# RESEARCH ON THE THREE ANGULAR RESOLUTION OF TERRESTRIAL LASER SCANNING

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**ABSTRACT:**

Terrestrial laser scanning technology has been applied more and more widely in the field of Surveying and mapping. Although requirements of the accuracy for different laser scanner survey may differ considerably, spatial resolution is an important aspect, which can be divided into range and angular components. The latter is a focus of this paper and is governed primarily by scanning interval, laser beam width and angle quantisation. An ultimate goal of this research is to derive the relationship and simplified formula between scanning interval and the angular quantisation when the EIFOV(Effective Instantaneous Field of View) is equal to the scanning interval; the relationship and simplified formula of scanning interval and the angular quantisation when the EIFOV is equal to the laser beam width, and the relationship and simplified formula of the theoretical minimum EIFOV and the angular quantisation. Firstly, this paper introduces the EIFOV model and the AMTF(Average Modulation Transfer Function) model. Secondly, the dimensionless AMTF and EIFOV generic model are proposed. Thirdly, the above relathionships are deduced，which are ellipse or hyperbola, and the three simplified formulas are proposed. The simplified formulas have direct significance on the angular resolution's calculation and the scanning interval setting.

## 1. INTRODUCTION

The emergence of the terrestrial laser scanning technology has broken the traditional mode of surveying data acquisition and processing, and has promoted the development of the objective surface characteristics'recovery techinque which is based on the measurement model of point cloud(Reshetyuk, 2009; Zhang Yi, 2008). Recovery degree of the objective surface minuitiae feature is described generally by the spatial resolution. In terms of terrestrial laser scanning technology, the spatial resolution designates the range and angular resolution of point cloud. The latter is the main factor to determine the objective details' recognition capability of point cloud (Lichti, 2006; Zhu Ling, 2008), which is governed primarily by scanning interval, laser beam width and angle quantisation. At present, Professor Lichi's *EIFOV*(Effective Instantaneous Field of View)model, which was deduced from *AMTF*(Average Modulation Transfer Function) model, is the only one involving above three aspects. In practical, the angle quantisation can be changed only by selecting different scanner.. Scanning interval and laser beamwidth are usually required to determine in advance through the formula of beam width, the relationship of the *EIFOV* and the scanning interval, and the *EIFOV* of the point cloud can be obtained. But no manufacturer of scanner provide the formula of the beam width and the range, meanwhile the relationship model among the *EIFOV*, scanning interval, and the angular quantisation is very complicated, so that we need to develop a simple method to calcuate the magnitude of scanning interval on the angular quantisation knowned. Furthermore, the magnitude of theoretical minimum angular resolution can be used to evaluate the instrument performance. However the theoretical minimum angular resolution is unavailable.

In order to resolve the above problems,Related research would be focused on the formulas of differenct scanner in detail:
1) The relationship and simplified formula of scanning interval and the angular quantisation when the *EIFOV* is equal to the scanning interval;
2) The relationship and simplified formula of scanning interval and the angular quantisation when the *EIFOV* is equal to the laser beam width;
3) The relationship and simplified formula of the theoretical minimum *EIFOV* and the angular quantisation.

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This paper is a further research with some addition of more introduction of theories, on the base of my Ph.D dissertation *Research on Point Cloud Angular Resolution And Processing Model of Terrestrial Laser Scanning* and an early paper in Chinese *Research on the point cloud angular resolution of terrestrial laser scanners*, which was accepted by Geomatics and Information Science of Wuhan University and is in publication plan.

The following organization of this paper is listed here：
In section 2, the basic theory is introduced including the *AMTF* and the *EIFOV* model, the three kind formulas to calculate laser beamwidth of various scanners, as well as the dimensionless *AMTF* and *EIFOV* general model.
In section 3, based on the above theory, the three angular resolution of terretrial laser scanner is researched, and it is concluded that the relationships and simplified formulas of scanning interval and the angular quantisation in two different *EIFOV* value, as well as the theoretical minimum *EIFOV* and the angular quantisation.
In section 4, the laser beamwidth and angular resolution of 29 kinds of commerical TLS systems is analysed based on the above theory.
Finally, a conclusion is drew in section 5.

## 2. THE METHOD OF CALCULATING BEAMWIDTH AND THE MODEL OF AMTF AND EIFOV

### 2.1 *AMTF* Model

*AMTF* model is computed by Fourier transfer—*APSF*(Average Point Spread Function), including the sampling $AMTF_s$, the beam width $AMTF_b$ and quantisation $AMTF_q$. The combined model is(Lichti, 2006; Yang Ronghua, 2011)

$$AMTF_{sbq}(u) = \left| \frac{\sin(\pi \Delta u)}{\pi \Delta u} \frac{2J_1(\pi w u)}{\pi w u} \frac{\sin(\pi \tau u)}{\pi \tau u} \right| \tag{1}$$

where $\Delta$ = scanning interval, which unit is millimeter
$w$ = diameter of beamwidth in the distance of $S$, which unit is millimeter
$\tau$ = angular quantisation, which unit is millimeter
$u$ = frequency, which unit is 1/mm.

### 2.2 *EIFOV* Model

*EIFOV* model is favoured for the analysis of electric-optical system resolution. The appropriate expression of the *EIFOV* extends to laser scanners as it quantifies the combined effects of sampling, beam width and quantisation. *EIFOV* model is computed via the cut-off frequency. It is(Lichti, 2006; Yang Ronghua, 2011)

$$EIFOV = \frac{1}{2u_c} \tag{2}$$

where $u_c$ = the cut-off frequency which satisfy the equation

$$AMTF_{sbq}(u_c) = \frac{2}{\pi}$$

### 2.3 Three Method of Calculating Beamwidth

The point cloud angular resolution is related with the laser beamwidth that is affected by several factors such as the scaning distance, the divergence characterization of laser beam, the diameter of the transmitting aperture and the inclination angles of the objective surface, etc(Zhang Yi, 2008;Lai Zhikai, 2004). However, no scanner manufacturer currently provides the formula used to calculate the laser beam width. Most manufacturers keep the value of the most laser characteristics parameters still as secret. So it is difficult to know how big is the beamwidth in any distance. In here three methods are given to calculate different scanner's beam width:

Firstly, the diameter of the transmitting aperture and three more diameters in different distances are given. The formula is(Reshetyuk, 2009; Zhang Yi, 2008)

$$w = \sqrt{w_0^2 + c^2 (S - R_0)^2} \tag{3}$$

where $R_0$ = the range between the beam waist and the transmitting aperture, which unit is meter
$w_0$ = diameter of beamwidth in the distance of $R_0$, which unit is millimeter
$c$ = constant variable, which unit is mm/m

Secondly, the diameter of the transmitting aperture and the beam divergence angle is given, or two diameters in different distances are given. The formula is(Reshetyuk, 2009; Zhang Yi, 2008)

$$w = 2S \cdot \tan(\frac{\gamma}{2} \cdot 10^{-6}) \cdot 10^3 + D_0 \tag{4}$$

where $\gamma$ = beam divergence angle, which unit is urad
$D_0$ = diameter of the transmitting aperture, which unit is millimeter

Thirdly, the diameter of the transmitting aperture and the diameter of beamwidth in a certain distance are given. The formula is(Reshetyuk, 2009; Zhang Yi, 2008)

$$w = \begin{cases} \sqrt{w_0^2 + c^2 \cdot (S - R_0)^2} & \text{when } S \le 2R_0 \\ 2 \cdot 10^3 (S - 2R_0) \tan(\frac{\gamma}{2} \cdot 10^{-6}) + D_0 & \text{when } S > 2R_0 \end{cases} \tag{5}$$

### 2.4 The Dimensionless *AMTF* And *EIFOV* Generic Model

To make the model more practical and more simple, the *AMTF* model eq.1 and the *EIFOV* model eq.2 can be transformed to dimensionless form by using variable substitution, which satisfies the equation $\Delta = kw$, $\tau = mw$, $u = \frac{U}{w}$, $EIFOV = Nw$.
The dimensionless *AMTF* and *EIFOV* generic model is(Yang Ronghua, 2011)

$$AMTF(U) = \left| \frac{\sin(\pi k U)}{\pi k U} \frac{2J_1(\pi U)}{\pi U} \frac{\sin(\pi m U)}{\pi m U} \right| \tag{6}$$

$$N = \frac{1}{2U_c} \tag{7}$$

where $k$ = the dimensionless scanning interval, which is the ratio of the scanning interval and the beamwidth
$m$ = the dimensionless angular quantisation, which is the ratio of the angular quantisation and the beamwidth
$U$ = the dimensionless frequency, which is the product of frequency and beamwidth
$U_c$ = the dimensionless cut-off frequency, which is the product of cut-off frequency and beamwidth
$N$ = the dimensionless *EIFOV*, which is the ratio of the *EIFOV* and the beamwidth

From the eq.6 and eq.7, the relationship graph of the dimensionless *EIFOV* $N$ and the dimensionless scanning interval $k$ (Fig.1) can be derived, which indicates the six relationship curve grahps of $N$ and $k$ under different dimensionless angular quantisations(assuming $m_1 = 0$, $m_2 = 0.5$, $m_3 = 1$, $m_4 = 1.5$, $m_5 = 2$, $m_6 = 2.5$). From Fig.1, we can see that $N$ is minimum on the condition of $k = 0$, which is described as theoretical minimum *EIFOV*. Furthermore, we can also see that the function of $N = f(k)$ is montone increasing function, which asymptotic line is the line of $N = k$.

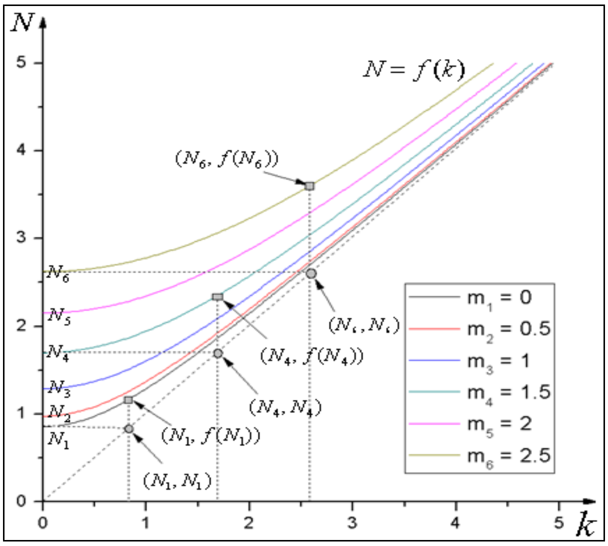


Figure 1. The rationship curve graph of $N$ and $k$

## 3. THREE KINDS RELATIONSHIP AND SIMPLIFIED FORMULA

In practice, the scanned point cloud is supposed to have the specific angular resolution which equals scanning interval or laser beamwidth. In addition, it is hoped to evaluate the scanner performance by its theoretical minimum EIFOV which is governed only by angle quantisation. From Fig.1, the condition of the $N = k$ is $k \gg 1$ (or $k \to +\infty$) can be shown, and the dimensionless *EIFOV* $N$ is equal to the theoretical minimum value when $k = 0$. Although Lichti(2006) have given that the condition of the $N = 1$ is $k = 0.545$, and $N_{\min} = 0.8594$ on the condition of ignoring angular quantisation, which can be effected when the size of angular quantisation is appropriate as scanning interval. However, more biases could arise in computing results and dimensionless variable $k$ could not be infinite in actual scanning parameter setting, the minimum $k$ value should be deduced( $k \gg 1$ ). Here, we assume that $k_1$ and $k_2$ are the dimensionless scanning interval variables, the dimensionless *EIFOV* $N$ is equal to $N_1$ when $k = k_1$, the dimensionless *EIFOV* $N$ is equal to 1 when $k = k_2$, the theoretical minimum dimensionless *EIFOV* is denoted by $N_{\min}$, and $N_1$ satisfy the equation $\frac{N_1 - k_1}{k_1} < 0.005$.

Assuming the regulations of relative approximate error is 0.005, which is that the condition of $N = k$ is $\frac{N-k}{k} < 0.005$. As the relationship of $N$ and $m$ is montone increasing function, which asymptote is $N = k$ and satisfy that $N > k$ and $k \in [0, +\infty)$ (Lichti, 2006). So the function of $y = \frac{N-k}{k}$ is also montone increasing function, and $\frac{N-k}{k} < 0.005$ when $k > k_1$, which is equivalent to $N = k$ when $k > k_1$. So we can obtain point cloud which angular resolution is closed to scanning interval by setting the scanning interval parameter more than $k_1$, and can obtain point cloud which angular resolution is equal to laser beamwidth by setting the scanning interval parameter at $k_2$. Furthermore, $N_{\min}$ can be used to estimate relationships of minimum theoretical angular resolution from different scanners，and use the scanner having smaller $N_{\min}$ to accomplish the task of obtaining higher angular resolution point cloud.

Analysed from above，it have great significance in deriving the relationship of $k_1$ and $m$, the relationship of $k_2$ and $m$, and the relationship of $N_{\min}$ and $m$. Moreover, we need to derive the simplified formula of calculating $k_1$, $k_2$ and $N_{\min}$ under knowing the value of the angular quantisation. The following is the three kinds of relationships and simplified formulas.

### 3.1 The Relationship & Simplified Formula of $k_1$ And $m$

Assuming $N_1 = \frac{1}{2U_c} = ak_1$, shown as Fig. 1, $a \geq 1$. With the equation (6) and (7), we can gain the relationship of $a$, $k_1$ and $m$ is

$$\left| \frac{\sin(\frac{\pi}{2a})}{\frac{\pi}{2a}} \frac{2J_1(\frac{\pi}{2ak_1})}{\frac{\pi}{2ak_1}} \frac{\sin(\frac{\pi m}{2ak_1})}{\frac{\pi m}{2ak_1}} \right| = \frac{2}{\pi} \tag{8}$$

As $\left|\frac{\sin(x)}{x}\right| \leq 1$, $\left|\frac{2J_1(x)}{x}\right| \leq 1$, with the equation (8), we can gain

$$\left| \frac{\sin(\frac{\pi}{2a})}{\frac{\pi}{2a}} \right| \geq \frac{2}{\pi}, \left| \frac{2J_1(\frac{\pi}{2ak_1})}{\frac{\pi}{2ak_1}} \right| \geq \frac{2}{\pi}, \left| \frac{\sin(\frac{\pi m}{2ak_1})}{\frac{\pi m}{2ak_1}} \right| \geq \frac{2}{\pi} \tag{9}$$

Note: $A_1 = \left| \frac{\sin(\frac{\pi}{2})}{\frac{\pi}{2}} \frac{2J_1(\frac{\pi}{2k_1})}{\frac{\pi}{2k_1}} \frac{\sin(\frac{\pi m}{2k_1})}{\frac{\pi m}{2k_1}} \right|$. With the equation (9), we can gain

$$\frac{2}{\pi} < \left| \frac{\sin(\frac{\pi}{2})}{\frac{\pi}{2a}} \frac{2J_1(\frac{\pi}{2k_1})}{\frac{\pi}{2ak_1}} \frac{\sin(\frac{\pi m}{2k_1})}{\frac{\pi m}{2ak_1}} \right| = a^3 A_1 \tag{10}$$

With the equation (10), we can gain

$$A_1 > \frac{1}{a^3} \cdot \frac{2}{\pi} \tag{11}$$

As $\frac{|N_1 - k_1|}{k_1} = a - 1 < 0.005$ , we can gain

$$\frac{1}{a^3} > 0.9851 \tag{12}$$

With the equation (11) and (12), we can gain

$$A_1 > \frac{1.9702}{\pi} \tag{13}$$

With keeping two digit of decimals, indicated from the monotonicity of $N$ and $m$ ,derived from above deduction, the condition which makes $\frac{N_1 - k_1}{k_1} < 0.005$ tenable is $A_1 > \frac{1.98}{\pi}$ , i.e.

$$\frac{\sin(\frac{\pi}{2})}{\frac{\pi}{2}} \frac{2J_1(\frac{\pi}{2k_1})}{\frac{\pi}{2k_1}} \frac{\sin(\frac{\pi m}{2k_1})}{\frac{\pi m}{2k_1}} > \frac{1.98}{\pi} \tag{14}$$

The above equation is very complicated. we need to get its simplified form for convenient calculation. Here, the least squares curve fitting method of 1000 uniform sampling points $(m,k_1)$ obtained by the equation (14) is used to derive the simplified formulas of $k_1$ and $m$ **.** As up to now, the highest precision in point cloud data processing of terrestrial laser scanning is 0.01 folds laser beamwidth(Zhang Yi, 2008), and the maximum angular quantisation of different scanner is 2.08 folds laser beamwidth(GIM, 2010). Therefore, we define that $m \in [0, 2.5]$ and fitting precision is 0.005. Then, we can get the relationship graph of $k_1$ and $m$ (Fig.2) and the fitting formulas of $k_1$ is

$$k_1 = \sqrt{a_1 + b_1 \cdot (m - g_1)^2} + h_1 \tag{15}$$

where $a_1$ = 30.8136
$b_1$ = 41.03034
$g_1$ = 0.0008
$h_1$ = 0.006

From the equation (15), we can see that the relationship graph of $k_1$ and $m$ is hyperbola, and the fitting errors of the equation (15) and $m$ is Fig.3. From the plot of Fig.2, we can see that fitting errors is less than 0.005 when $m > 0.01$ . so we can think that the equation (14) is approximately equaivalent with the equation (15).

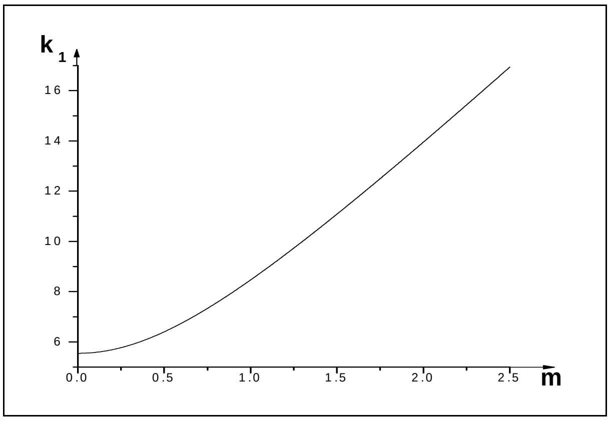


Figure 2. The rationship curve graph of $k_1$ and $m$

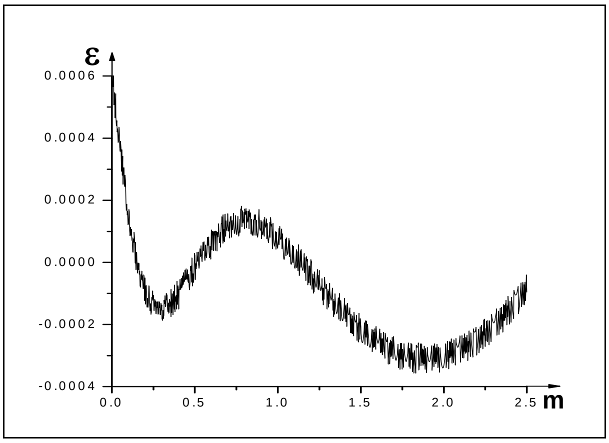


Figure 3. The rationship curve graph of fitting errors $\varepsilon$ (equation (15)) and $m$

### 3.2 The Relationship & Simplified Formula of $k_2$ And $m$

With the equation (6), (7) and $N = 1$ , we can gain the relationship of $k_2$ and $m$ that is

$$\left| \frac{\sin(\frac{\pi k_2}{2})}{\frac{\pi k_2}{2}} \frac{\sin(\frac{\pi m}{2})}{\frac{\pi m}{2}} \right| = \frac{1}{2J_1(\frac{\pi}{2})} \tag{16}$$

where $0 \le k_2 \le 0.545$
$0 \le m \le 0.545$

The above equation is still very complicated. Its simplified form can be derived through the same method as above. we can get the relationship graph of $k_2$ and $m$ **(** Fig.4**)** and the fitting formulas of $k_2$ and $m$ that is

$$k_2 = \sqrt{a_2 - b_2 \cdot (m + g_2)^2} + j_2 \cdot m - h_2 \tag{17}$$

where $a_2$ = 0.35426
$b_2$ = 0.99264
$g_2$ = 0.0521
$j_2$ = 0.085793
$h_2$ = 0.047672

From the equation (17), the relationship graph of $k_1$ and $m$ is ellipse, and the fitting errors of the equation (17) and $m$ is Fig.5. From the plot of Fig.5, we can see that fitting errors is less than 0.0003. so we can think that the equation (16) is approximately equaivalent with the equation (17).

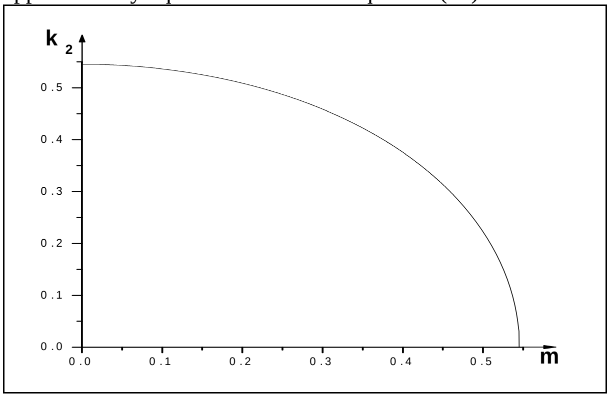


Figure 4. The rationship curve graph of $k_2$ and $m$

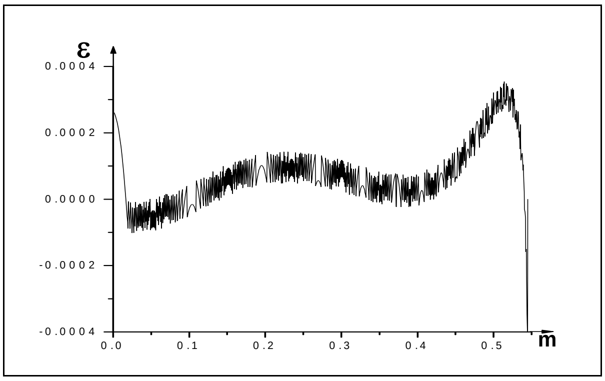


Figure 5. The rationship curve graph of fitting errors $\varepsilon$ (equation (17)) and $m$

### 3.3 The Relationship & Simplified Formula of $N_{\min}$ And $m$

With the equation (6), (7) and $k=0$, we can gain the relationship of $N_{\min}$ and $m$ that is

$$\left| \frac{2J_1(\frac{\pi}{2N_{\min}})}{\frac{\pi}{2N_{\min}}} \frac{\sin(\frac{\pi m}{2N_{\min}})}{\frac{\pi m}{2N_{\min}}} \right| = \frac{2}{\pi} \tag{18}$$

The equation (18) is needed to simplify using the same method as above. we can also get the relationship graph of $N_{\min}$ and $m$ **(** Fig.6**)** and the fitting formulas of $N_{\min}$ and $m$ that is

$$N_{\min} = \sqrt{a_3 + b_3 \cdot (m - g_3)^2} - h_3 \tag{19}$$

where $a_3$ = 0.82102

$b_3$ = 1.03814

$g_3$ = 0.0371

$h_3$ = 0.0437

From the equation (19), the relationship graph of $k_1$ and $m$ is hyperbola, and the fitting errors of the equation (19) and $m$ is Fig.7. From the plot of Fig.7, we can see that fitting errors is less than 0.004. So the equation (18) is approximately equaivalent with the equation (19).

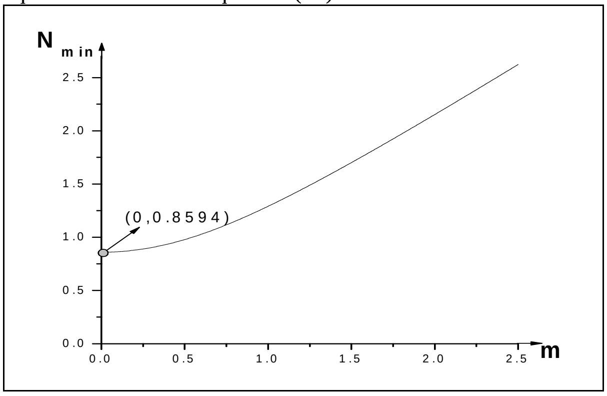


Figure 6. The rationship curve graph of $N_{\min}$ and $m$

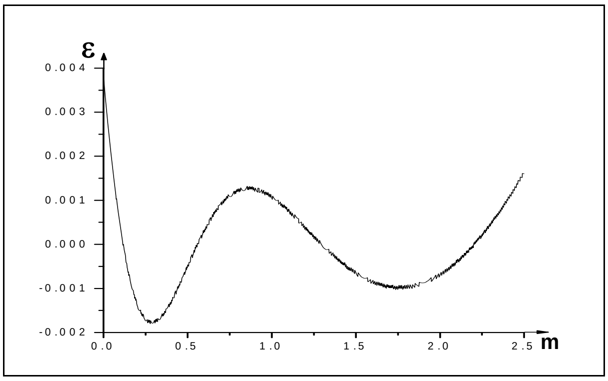


Figure 7. The rationship curve graph of fitting errors $\varepsilon$ (equation (19)) and $m$

## 4. BEAMWIDTH AND RESOLUTION OF TLS SYSTEM

The laser beam width and angular resolution of 29 commerically available TLS systems(GIM, 2010) is analysed using the three methods of calculating beamwidth diameter and the equation (15), (17), (19). To facilitate the comparision, each vendor's reported finest angular sampling interval and beamwith and calculated *EIFOV* have been reduced to linear spatial units at a range of 50m. The results about the coefficients of different scanner's beamwidth formula are given in Table 1, and the results about $k_1$, $k_2$ and $N_{\min}$ of different scanner are given in Table 2.

From Table 1, we can see that the three methods of equation (3), (4) and (5) can solve the problem of calculating all TLS systems' laser beamwidth diameter.

Shown as Tab. 2：

1) The 29 systems can be classified into three groups according to $EIFOV_{\min}$ (the theoretical minimum *EIFOV*): fine resolution scanners; medium resolution instruments; and coarse resolution instruments.
2) For Trimble GS、Basis Software Surphaser 25HS、Z+F PROFILER 5006h 和 Z+F Imager 5003: The theoretical minimum dimensionless *EIFOV* N(>1); The dimensionless angular quantisation(≤0.545); The dimensionless scanning interval $k_2$ (N/A);

3) For other scanners: The theoretical minimum dimensionless *EIFOV* N(<1); The dimensionless angular quantisation(≤0.545);
4) For Riegl LMS-Z390, Riegl VZ-1000, Riegl VZ-400, Callidus CPW8000, Callidus CP320: $N_{\min}$ and $k_1$ achieve minimum values, $k_1 = 5.56$, $N_{\min} = 0.86$;
5) For Basis Software Surphase： $N_{\min}$ and $k_1$ achieve maximum values, $k_1 = 14.43$, $N_{\min} = 2.23$;
6) For Faro LS880: $k_2$ achieve minimum values, $k_2 = 0.2$;
7) The size of spot diameter affects a lot on minimum angular resolution, which arises the most in low-precision scanners. Furthermore, angular resolution of point cloud is as an integrated result of scanning interval, angular accuracy and spot diameter. Meanwhile, the method of spot-overlay can improver angular resolution of point cloud, with a maximum value of 0.86 times of spot diameter.

Table 1. The coefficient of the formulas of different scanner’s beamwidth diameter

| Method | TLS System | $D_0$ | $\gamma$ | $R_0$ | $w_0$ | $c$ |
|---|---|---|---|---|---|---|
| Firtst | Basis Software Surphaser 25HS | 2.8 | | 4.52 | 2.32 | 0.3481 |
| | BasisSoftware Surphaser 25HSX | 2.8 | | 4.52 | 2.32 | 0.3481 |
| | Leica ScanStation 2 | 6 | | 25 | 4.00 | 0.1789 |
| | Leica ScanStation C10 | 6 | | 25 | 4.00 | 0.1789 |
| | Leica HDS3000 | 6 | | 25 | 4.00 | 0.1789 |
| Third | I-Site 4400-LR | 15 | 1400 | 5.36 | 7.50 | 2.4249 |
| | I-Site 4400-CR | 15 | 1400 | 5.36 | 7.50 | 2.4249 |
| | I-Site 8800 | 8 | 250 | 17 | 4.30 | 0.3968 |
| | OptechILRIS-3DER | 14 | 170 | 50 | 8.00 | 0.2298 |
| | Leica HDS4400 | 20 | 1400 | 7.14 | 10.00 | 2.4248 |
| | Riegl LPM-321 | 60 | 800 | 50 | 40.00 | 0.8944 |
| | Riegl LMS-Z420i | 8 | 300 | 11.67 | 3.50 | 0.6166 |
| | Riegl LMS-Z620 | 14 | 150 | 18.33 | 2.80 | 0.7482 |
| | Riegl VZ-400 | 7 | 300 | 6.67 | 2.00 | 1.0062 |
| | Riegl VZ-1000 | 7 | 300 | 6.67 | 2.00 | 1.0062 |
| Second | Callidus CPW 8000 | 3 | 200 | | | |
| | Z+F Imager 5003 | 3.3 | 214 | | | |
| | Z+F Imager 5006 | 3 | 220 | | | |
| | Faro LS 420 | 3 | 250 | | | |
| | Faro LS 840 | 3 | 250 | | | |
| | Faro LS 880 | 3 | 250 | | | |
| | Faro Photo 120 | 3.3 | 320 | | | |
| | Faro Photo 20 | 3.3 | 320 | | | |
| | Optech ILRIS-HD | 9.2 | 150 | | | |

Table 2. The value of the scanning interval $k_1$ and $k_2$ as well as the dimensionless theoretical minimum angular resolution

| Class | TLS System | $m$ | $k_1$ | $k_2$ | $N_{\min}$ | $EIFOV_{\min}$ |
|---|---|---|---|---|---|---|
| Fine | Trimble GX | 0.50 | 6.41 | 0.22 | 0.98 | 2.93 |
| | Trimble GS | 1.14 | 9.17 | / | 1.4 | 4.20 |
| | Leica ScanStation 2 | 0.50 | 6.41 | 0.22 | 0.98 | 5.87 |
| | Leica HDS3000 | 0.50 | 6.41 | 0.22 | 0.98 | 5.87 |
| | Leica HDS4500 | 0.30 | 5.88 | 0.46 | 0.9 | 5.41 |
| | Basis Software Surphaser25HS | 2.08 | 14.5 | / | 2.23 | 9.35 |
| Medium | Trimble CX | 0.27 | 5.82 | 0.48 | 0.89 | 11.61 |
| | Z+F Imager 5006 | 0.44 | 6.23 | 0.33 | 0.95 | 13.31 |
| | Leica HDS6100 | 0.45 | 6.26 | 0.31 | 0.96 | 13.37 |
| | Riegl LMS-Z420i | 0.14 | 5.63 | 0.53 | 0.87 | 13.90 |
| | Leica HDS6000 | 0.45 | 6.26 | 0.31 | 0.96 | 13.37 |
| | BasisSoftwareSurphaser25HSX | 0.17 | 5.66 | 0.52 | 0.87 | 13.96 |
| | Riegl LMS-Z390 | 0.03 | 5.56 | 0.54 | 0.86 | 15.52 |
| | Riegl VZ-1000 | 0.03 | 5.56 | 0.54 | 0.86 | 15.52 |
| | Riegl VZ-400 | 0.03 | 5.56 | 0.54 | 0.86 | 15.52 |
| | Faro LS 880 | 0.51 | 6.44 | 0.20 | 0.98 | 15.23 |
| | Z+F PROFILER 5006h | 0.63 | 6.87 | / | 1.05 | 14.63 |
| | Trimble FX | 0.39 | 6.09 | 0.39 | 0.93 | 16.85 |
| | Optech ILRIS-HD | 0.13 | 5.62 | 0.53 | 0.87 | 18.65 |
| | Z+F Imager 5003 | 0.63 | 6.87 | / | 1.05 | 14.63 |
| | Faro Photo 120 | 0.41 | 6.14 | 0.36 | 0.94 | 18.12 |
| | OptechILRIS-3DER | 0.18 | 5.67 | 0.52 | 0.87 | 19.67 |
| Coarse | Riegl LPM-321 | 0.20 | 5.70 | 0.51 | 0.88 | 35.10 |
| | 3rdTech DeltaShpere-3000 | 0.34 | 5.97 | 0.43 | 0.91 | 35.72 |
| | 3rdTech DeltaShpere-3000IR | 0.34 | 5.97 | 0.43 | 0.91 | 35.72 |
| | Callidus CPW 8000 | 0.02 | 5.56 | 0.54 | 0.86 | 88.84 |
| | I-Site 4400-LR | 0.25 | 5.78 | 0.49 | 0.89 | 62.16 |
| | Leica HDS4400 | 0.50 | 6.41 | 0.22 | 0.98 | 68.45 |
| | Callidus CP 3200 | 0.02 | 5.56 | 0.54 | 0.86 | 200.12 |

## 5. CONCLUSIONS

Spatial resolution governs the level of identifiable detail within a scanned point cloud and is particularly important for recording of objective features with fine details(Lichti,2006). The angular resolution of laser scanners is affected by sampling interval, laser beamwidth and angular quantisation. *EIFOV* is regarded as a more appropriate measure of the angualr resolution. To quickly obtain scanning interval corresponding with the known angular resolution, here we present the dimensionless *AMTF* and *EIFOV* generic model, the three kind methods of calculating beamwidth diameter, and the three kind functional relationship that is the relationship of $k_1$ and $m$ where $N = k$, the relationship of $k_2$ and $m$ where $N = 1$, and the relationship of $N_{\min}$ and $m$ where $k = 0$. In addition, we derive the above relationsips’ simplified formula, give the definition of the optimal sampling interval, and analyse 29 available TLS systems’ laser beamwidth diameter and variables $k_1$, $k_2$ and $N_{\min}$. The results shows that the simplified formulas have direct significance on the angular resolution’s calculation and the scanning interval setting.

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