Chapter 2
Basic Structures: Sets, Functions, Sequences and Sum

2.2 Set Operations

1. Introduction

- Definition 1 (page 86)
  - Let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that either in A or in B, or in both.

    $A \cup B = \{ x \mid x \in A \vee x \in B \}$

    - $\{1,3,5\} \cup \{1,2,3\} = \{1,2,3,5\}$
- Definition 2 (intersection)
  - Let A and B be sets. The intersection of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

    $A \cap B = \{ x \mid x \in A \wedge x \in B \}$

    - $\{1,3,5\} \cap \{1,2,3\} = \{1,3\}$

1. Introduction

- Definition 3 (page 127)
  - Two sets are called disjoint if their intersection is the empty set.
  - Example 5 (page 127)
    - Let A={1,3,5,7,9} and B={2,4,6,8,10}. Since $A \cap B = \varnothing$, A and B are disjoint
- For two finite sets A and B, we have:

  $|A \cup B| = |A| + |B| - |A \cap B|$
  - Please explain it via Venn Diagram

1. Introduction

- Definition 4 (the difference of two sets, 差集)
  - Let A and B be sets. The difference of A and B, denoted by A-B, is the set that containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A (关于A的集合B的补集)

    $A-B=\{x \mid x \in A \land x \text{ notin } B\}$

  - Please explain it via Venn Diagram.
- Example 6 (page 88)
  - {1,3,5}-{1,2,3} = ?
  - {1,2,3}-{1,3,5} = ?

1. Introduction

- Definition 5 (complement of a set, 补集)
  - Let U be universal set (全集). The complement of the set A, denoted by ~A, is the complement of A with respect to U.

    $\sim A = U - A$

    $\sim A = \{ x \mid x \notin A \}$
  - Please explain it via Venn Diagram.

1. Introduction

- Example 8 (page 124)
  - A={a,e,i,o,u}
  - U: the set of letters of the English alphabet
  - ~A = ?
- Example 9 (page 124)
  - A: the set of positive integers greater than 10
  - U: all positive integers
  - ~A = ?

2. Set identities (集合的恒等式)

- Table 1 (page 124)
  1. Identity law (同一律)
  2. Domination Law (零律)
  3. Idempotent law (幂等律)
  4. Complementation law (双重否定律)
  5. Commutative law (交换律)
  6. Associative law (结合律)
  7. Distributive law (分配律)
  8. De Morgan’s law (德摩根律)
  9. Absorption laws (吸收律)
  10. $A \cup \sim A = U$ (排中律)
  11. $A \cap \sim A = \varnothing$ (矛盾律)

2. Set identities (集合的恒等式)

- Example 10 (page 125)
  - Prove that $\sim(A \cap B) = \sim A \cup \sim B$.
  - Solution:
    1. Left $\subseteq$ Right
    2. Right $\subseteq$ Left
- Use set builder notation and logical equivalence to show that

  $\sim(A \cap B) = \sim A \cup \sim B$

  - Proof: See page 125

2. Set identities (集合的恒等式)

- Example 12 (page 125)
  - Prove for all sets A, B, C that:

    $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

    - Proof: See book.
  - Use a membership table to show that

    $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

    - Proof:See book (page 126).

2. Set identities (集合的恒等式)

- Let A, B, and C be sets. Show that

  $\sim(A \cup (B \cap C)) = (\sim C \cup \sim B) \cap \sim A$

- Proof:
  - By using the set identities proved previously.
  - See book (page 126)

3. Generalized unions and intersections

- Introduction
  - The well-definedness of "$A \cup B \cup C$" and "$A \cap B \cap C$"
  - Why?
    - Reason:
      - the associative law of $\cup$ and $\cup$
      - $(A \cup B) \cup C = A \cup (B \cup C)$
      - $A \cap (B \cap C) = (A \cap B) \cap C$

3. Generalized unions and intersections

- Example 15
  - Let $A=\{0,2,4,6,8\}$, $B=\{0,1,2,3,4\}$, and $C=\{0,3,6,9\}$.
  - What are $A \cup B \cup C$ and $A \cap B \cap C$?
    - Solution:

1. $A \cup B \cup C = ?$
2. $A \cap B \cap C = ?$

3. Generalized unions and intersections

- Definition 6
  - The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection.
  - The notation:

$$A_1 \cup A_2 \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

3. Generalized unions and intersections

- Definition 7
  - The intersection of a collection of sets is the set that contains those elements that are members of all the sets in the collection.
  - The notation:

$$A_1 \cap A_2 \cap \ldots \cap A_n = \cap_{i=1}^{n} A_i$$

- Let $A_i=\{i, i+1, i+2, \ldots\ldots\}$.
  - What are $\cup_{i=1}^{n} A_i$ and $\cap_{i=1}^{n} A_i$?
    - Solution: See blackboard.

Homework

- Page 130~133
  - 12, 13 (read), 14, 15 (read), 16(e),17 (read), 18(a)(e), 20, 24, 30, 35(read), 36, 46