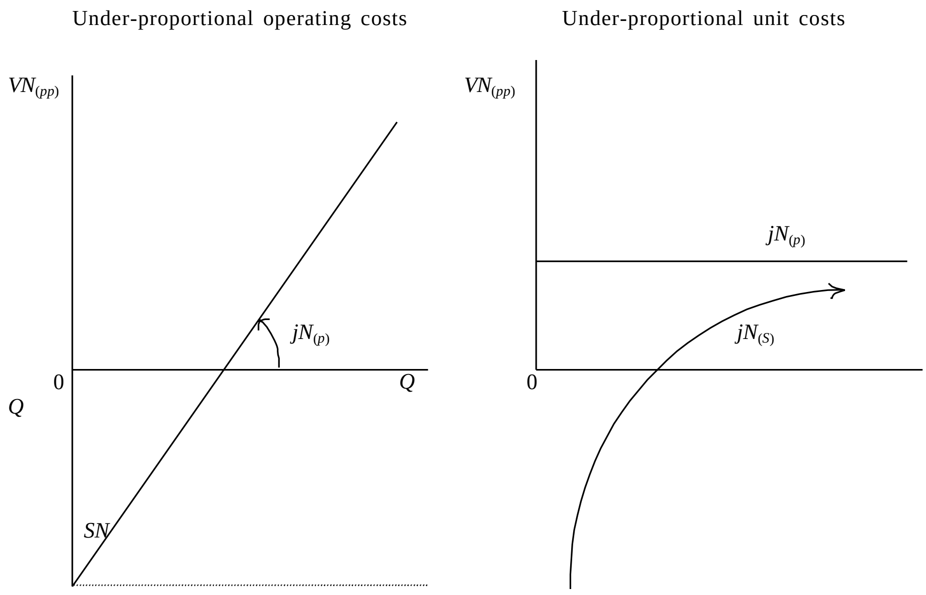


Figure 6. Development of under-proportional costs

Comparison of revenues and costs dynamism

To find a minimum volume of production, it is necessary to specify the revenues function and the cost function. It is possible to define the revenues, with a certain simplification, as a function of price and of the volume of production in the in-kind units.

$T = c \times Q$

T = total revenues volume
c = average market price
Q = volume of production in in-kind units

The revenues function is a proportional function which goes through the origin where the average exercise price is a constant of proportion. The higher the average market price, the higher the company revenues at the same volume of production in the in-kind units. When the volume of production is rising, the volume of sales is rising as well (Figure 7).

The development of costs in dependence on the volume of production may have different shapes. With some simplification, it can be said that it is given by the development of fixed and variable costs.

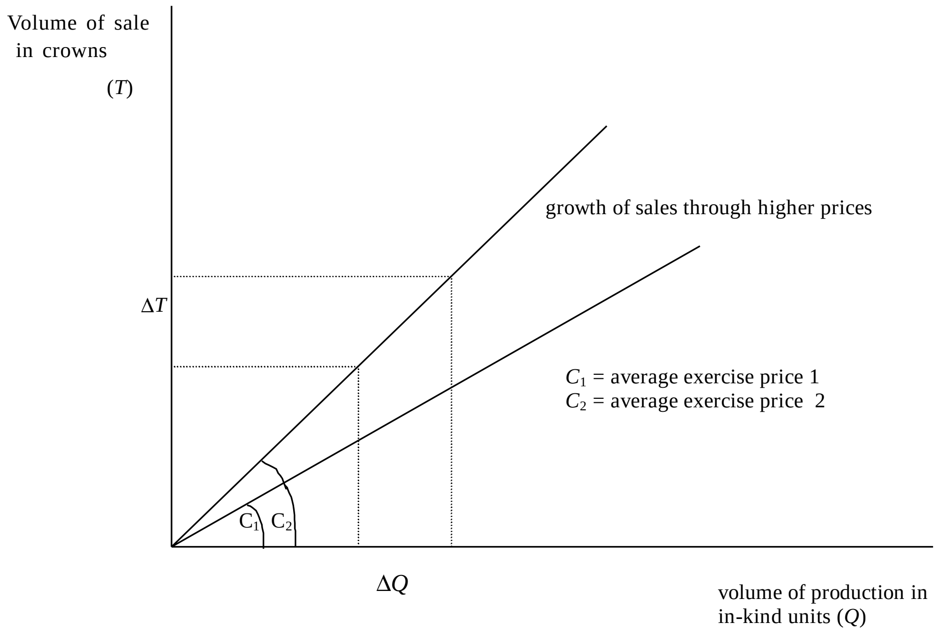


Figure 7. Development of the volume of sales

$VN = f(VN_{(S)}, VN_{(V)})$

$VN_{(S)}$ = fixed costs
$VN_{(V)}$ = variable costs

Provided that operating costs are identified with over-proportional costs, the cost function is of the following shape:

$$VN_{(np)} = SN + jN_{(V)} \times Q$$

The break-even point can be calculated by comparing the function of revenues dynamism and costs function dynamism.

THE BREAK-EVEN POINT – COMPARISON OF SALES DYNAMISM AND COSTS DYNAMISM

The break-even point is defined as the point in which the company reaches zero profitability of production and the revenues for goods are equal to costs on this production.

The break-even point then equals the volume of production in the in-kind units for which the volume of revenues equals to the volume of costs (Figure 8 and 9).

$$T = VN$$

$$c \times Q = SN + jN_{(p)} \times Q$$

$$c \times Q - jN_{(p)} \times Q = SN$$

$$Q = \frac{SN}{c - jN_{(p)}}$$

The area of unprofitable production is defined for the volume of production Q for which $Q < Q_0$.

The break-even point (the volume of production of zero profitability) is given by the volume of production $Q = Q_0$.

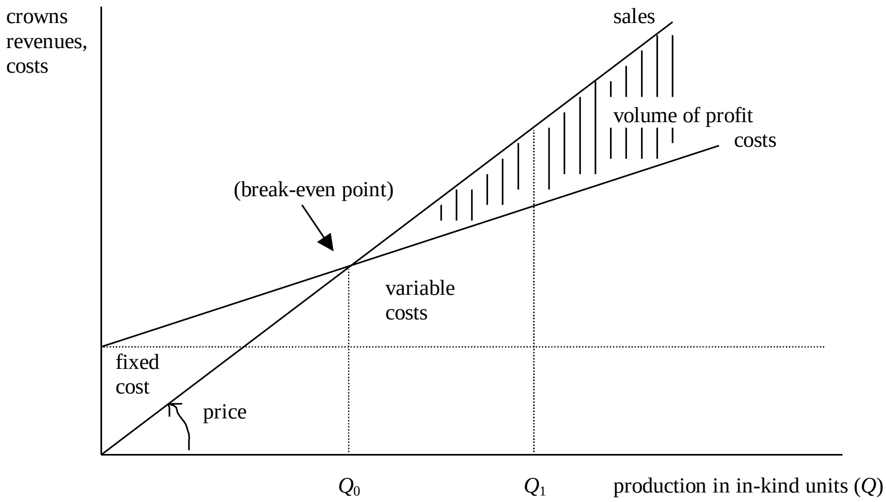
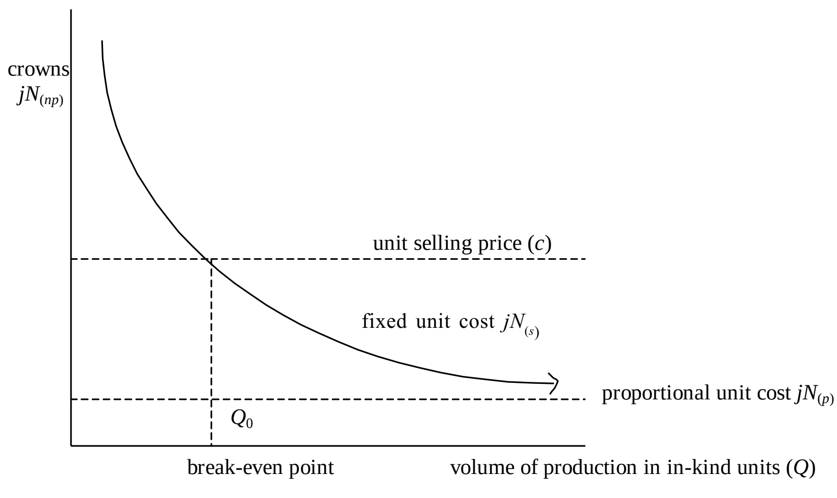


Figure 8. The break-even point in relation to revenues and costs of the production



Q_0 – break-even point

Figure 9. The break-even point and unit costs

The area of profitable production is given for the volume of production $Q > Q_0$.

The break-even point is therefore often called the threshold value of the profit.

Every company requires a certain minimum profitability which can ensure successful re-production of its production process.

Relations defining the break-even point Q_0

1. If costs are defined as over-proportional costs, the price of a unit of production must be higher than the proportional unit cost. If $c \leq jN_{(p)}$, the production is permanently unprofitable.
2. If costs are defined as proportional costs, then also $c > jN_{(p)}$.
3. The lower the definition of the break-even point for the lower volume of production, the higher the volume of profit gained by the company at the increased level of production.
4. The higher the costs, the higher the volume of production for zero profitability at the same conditions.
5. The higher the difference between the price and the unit costs, the lower the volume of production for which the break-even point is defined.

   These relations should be respected when production is being prepared.

The break-even point for the minimum profitability of the production

The minimum profitability can be given as a minimum profit from an in-kind unit of production.

- jZ_0 = the profit from the in-kind unit of production for the break-even point Q_0. It is obvious that $jZ_0 = 0$
- VN = operating cost
- jZ_{MIN} = the profit from the in-kind unit of production which provides the minimum profitability of production (minimum unit profit)
- Q_0 = the volume of production for the break-even point ($jZ_0 = 0$)
- Q_{MIN} = the volume of production which ensures the minimum profitability of production (minimum unit profit)
- Z_{MIN} = the minimum volume of profit
- T = sales

$$Z_{MIN} = jZ_{MIN} \times Q_{MIN}$$

The volume of production which ensures the minimum profitability of production can be derived from the following equations:

$$T = VN + Z_{MIN}$$

$$c \times Q = SN + jN_{(p)} \times Q + jZ_{MIN} \times Q$$

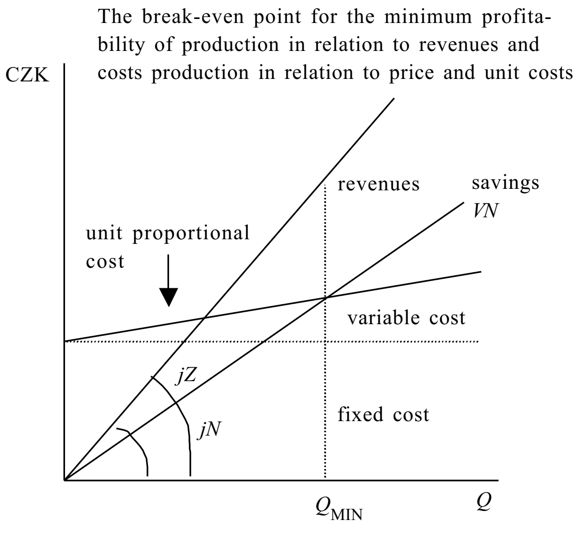
$$c \times Q - jN_{(p)} \times Q - jZ_{MIN} \times Q = SN$$

$$(c - jN_{(p)} - jZ_{MIN}) \times Q = SN$$

$$Q = \frac{SN}{c - (jN_{(p)} + jZ_{MIN})}$$

The minimum unit profit reduces the range between the price and the unit costs and at the same size of the fixed cost requires a bigger volume of production (Figure 10).

In the graphic method, we suppose that the profit per an in-kind unit of production (jZ) is constant and therefore the profit function is given by the profit per in-kind unit of production and by the volume of production in the in-kind units (according to the equation $Z = jZ \times Q$).



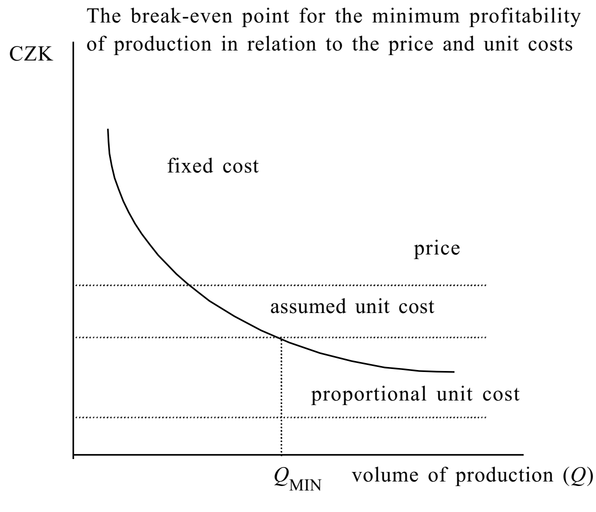


Figure 10. Graphic method for finding the break-even point for the minimum profitability of production

Under the same conditions, the higher the minimum profit we demand, the bigger volume of production is necessary.

It is obvious that it is impossible to increase the volume of production in the in-kind units endlessly, but it is necessary to confront the demands on the minimum volume of production with the technological and organisational possibilities as well as with the sales possibilities of the product.

The above-mentioned confrontation can evaluate whether the minimum volume of production is a real one and whether the amount of the minimum profit is real.

Alternative solution of the break-even point for the minimum profitability of production

The break-even point for the minimum profitability of production can be also solved in another way: the demand on proportional costs development is given by the function of under-proportional costs according to Figure 11.

The alternative solution of the break-even point for the minimum profitability of production:

1. At first, we must specify the sales function. It is given by the equation $T = c \times Q$.
2. We subtract the function of the required minimum profit from the sales function; the result is the cost function given by the equation $VN = (c - jZ) \times Q$.
3. We subtract fixed costs from the fixed cost function; it results in the function of the under-proportional costs showing the required development of variable costs.
4. This function is compared with the real function of variable costs (in our case the real function of proportional costs).
5. On the basis of this comparison, three stages can be seen:
   - The stage of unrealistic volume of production. The function of the required variable costs is situated below the *X*-axis, which is typical for this stage.
   - The stage of unprofitable production. The function of the required variable costs is a plus function, but is situated below the function of real proportional costs.
   - The stage of profitable production. The function of the required variable costs is situated above the function of the real proportional costs. The difference between these two functions shows the profit.

The equation which specifies the minimum volume of production is the same both in the case of the alternative way of solution and in the case of the original way. It can be derived in the following way:

$$(c - jZ) \times Q = SN + jN_{(p)} \times Q$$

$$Q_{\text{MIN}} = \frac{SN}{c - (jZ + jN_{(p)})}$$

The break-even point for non-linear costs and sales functions

We can often face real cases when the cost function and sales function are not linear. To ensure growing dis-

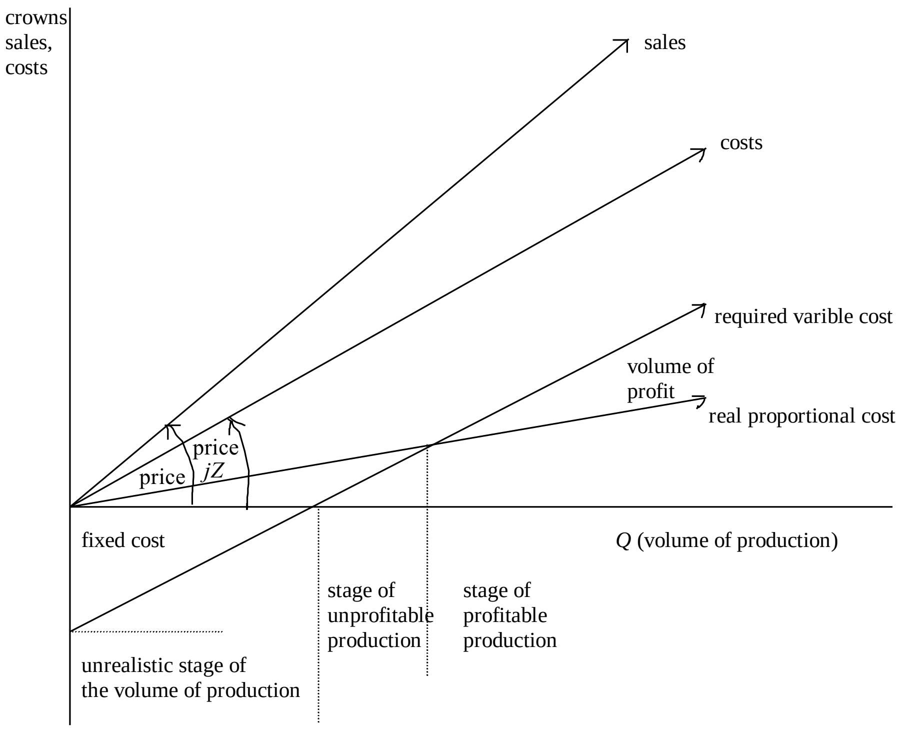


Figure 11. Graphical derivation of the alternative solution for the minimum profitability of production

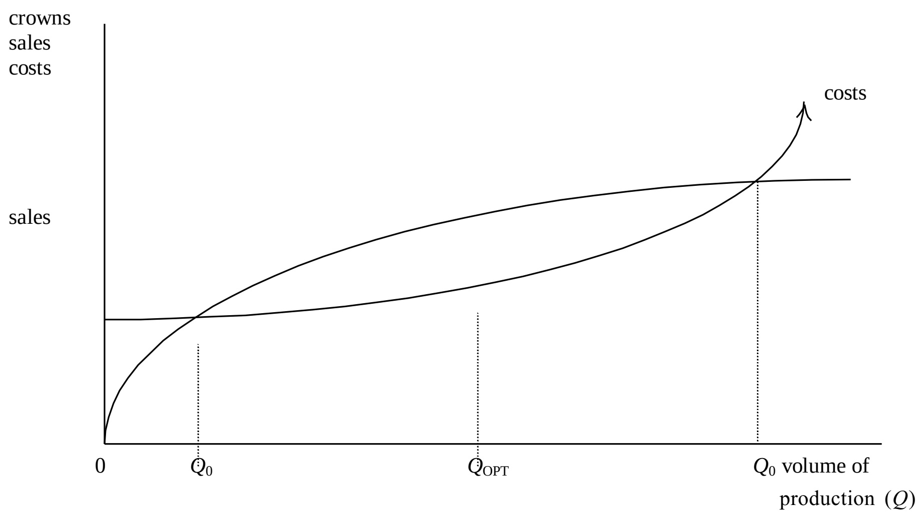


Figure 12. The break-even point for non-linear sales function and costs function

tribution, the supplier can provide bulk discounts that rise together with the volume of production.

In such a case, the sales function can be a degressively increasing one.

Such a cost function need not be a linear one, but the growing volume of production requires that costs grow progressively. One of these alternatives is shown in Figure 12.

Figure 12 shows that two or more break-even points can be found in non-linear costs and sales function. The maximum volume of profit need not be necessarily connected with the maximum of production. The optimal production, which is connected with the maximum profit, can be reached also at production which is lower than the maximum one.

The analysis of the break-even point follows the same rules as in the case of linear functions. However, the procedure may be more complicated.

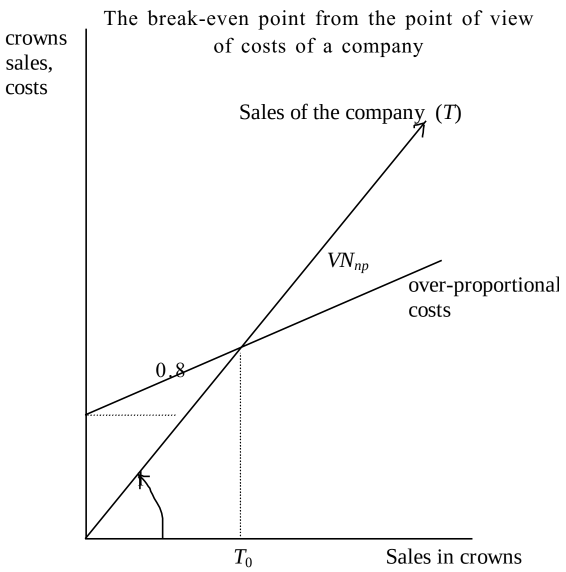
The break-even point in relation to cost revenue ratio

The break-even point can be analysed also at the aggregate level of the whole company. An example of the break-even point analysis for over-proportional costs will be shown.

The sales function and the X-axis contain the angle of 45°. The following equation can be used: $T = 1 \times T$. Therefore, from the point of view of the costs, the unit-sales in the diagram are given by a line parallel with the x-axis at the point 1 – for the break-even point.

Over-proportional cost-revenue ratio ($n_{(np)}$) is given by the following equations:

$$n_{(np)} = \frac{SN + VN(p)}{T} \qquad n_{(np)} = \frac{SN}{T} + \frac{VN(p)}{T}$$



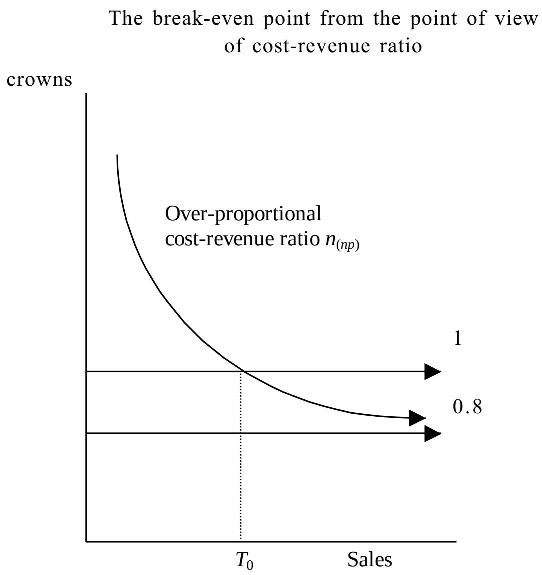


Figure 13. The break-even point for the aggregated data using the indicators of the cost-revenue ratio

The cost-revenue ratio of the over-proportional costs ($n_{(np)}$) creates the cost-revenue ratio of the fixed costs and cost-revenue ratio of the proportional costs (Figure 13).

The fixed costs cost-revenue ratio of ($n_{(SN)}$) is given by the following equations:

$$n_{(SN)} = \frac{SN}{T} \qquad 0 < n_{(SN)} < 1$$

$$\lim_{T \to \infty} n_{(SN)} = 0$$

The proportional costs cost-revenue ratio ($n_{(p)}$) is given by the following equation:

$$n_{(p)} = \frac{VN_{(p)}}{T} = \text{invariable}$$

Derived from this formula, the cost-revenue ratio of the over-proportional costs ($n_{(np)}$) is given by the following equation:

$$n_{(np)} = \frac{SN}{T} + n_{(p)}$$

It is given by a hyperbola with the limit:

$$\lim_{T \to \infty} n_{(np)} = n_{(p)}$$

It is obvious that under these presumptions the break-even point is defined by the equation:

$$n_{(np)} = 1$$

For the volume of sales where $n_{(np)} < 1$, the production is profitable and for the volume of sales where $n_{(np)} > 1$, the production is unprofitable.

The break-even point in relation to the profit-revenue ratio indicators for the minimum profitability of production

The minimum profitability of production can be shown by the rate of profitability (r), rate of profit (v) or by the cost-revenue ratio of production (n).

These indicators can be described by the following equations:

$$v = 1 - n \qquad v = \frac{Z}{T}$$

$$n = 1 - v \qquad n = \frac{VN}{T}$$

$$n = \frac{1}{r+1} \qquad r = \frac{Z}{VN}$$

$$v = 1 - \frac{1}{r+1}$$

Z = the volume of the profit of the company
VN = costs of the company
T = volume of the company revenues

At this break-even point, the sales function is modified into the costs function according to the equation $VN = (1 - v) \times T$.

It is obvious, that some other indicators defined above can be used instead of the rate of profit.

Then the break-even point can be calculated from the following equation:

$$T_{\text{MIN}} \times (1 - v) = VN$$

$$T_{MIN} = \frac{VN}{1 - v}$$

Provided that the costs are over-proportional, the following equation can be used:

$$VN = SN + n_{(p)} \times T$$

where $n_{(p)}$ = the cost-revenue ratio of the proportional cost

$$T_{MIN} = \frac{SN}{(1 - v) - n_{(p)}}$$

The company revenues for the expected minimum profit equal to the share of cost and the difference of production cost-revenue ratio, which ensures the minimum profit and proportional cost-revenue ratio.

Similarly, the break-even point could be derived for the minimum profitability of production and a non-linear cost function.

From this break-even point the prices, bulk discounts and other factors influence the indicators of cost-revenue ratio. Therefore, all these calculations must arise from the regressive evaluation of the indicators of the cost-ratio function in dependence on the volume of sales.

CONCLUSION

Classification of costs according to basic types of cost functions creates a model environment where the minimum volume of production, with respect to the profitability of production – to the break-even point, can be defined.

The break-even point can be defined, from the view of the costs indicators, at several levels. Each of these levels has its own interpretation and a different aggregation level. The above mentioned model situations represent a recommendable methodical tool how to find the break-even point in a particular situation of a company.

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