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DESIGNING OPTIMAL TAXES WITH A MICROECONOMETRIC MODEL OF HOUSEHOLD LABOUR SUPPLY

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Designing Optimal Taxes with a Microeconometric Model of Household Labour Supply

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Abstract

The purpose of this paper is to present an exercise where we identify optimal income tax rules according to various social welfare criteria, keeping fixed the total net tax revenue. Empirical applications of optimal taxation theory have typically adopted analytical expressions for the optimal taxes and then imputed numerical values to their parameters by using "calibration" procedures or previous econometric estimates. Besides the restrictiveness of the assumptions needed to obtain analytical solutions to the optimal taxation problem, a shortcoming of that procedure is the possible inconsistency between the theoretical assumptions and the assumptions implicit in the empirical evidence. In this paper we follow a different procedure, based on a computational approach to the optimal taxation problem. To this end, we estimate a microeconomic model with 78 parameters that capture heterogeneity in consumption-leisure preferences for singles and couples as well as in job opportunities across individuals based on detailed Norwegian household data for 1994. For any given tax rule, the estimated model can be used to simulate the labour supply choices made by single individuals and couples. Those choices are therefore generated by preferences and opportunities that vary across the decision units. We then identify optimal tax rules – within a class of 9-parameter piece-wise linear rules - by iteratively running the model until a given social welfare function attains its maximum under the constraint of keeping constant the total net tax revenue. The parameters to be determined are an exemption level, four marginal tax rates, three "kink points" and a lump sum transfer that can be positive (benefit) or negative (tax). We explore a variety of social welfare functions with differing degree of inequality aversion. All the social welfare functions imply monotonically increasing marginal tax rates. When compared with the current (1994) tax systems, the optimal rules imply a lower average tax rate. Moreover, all the optimal rules imply – with respect to the current rule – lower marginal rates on low and/or average income levels and higher marginal rates on relatively high income levels. These results are partially at odds with the tax reforms that took place in many countries during the last decades. While those reforms embodied the idea of lowering average tax rates, the way to implement it has typically consisted in reducing the top marginal rates. Our results instead suggest to lower average tax rates by reducing marginal rates on low and average income levels and increasing marginal rates on very high income levels.

Keywords: Labour supply, optimal taxation, random utility model, microsimulation.

JEL classification: H21, H31, J22.

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1. Introduction

This paper presents an empirical analysis of optimal taxation. The purpose is not new, but the exercise illustrated here differs in many important ways from previous attempts to empirically compute optimal taxes. The standard procedure adopted in the literature starts with some version of the optimal taxation framework originally set up in the seminal paper by Mirrlees (1971). The next step typically consists of feeding with numbers – taken from some previous empirical analysis - the formulas produced by the theory. This literature is surveyed by Tuomala (1990). A recent strand of research adopts the same approach to address the inverse optimal taxation problem, i.e. retrieving the social welfare function that makes optimal a given tax rule (Bourguignon and Spadaro, 2005). There are two main problems with this literature: 1) The theoretical results become amenable to an operational interpretation only by adopting some special assumptions concerning the preferences, the composition of the population and the structure of the tax rule; 2) The empirical measures used as counterparts of the theoretical concepts are usually derived from previous estimates obtained under assumptions that may be different from those used in the theoretical model. As a consequence the consistency between the theoretical model and the empirical measures is dubious and the significance of the numerical results remains uncertain. The typical outcome of these exercises envisages a lump-sum transfer which is progressively taxed away by very high marginal and decreasing tax rates on lower incomes (i.e. a negative income tax mechanism); beyond the "beak-even point" (i.e. the income level where the transfer is completely exhausted), the marginal tax rates become constant or slightly increasing. Recent papers by Tuomala (2006, 2008) show however that these results are essentially forced by the restrictive assumption typically made upon preferences, elasticities and distribution of productivities (or wage rates). Interestingly, when Tuomala (2008) adopts a more flexible specification of the utility function he find that the optimal system is progressive with monotonically increasing marginal tax rates.

While most of the studies mentioned above were essentially illustrative numerical exercises, several recent contributions have attempted to use optimal taxation results in the empirical evaluation or design of tax-transfer reforms. Saez (2001) makes Mirrlees's results more easily interpretable by reformulating them in terms of labour (or income) supply elasticities in order to provide a more direct link between theoretical results and empirical measures. Saez (2002) develops a model amenable to empirical implementation that focus on the relative magnitude of the labour supply elasticities at the extensive and intensive margin. Immervoll et al. (2007) adopt Saez's model (2002) to evaluate alternative income support policies in European countries. Blundell et al. (2006) and Haan and Wrohlich (2007) also use Saez (2002) to evaluate taxes and transfers for lone mothers in Germany and UK, whereas Kleven et al.(2007) provide results on the taxation of couples. Although these new contributions are interesting attempts to advance towards the empirical implementation of theoretical optimal taxation results, they still rely on very restrictive assumptions and moreover might suffer from a possible inconsistency between the theoretical model and the empirical measures used to implement it. To escape these problems we follow here a completely different approach. We do not start from theoretical results dictating conditions for optimal tax rules under various assumptions. Instead we use a microeconometric model of labour supply in order to identify by simulation the tax rule that maximizes a social welfare function under the constraints that the households maximize their own utility and total net tax revenue remains constant. The complex specification we adopt for representing preferences and opportunity sets does not permit an analytical solution of the maximization problems, which are therefore solved computationally. The microeconometric simulation approach is common in evaluating tax reforms, but has not been much used in empirical optimal taxation studies.[1] The closest previous example adopting a similar approach is probably represented by Fortin, Truchon and Beauséjour (1993), who however use a calibrated (not estimated) model with rather restrictive (Stone-Geary) preferences and focus on alternative income support schemes rather than on the whole tax rule. By contrast, we develop a microeconometric model of labour supply that allows for a rather flexible representation of preferences, embodies an exact representation of taxes and transfers, and represents simultaneous decisions of household members and accounts for quantity constraints on labour supply choices. As explained in Appendix A, the empirical labour supply model contains 78 parameters that capture the heterogeneity in preferences and opportunities among households and individuals. The estimated model is used to simulate the choices given a particular tax rule. Those choices are therefore generated by preferences and opportunities that vary across the decision units. However, since preferences are heterogeneous and some individuals live as singles whereas others form families and live together, when it comes to social evaluation it does not make sense to treat the estimated utility functions as comparable individual welfare functions. To solve the interpersonal comparability problem we adopt a method that consists of using a common utility function in order to produce interpersonally comparable individual welfare measures. The common utility function is justified as a normative standard where the social planner treats individuals symmetrically and it is only used to compute and compare the individual welfare levels that provide the basis for the social welfare evaluation of tax reforms; it is not used for simulating household behaviour (where instead the estimated individual utility functions are used). This procedure, which circumvents the problem of interpersonal comparability of heterogeneous preferences, is well-established in the empirical public economics literature. It is proposed in Deaton and Muellbauer (1980) and in Hammond (1991), and it forms the basis for the definition and measurement of a money-metric measure of utility in King (1983) and in Aaberge, Colombino and Strøm (2004). Moreover, it has been applied for example by

[1] A recent survey of microsimulation analyses of tax systems is provided by Bourguignon and Spadaro (2006).

Fortin, Truchon and Beauséjour (1993). As a practical matter, an average of the estimated individual utility functions or an estimated utility function (individual welfare function) with common parameters (as in our case) is typically used. Note, however, that the procedure traditionally followed by large part of the theoretical and empirical literature consists in simply ignoring the interpersonal comparability problem, either because consumption-leisure preferences are assumed homogeneous or because heterogeneous utility functions are cavalierly aggregated forgetting that they are not comparable.

The microeconometric model is briefly presented in Section 2, while in Appendix A we present the detailed empirical specifications of the utility functions and of the choice sets, and we provide the estimation results based on Norwegian data from 1994. In order to illustrate the behavioural implications of the estimates, Section 3 reports wage and income elasticities of labour supply. Section 4.1 introduces measures of individual welfare that allow interpersonal comparisons. As explained in Section 4.2, aggregation of welfare levels across individuals is made by using four alternative rank-dependent social welfare functions with varying degree of inequality-aversion. Finally, we identify optimal tax rules – within a class of 9-parameter piece-wise linear rules - by iteratively running the model until a given social welfare function attains its maximum under the constraint of keeping constant the total net tax revenue. The parameters to be determined are an exemption level, four marginal tax rates, three "kink points" and a lump-sum transfer that can be positive or negative The resulting optimal tax rules are presented in Section 5. Section 6 contains the final comments.

Since the microeconometric model, once estimated, is then used for a rather ambitious purpose – i.e. simulating choices in view of identifying optimal tax rules – it is important to check its reliability, besides reporting standard tests on parameters estimates. Ultimately, the model should be judged in its ability to do the job it is built for, i.e. predicting the outcomes of policy changes. We therefore perform an out-of-sample prediction exercise. Namely, we use the model (estimated on 1994 data) to predict household-specific distributions of income in Norway in 2001. We then compare the predicted distributions to the observed ones. The prediction performance turns out to be very satisfactory. Details on the exercise are given in Appendix C.

2. The microeconometric labour supply model

The labour supply model used in this study is detailed described in Appendix A. Here we give a bird-eye presentation. The model can be considered as an extension of the standard multinomial logit model, and differs from the traditional models of labour supply in several respects.[2] First, it accounts for observed as well as unobserved heterogeneity in tastes and choice constraints, which means that it is able to take into account the presence of quantity constraints in the market. Second, it includes both single person households and married or cohabiting couples making joint labour supply decisions. A proper model of the interaction between spouses in their labour supply decisions is important as most of the individuals are married or cohabiting. Third, by taking all the details of the tax system into account, the budget sets become complex and non-convex in certain intervals.

For expository simplicity we consider in this section only the behaviour of a single person household. The extension to couples is fully explained in Appendix A. In the model, agents choose among jobs characterized by the wage rate w, hours of work h and other characteristics. The problem solved by the agent looks like the following:

$$\text{(2.1)} \qquad \begin{aligned} &\max_{(w,h,s,j)\in B} U(c,h,s,j) \\ &\text{s.t.} \\ &c = f(wh, I) \end{aligned}$$

where

h = hours of work,

w = the pre-tax wage rate,

s = observed job characteristics (besides h and w),

j = unobserved (by the analyst) job and/or household characteristics,

I = the pre-tax non-labour income (exogenous),

c= disposable income (income after tax),

f = tax rule that transforms pre-tax incomes (wh,I) into disposable income c,

B= the set of all opportunities available to the household (including non-market opportunities, i.e. a "job" with $w=0$ and $h=0$).

Agents can differ not only in their preferences and in their wage (as in the traditional model) but also in the number of available jobs of different types. Moreover, for the same agent, wage rates (unlike in the traditional model) can differ from job to job. As analysts we observe the chosen h, w and s, but we do not know exactly what opportunities are contained in B. Therefore we use a probability

[2] Examples of previous applications of this approach are found in Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999, 2000). The modeling approach used in these studies differs from the standard labour supply models by characterizing behaviour in terms of a comparison between utility levels rather than between marginal variations of utility. These models are close to other recent contributions adopting a discrete choice approach such as Dickens and Lundberg (1993), Euwals and van Soest (1999), Flood, Hansen and Wahlberg (2004) and Labeaga, Oliver and Spadaro (2007).

density function to represent *B*. Let $p(h,w,s)$ denote the density of available jobs of type (h,w,s). By specifying a probability density function on B we can for example allow for the fact that jobs with hours of work in a certain range are more or less likely to be found, possibly depending on agents' characteristics; or for the fact that for different agents the relative number of market opportunities may differ. We assume that the utility function can be factorised as

$$U\left(f(wh,I),h,s,j\right)=v\left(f(wh,I),h,s\right)e(j), \tag{2.2}$$

where v and e are respectively the systematic and the random component. The term e is a random taste-shifter that accounts for the effect on utility of all the characteristics of the household-job match observed by the household but not by us. Moreover, we assume that e is i.i.d. according to Type III Extreme Value distribution:

$$\Pr\left(e\le q\right)=\exp\left(-q^{-1}\right). \tag{2.3}$$

We observe the chosen h , w and s. Therefore we can specify the probability that the agent chooses a job with observed characteristics (h,w,s). It can be shown that under the assumptions (2.1), (2.2) and (2.3) we can write the probability density function of a choice (h,w,s) as[3]

$$j(h,w,s)=\frac{v(f(hw,I),h,s)p(h,w,s)}{\iiint_B v(f(xy,I),x,z)p(x,y,z)dxdydz}. \tag{2.4}$$

Opportunities with $h=0$ (and $w=0\ and\ s=0$) are non-market opportunities (i.e. alternative allocations of "leisure"). The density (2.4) is the contribution of an observation (h, w,s) to the likelihood function, which is then maximized in order to estimate the parameters of $v(f(hw,I),h,s)$ and of $p(h,w,s)$. The intuition behind expression (2.4) is that the probability of a choice (h,w,s) can be expressed as the relative attractiveness – weighted by a measure of "availability" $p(h,w,s)$ – of jobs of type (h,w,s). A convenient parametric specification allows us to estimate the parameters of (the systematic part of) the utility function $v(\cdot)$ and of the opportunity density function $p(\cdot)$.

In order to estimate the model we use a sample, extracted from the Norwegian 1995 Survey of Level of Living, containing 309 single females, 312 single males, and 1842 couples, where

[3] For the derivation of the choice density (2.4), see Aaberge et al. (1999). Note that (2.4) can be considered as a special case of the more general multinomial type of framework developed by Dagsvik (1994). A more specialized type of continuous multinomial logit was introduced by Ben-Akiva and Watanatada (1981).

spouses as well as singles are between 20 and 62 years old. Self-employed as well as individuals receiving permanent disability benefits are excluded from the sample. To capture the heterogeneity in preferences we estimate simultaneously three separate utility functions: one for single females, one for single males and one for couples.

Appendix A reports the exact specification of $v(\cdot)$ in (A2) and (A10), and of $p(\cdot)$ in (A7), A(8), (A9), (A11), (A13) and (A19). The corresponding parameter estimates are displayed in Tables A1 and A2.

Although the random utility specification (2.2) is by now rather common in labour supply analyses, its implications (in view of interpreting households' behaviour and simulation results) have not been fully clarified in the applied literature. Let us write *U(1) = v(1)e(1)* and *U(2) = v(2)e(2)* to denote the utility attained respectively at job 1 and at job 2. Then it is easily seen that it may happen that job 1 is preferred to job 2, although the observed characteristics may make job 2 look more desirable than job 1. Namely, it may happen that *U(1) > U(2)* even though *v(1) < v(2)*, simply because *e(1)/e(2) > v(2)/v(1)*. As a specific consequence of this, it may happen that the household optimizes on a "flat" segment of the budget line. This could never happen in a standard model where utility only depends on income and leisure (which is the reason why in that kind of model one is typically forced to introduce "optimization errors" to rationalize the data).

It is important to stress that household members choose among jobs (characterized by *h*, *w* and other characteristics *s and j*), not just among different values of *h*. Theoretical optimal taxation models typically consider effort as the agents' choice variable. Effort does not coincide with hours of work; it might include searching for jobs of better quality etc. On the other hand, empirical models of labour supply used for tax reform evaluations have traditionally considered hours of work as the sole choice variable, implicitly equating hours of work and effort. An exception is provided by Bourguignon and Spadaro (2005), who under rather special assumptions are able to impute to each agent an effort value. In our model we do not strictly identify effort as hours of work, since the agent chooses a package that includes not only hours but also wage rates and other observed and unobserved job characteristics. A related concept – taxable income – has been used by Feldstein (1995) and Gruber and Saez (2002). The idea is that in evaluating the effects of changes in taxes one should not just look at hours of work (and participation), since households' response include many other dimensions (effort, wage rates, job content etc.). At least part of these multi-dimensional responses is reflected in taxable income. Our model is consistent with this argument, since households – as a response to a change in the tax system – might choose a new job that differs from the previous one not only with respect to hours of work but also with respect to the wage rate and other job characteristics.

3. Labour supply elasticities

In this section we report wage and income elasticities of labour supply both to illustrate the behavioural implications of the microeconometric model and because they are useful for the understanding and the interpretation of the optimal taxation results that will be presented in Section 6.

The wage elasticities are computed by means of stochastic simulation. Wage rates are incremented by 1 percent. Draws are made from the distributions related to preferences and opportunities. Given the responses of each individual, we aggregate them to compute the aggregate elasticities. Table 3.1 displays these elasticities. Since many individuals in this labour supply model of discrete choice will not react to small exogenous changes, the elasticities in Table 3.1 have been computed as an average of the percentage changes in labour supply from a 10 percent increase in the wage rates. By exact aggregation we find that the overall wage elasticity is equal to 0.12, which suggests rather low behavioural responses from wage and tax changes. At least, this would be the case if we used a representative agent model with wage elasticity equal to 0.12. However, by looking behind the aggregate elasticity the picture, as demonstrated by Table 3.1, changes substantially. Note that the third and the sixth panel of Table 3.1 give the unconditional elasticities of labour supply, which means that both the impact on participation and hours supplied is accounted for.

In principle, elasticities such as those illustrated above might be used to compute optimal tax-transfer rules, e.g. by following the line developed – among others - by Diamond (1988), Saez (2001), Saez (2002), Blundell et al. (2006) and Kleven et al. (2007). As we explained in Section 1, we think that this procedure is not totally satisfactory, due to the possible inconsistency between the assumptions adopted by the theoretical optimal taxation model and the assumptions adopted in producing the empirical evidence. Our microeconometric estimates are based on assumptions that are much more flexible and general than those leading to the theoretical results for example of Diamond (1988) and Saez (2001, 2002). We follow a different approach and obtain the optimal tax-transfer rule computationally, i.e. we iteratively run the microeconometric model of household behaviour until the social welfare function is maximized under the constraint of total tax revenue.

Table 3.1. Labour supply elasticities with respect to wage for single females, single males, married females and married males by deciles of household disposable income*. Norway 1994

Family status	Type of elasticity	Income decile under the 1994 tax system	Female elasticities: *Own wage elasticities*	Female elasticities: *Cross elasticities*	Male elasticities: *Own wage elasticities*	Male elasticities: *Cross elasticities*
Single females and males	Elasticity of the probability of participation	I	0.59		0.00	
		II	0.45		0.00	
		III-VIII	0.06		0.06	
		IX	0.00		0.00	
		X	0.00		0.00	
		All	0.12		0.04	
	Elasticity of the conditional expectation of total supply of hours	I	-0.17		0.77	
		II	-0.04		0.00	
		III-VIII	-0.08		-0.08	
		IX	-0.07		0.00	
		X	0.00		0.00	
		All	-0.09		-0.02	
	Elasticity of the unconditional expectation of total supply of hours	I	0.42		0.77	
		II	0.42		0.00	
		III-VIII	-0.02		-0.02	
		IX	-0.07		0.00	
		X	0.00		0.00	
		All	0.02		0.02	
Married/cohabitating females and males	Elasticity of the probability of participation	I	1.03	-0.28	0.90	-0.23
		II	0.35	-0.14	0.79	0.00
		III-VIII	0.14	-0.23	0.13	-0.10
		IX	0.12	-0.12	0.06	-0.06
		X	0.07	0.00	0.06	-0.19
		All	0.21	-0.19	0.23	-0.11
	Elasticity of the conditional expectation of total supply of hours	I	1.51	-0.01	0.87	0.11
		II	0.62	-0.53	0.38	-0.08
		III-VIII	0.27	-0.24	0.18	-0.14
		IX	0.08	-0.22	0.02	-0.09
		X	0.19	-0.10	-0.02	-0.23
		All	0.31	-0.25	0.16	-0.13
	Elasticity of the unconditional expectation of total supply of hours	I	2.54	-0.29	1.77	-0.12
		II	0.97	-0.67	1.17	-0.08
		III-VIII	0.41	-0.47	0.31	-0.24
		IX	0.20	-0.34	0.08	-0.14
		X	0.26	-0.10	0.05	-0.42
		All	0.52	-0.42	0.39	-0.23

Table 3.1 demonstrates that all own wage elasticities of married females and married males (except for the upper decile) are positive, whereas single females and males located in the central part of the income distribution exhibit a weakly negative response to a wage increase, due to the prevalence of the income effect. Second, we observe that almost all cross wage elasticities are negative. Thus, an increase in, say, the wage rate for males implies that the labour supply of his spouse goes down. The negative cross wage elasticities mean that an overall wage increase gives far weaker impact on labour supply, both for males and females, than partial wage increases for the two genders. For couples belonging to the ninth decile of the couples' income distribution this counteracting effect is so strong that labour supply of these couples' declines from an overall wage increase. From each of the panels of Table 3.1 we observe that the labour supply of the 10-20 percent poorest are far more responsive to changes in economic incentives than the 10-20 percent richest. For single females and males in the 3-8 deciles of their corresponding income distributions we observe backward bending labour supply curves as income effects dominate over substitution effects. By comparing the fourth and fifth panel of Table 3.1 we see for married/cohabitating females that hours supplied (given participation), in particular for those belonging to the poorest couples, is by far more responsive than participation. This result reflects the flexibility of the Norwegian labour market, where jobs with part-time working hours are rather common. Moreover, generous maternity leave arrangements and high coverage of subsidized kindergartens makes it attractive for women to combine raising children and participating in the labour market. By contrast, for single females we find that participation increases when wages increase, whereas hours supplied (given participation) decrease.

The major feature of the estimated labour supply elasticities can be summarized as follows: (a) labour supply of married women is far more elastic than for married men; (b) individuals belonging to low-income households are much more elastic than individuals belonging to high-income households. As demonstrated by the review of Røed and Strøm (2002) these findings are consistent with the findings in many recent studies. In order to complement the information provided by the wage elasticities Tables 3.2 and 3.3 display information for income elasticities. Non-labour income comprehends several categories. Table 3.2 shows how the elasticity of labour supply varies with respect to changes in these income categories and how it depends on gender, household type and location in the income distribution.

Table. 3.2. Labour supply elasticities with respect to non-labour income for single females, single males, married females and married males by deciles of household disposable income. Norway 1994

Family status	Type of elasticity	Income decile under the 1994 tax system	Female elasticities: *Non-labour income (cap. income + cash transfers)*	Female elasticities: *Capital income*	Female elasticities: *Cash trans-fers*	Male elasticities: *Non-labour income (cap. income + cash transfers)*	Male elasticities: *Capital income*	Male elasticities: *Cash trans-fers*
Single females and males	Elasticity of the probability of participation	I	-0.59	0.59	-0.59	0	0	0
		II	0	0	0	0	0	0
		III-VIII	-0.71	-0.13	-0.64	-0.12	-0.12	-0.06
		IX	-1.38	-0.34	-1.38	-0.33	0	-0.33
		X	-1.33	-1.00	-1.00	-0.83	-0.83	0
	Elasticity of the conditional expectation of total supply of hours	I	0.43	-0.16	0.43	0	0	0
		II	0	0	0	0	0	0
		III-VIII	0.08	0.02	0.09	0.05	0.05	0.05
		IX	-0.21	-0.04	-0.21	0.05	0	0.05
		X	-0.51	0.16	-0.47	-0.42	0.01	-0.40
	Elasticity of the unconditional expectation of total supply of hours	I	-0.18	0.42	-0.18	0	0	0
		II	0	0	0	0	0	0
		III-VIII	-0.63	-0.11	-0.56	-0.07	-0.07	-0.01
		IX	-1.56	-0.22	-1.42	-0.29	0	-0.29
		X	-1.81	-0.86	-1.42	-1.22	-0.82	-0.40
Married/cohab. females and males	Elasticity of the probability of participation	I	0	0	0	0	0	0
		II	0	0	0	0.07	0.14	0.07
		III-VIII	-0.16	-0-06	-0.11	-0.17	-0.17	-0.10
		IX	-0.23	-0.12	0	-0.46	-0.29	-0.17
		X	-0.81	-0.54	-0.27	-0.82	-0.57	-0.25
	Elasticity of the conditional expectation of total supply of hours	I	0	0	0	0	0	0
		II	-0.05	-0.10	-0.10	-0.08	0.01	-0.12
		III-VIII	-0.05	0.01	-0.03	-0.03	0	-0.03
		IX	-0.14	-0.06	0	-0.01	-0.01	0.03
		X	-0.22	-0.22	0.10	-0.32	-0.13	-0.13
	Elasticity of the unconditional expectation of total supply of hours	I	0	0	0	0	0	0
		II	-0.05	-010	-0.10	-0.01	0.16	-0.04
		III-VIII	-0.21	-0.05	-0.13	-0.20	-0.07	-0.13
		IX	-0.37	-0.18	0	-0.47	-0.30	-0.14
		X	-1.01	-0.75	-0.17	-1.11	-0.69	-0.38

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Table 3.3. Aggregate labour supply elasticities with respect to non-labour income for single and married individuals. Norway 1994

Family status	Type of elasticity	Female elasticities			Male elasticities		
		Non-labour income (cap. income + cash transfers)	*Capital income*	*Cash trans-fers*	*Non-labour income (cap. income + cash transfers)*	*Capital income*	*Cash trans-fers*
Single females and males	Elasticity of the probability of participation	-0.79	-0.20	-0.71	-0.19	0	-0.08
	Elasticity of the conditional expectation of total supply of hours	-0.09	-0.03	-0.06	-0.05	-0.15	-0.02
	Elasticity of the unconditional expectation of total supply of hours	-0.89	-0.23	-0.77	-0.23	-0.16	-0.09
Married/coh females and males	Elasticity of the probability of participation	-0.20	-0.11	-0.09	-0.23	-0.12	-0.10
	Elasticity of the conditional expectation of total supply of hours	-0.09	-0.04	-0.02	-0.10	-0.04	-0.05
	Elasticity of the unconditional expectation of total supply of hours	-0.30	-0.15	-0.11	-0.32	-0.16	-0.15

4. The framework of the social planner

Since the microeconomic model that is used in this study allows heterogeneous preferences for leisure and consumption and moreover some individuals live as singles whereas others live in a couple, it does not make sense to treat the estimated utility functions as comparable individual welfare functions. Thus, it is necessary to introduce measures of individual welfare that permit interpersonal comparisons. [4] Section 4.1 explains the method used for dealing with this problem, whereas Section 4.2 discusses the methods that will be used for aggregating individual welfare levels into a social welfare function.

4.1. Individual welfare functions

A social planner wants to compare gains in welfare of some households to losses in welfare of others households as part of the evaluation of a tax reform. Unless one is prepared to assert that heterogeneous consumption-leisure preferences are comparable, one has somehow to solve the interpersonal comparability problem. In the context of empirical applications, there is only one type of solution convincingly elaborated in the literature, consisting in using a common utility function to

[4] See Boadway et al. (2002) and Fleurbaey and Maniquet (2006) for a discussion of interpersonal comparability of utility when preferences for leisure differ between individuals.

evaluate the bundles chosen by households according to their own preferences. This approach is advocated, among others, by Deaton and Muellbauer (1980), Hammond (1991) and King (1983). There are different versions, differing essentially in the way the common utility is specified. The common utility function is to be determined by the social planner based on her/his ethical judgements, and contains within it interpersonal comparability of both welfare levels and welfare differences. The common utility function (individual welfare function) *V* is to be interpreted just as the input of a social welfare function. It is not used to simulate behaviour; it is only used to evaluate – in a comparable way – the results of choices made according to the actual individual utility functions. The different roles played by the actual utility function *U* and the individual welfare function *V* are also explained in Section 5 where we specify the various steps of the simulation used to identify the optimal tax rules. Appendix B reports the parametric specification adopted for V and the corresponding parameter values.

A different way to circumvent the interpersonal comparability problem consists in avoiding interpersonal comparisons altogether and basing the social evaluation exclusively on ordinal comparisons. We provide an example of this method in Table 5.6, where we presents the number of "winners" under the optimal tax rules. This is just an illustration, whereas a proper application of the ordinal criterion would require defining the optimal tax in a different way; for example the rule that maximizes the number of winners.

4.2. Social Welfare Functions

The informational structure of the individual welfare functions (common utility function) defined by (B.1) in Appendix B allows comparison of welfare gains and losses of different individuals due to a policy change. When evaluating the distribution of individual welfare effects of a tax system and/or a tax reform it is required to summarize the gains and losses by a social welfare function. The simplest welfare function is the one that adds up the comparable welfare gains over individuals. The objection to the linear additive welfare function is that the individuals are given equal welfare weights, independent of whether they are poor or rich. Concern for distributive justice requires, however, that poor individuals are assigned larger welfare weights than rich individuals. This structure is captured by the following family of rank-dependent welfare functions [5],

[5] Several other authors have discussed rationales for rank-dependent measures of inequality and social welfare, see e.g. Sen (1974), Hey and Lambert (1980), Donaldson and Weymark (1980, 1983), Weymark (1981), Ben Porath and Gilboa (1992) and Aaberge (2001).

(4.1) $$W = \int_0^1 p(t) F^{-1}(t)\,dt, \quad i = 1,2,...,$$

where F^{-1} is the left inverse of the cumulative distribution function of the individual welfare levels V with mean m, and $p(t)$ is a positive weight-function defined on the unit interval. The social welfare functions (4.1) can be given a similar normative justification as it is made for the "expected utility" social welfare functions introduced by Atkinson (1970). Given suitable continuity and dominance assumptions for the preference ordering $\succeq$ defined on the family of income distributions $\boldsymbol{F}$, Yaari (1988, 1989) demonstrated that the following axiom,

Axiom (Dual independence). *Let F_1, F_2 and F_3 be members of $\boldsymbol{F}$ and let $a \in [0,1]$ Then $F_1 \succeq F_2$ implies*

$$\left(a F_1^{-1} + (1-a) F_3^{-1}\right)^{-1} \succeq \left(a F_2^{-1} + (1-a) F_3^{-1}\right)^{-1},$$

characterizes the family of rank-dependent measures of social welfare functions (4.1) where $p(t)$ is a positive non-decreasing function of t. We refer to Yaari (1987, 1988) for a discussion of the difference between the dual independence axiom and the conventional independence axiom that justifies the "expected utility" social welfare functions.

In this paper we use the following specification of $p(t)$,

(4.2) $$p_i(t) = \begin{cases} -\log t, & i = 1 \\ \frac{i}{i-1}\left(1 - t^{i-1}\right), & i = 2,3,.... \end{cases}$$

Note that the inequality aversion exhibited by the social welfare function W_i (associated with $p_i(t)$) decreases with increasing i. As $i \to \infty, W_i$ approaches inequality neutrality and coincides with the linear additive welfare function defined by

(4.3) $$W_\infty = \int_0^1 F^{-1}(t)\,dt = m \ .$$

It follows by straightforward calculations that $W_i \le m$ for all i and that W_i is equal to the mean m for finite i if and only if F is the egalitarian distribution. Thus, W_i can be interpreted as the equally distributed individual welfare level. As recognized by Yaari (1988) this property suggests that C_i, defined by

$$C_i = 1 - \frac{W_i}{m}, \quad i = 1,2,... \tag{4.4}$$

can be used as a summary measure of inequality and moreover can be proved to be a member of the "illfare-ranked single-series Ginis" class introduced by Donaldson and Weymark (1980)[6]. Thus, as was recognized by Ebert (1987) the justification of the social welfare function $W_i = m\left(1 - C_i\right)$ can also be made in terms of a value judgement of the trade-off between the mean and (in)equality in the distribution of welfare.

As noted by Aaberge (2000, 2007), C_1 is actually equivalent to a measure of inequality that was proposed by Bonferroni (1930), whilst C_2 is the Gini coefficient. As demonstrated by Aaberge (2000, 2007) C_1 exhibits strong downside inequality aversion and is particularly sensitive to changes that concern the poor part of the population, whilst C_2 normally pays more attention to changes that take place in the middle part of the income distribution. The C_3-coefficient exhibits upside inequality aversion and is thus particularly sensitive to changes that occur in the upper part of the income distribution. Due to the close relationship between C_1, C_2 and C_3 Aaberge (2007) proposed to treat them as a group and call them Gini's Nuclear Family of inequality measures.

To ease the interpretation of the inequality aversion profiles exhibited by W_1, W_2, W_3 and W_∞ Table 4.1 provides ratios of the corresponding weights – as defined by (4.2) – of the median individual and the 5 per cent poorest, the 30 per cent poorest and the 5 per cent richest individual for different social welfare criteria. As can be observed from the weight profiles provided by Table 4.1 W_1 will be particular sensitive to changes in policies that affect the welfare of the poor.

Table 4.1. Distributional weight profiles of four different social welfare functions

	W_1 (Bonferroni)	W_2 (Gini)	W_3	W_∞ (Utilitarian)
p(.05)/p(.5)	4,32	1,90	1,33	1
p(.30)/p(.5)	1,74	1,40	1,21	1
p(.95)/p(.5)	0,07	0,10	0,13	1

[6] Note that Aaberge (2001) provides an axiomatic justification for using the C_k – measures as criteria for ranking Lorenz curves.

4.3. The Optimal Taxation Problem

The optimal taxation problem considered in this exercise can be formulated as follows. For the sake of simplicity – as we did in Section 2 – the formulation illustrated here assumes that the *N* households are single individuals, while in fact we consider both couples and singles.

$$
\begin{aligned}
&\max_{J} W\left(V\left(c_1, h_1, s_1, j_1\right), V\left(c_2, h_2, s_2, j_2\right), \ldots, V\left(c_N, h_N, s_N, j_N\right)\right) \\
&\text{s.t.} \\
&\left(c_n, h_n, s_n, j_n\right) = \underset{(w,h,s,j)\in B}{\arg\max} U_n\left(c, h, s, j\right) \text{ s.t. } c = f(wh, I_n; J), \forall n \qquad (4.5) \\
&\sum_{n=1}^{N}\left(w_n h_n + I_n - f(w_n h_n, I_n; J)\right) \geq R.
\end{aligned}
$$

where all the variables are the same as those appearing in expression (2.1) in Section 2 and R is the current (1994) total net tax revenue.

The function $f(wh, I_n; J)$ - which transforms gross incomes (*wh, I)* into net available income c – denotes a class of tax rules defined up to a vector of parameters J . As we will explain in Section 5, we will consider a class of piecewise-linear tax rules with a lump-sum tax or transfer. Therefore the parameters will be the amount of the lump-sum tax or transfers, the lower and upper limits of the tax brackets and the marginal tax rates applied to the tax brackets. Household *n* maximizes her own utility given the tax rule $f(wh, I_i; J)$ by choosing the "job" $\left(c_n, h_n, s_n, j_n\right)$. Taking the individual choices into account as a constraint, the social planner searches for the tax rule – i.e. the parameter vector J – that maximizes the social welfare function W, subject to the constraint that the total net tax revenue must be at least as large as *R*. The social welfare function W takes as arguments the evaluations – according to the common utility function V – of the N chosen "jobs". Given the very flexible and general specifications adopted for the random utility functions and the opportunity sets, problem (4.5) cannot be solved analytically. The maximization of W is performed by a global maximization procedure that efficiently scans the parameter space. At each run of the iterative procedure, the maximization of the individual utility function is simulated by the microeconometric model described in Section 2 and in Appendix A. Section 5 describes how problem (4.5) is solved in practice and presents the results.

5. Optimal tax rules

The search for the optimal tax rule is limited to the class of piecewise-linear rules, with five brackets:

(5.1)
$$y=\begin{cases} Z+T \ if \ Z \le E \\ Z+T-t_1(Z-E) \ if \ E < Z \le Z_1 \\ Z+T-t_1(Z_1-E)-t_2(Z-Z_1) \ if \ Z_1 < Z \le Z_2 \\ Z+T-t_1(Z_1-E)-t_2(Z_2-Z_1)-t_3(Z-Z_2) \ if \ Z_2 < Z \le Z_3 \\ Z+T-t_1(Z_1-E)-t_2(Z_2-Z_1)-t_3(Z_3-Z_2)-t_4(Z-Z_3) \ if \ Z_3 < Z \end{cases}$$

where y is income after tax, Z is the sum of gross market income (earnings plus capital income) and taxable public transfers, T is a tax-free public transfer (positive or negative), E is the exemption level, (t_1, t_2, t_3, t_4) are the marginal tax rates applied to the four brackets of income above the exemption level, Z_1 is the upper limit of the first bracket, Z_2 is the upper limit of the second bracket, Z_3 is the upper limit of the third bracket and T is a lump-sum that can be positive (i.e. a lump-sum transfer) or negative (i.e. a lump-sum tax). Thus, each particular tax rule is characterized by the nine parameters: E, t_1, t_2, t_3, t_4, Z_1, Z_2, Z_3 and *T*. In the exercise presented hereafter the top marginal tax rate is constrained to be $t_4 \le 0.75$.[7]

The tax rule specified by (5.1) replaces the current rule as of 1994, which is described by the example of Table 5.1 and also belongs to the class of piece-wise linear tax rules,[8] where M denotes earnings. In this paper we focus on the effect of the tax system on labour supply. Thus, individuals receiving income support related to health or disability (which represents a major part of welfare policies) are not included in the sample that forms the basis of this study. The most important welfare policies addressed to the employed in 1994 were tax-free transfers related to children. These are kept unchanged.

[7] This upper limit is imposed for the sake of realism, since it is the highest top marginal tax rate on personal income reached in Norway in the period 1980 – 2000.

[8] Taxes include the part of social security contributions paid by the employee.

Table 5.1. Current tax rule in Norway as of 1994 for singles without children and couples without children and with two wage earners

Gross earnings (NOK 1994)	Tax
(0 – 17000)	0
(17000 – 24709)	0.25M - 4250
(24709 – 28250)	0.078M
(28250 – 140500)	0.302M - 6328
(140500 – 208000)	0.358M - 14196
(208000 – 234500)	0.453M - 33956
(234500 –)	0.495M - 43804

The identification of the optimal tax rules consists of four steps:

1. First, for each household we simulate the opportunity set, which contains the observed job plus 199 market and non-market alternatives drawn from the estimated *p*-densities defined in Appendix A, - expressions (A7), A(8), (A9), (A11), (A13) and (A19). Second, for each household and each alternative in the opportunity set we draw a value ε from the distribution (2.3). Next, the new tax rule is applied to individual earners' gross incomes in order to obtain disposable incomes (income after tax) corresponding to each alternative in the choice set.[9] For each household a new choice (h,w,s), in view of a new tax rule, is given by the alternative that maximizes the household-specific utility functions U defined by (2.2) where v is defined by (A2) for singles and by (A10) for couples (Appendix A).
2. To each decision maker (wife or husband) an *equivalent income* (y) is imputed. The equivalent income is computed as total disposable household income (c) divided by the square root of the number of household members. The purpose of this procedure is to convert the distribution of incomes (c) across heterogeneous families into a distribution of (equivalent) incomes (y) across adult individuals.
3. As a result of the previous steps, we now have *for* each individual a simulated pair (y, h). As explained in Section 4, we compute the individual welfare levels by applying to the chosen (y, h) the individual welfare (common utility) function – specified as (B.1) of Appendix B.
4. We then compute W_i for $i = 1,2,3$ and ∞.

[9] We also account for the fact that couples with one wage earner face milder taxation in the sense that all tax brackets above the second bracket in Table 5.1 are widened.

Optimization is performed by iterating the steps 1- 4 in order to find the tax rule from the class (5.1) that produces the highest value of W_i for each value of i, under the constraint of constant total tax revenue. The results are reported in Tables 5.2 - 5.6.

Table 5.2 Optimal tax rules according to alternative social welfare criteria(*). (t_4 constrained to be ≤ 0.75)

	Social welfare function			
	W_1 (Bonferroni)	W_2 (Gini)	W_3	W_∞ (Utilitarian)
t_1	0.06	0.16	0.21	0.23
t_2	0.30	0.26	0.25	0.28
t_3	0.39	0.38	0.37	0.33
t_4	0.75	0.75	0.75	0.75
T	-11 900	-6 000	-2 800	-2 800
E	29 000	21 000	23 000	24 000
Z_1	120 000	130 000	140 000	210 000
Z_2	220 000	230 000	230 000	280 000
Z_3	720 000	710 000	710 000	740 000

(*) E, T, Z_1, Z_2 and Z_3 are measured in thousands of 1994 NOK

Table 5.2 displays the optimal tax systems from the optimization exercise. In order to ease the comparability of the behavioural responses to the 1994 tax system and the various optimal tax systems we report proportions of individuals by family status in specific tax income brackets in Table 5.3. Tables 5.4 and 5.5 provide additional information of the behavioural implications of the optimal tax rules. Table 5.6 displays the percentages of winners under the optimal rule by income deciles of the 1994 income distribution.

a) Under any social welfare function, the marginal tax rates are continuously increasing for all level of income. The lump-sum transfer turns out to be a (modest) tax, which implies a reduction of the universal transfers that characterize the current system (essentially child benefits). Altogether then the optimal tax-transfer rule envisages a universal transfer and a sequence of continuously increasing marginal tax rates starting from 0 up to 75%. This picture is in sharp contrast with most of the results obtained by the numerical exercises based on Mirrlees's optimal tax formulas. The typical outcome of those exercises envisages a lump-sum transfer which is progressively taxed away by very high marginal and decreasing tax rates on lower incomes (i.e. a negative income tax mechanism); after the income level where the transfer is exhausted the marginal tax rates remains constant or slightly increasing.[10] Recent papers by Tuomala (2006, 2008) show however that these results are essentially forced by the restrictive assumption typically made upon preferences, elasticities and distribution of productivities (or wage rates). Interestingly, when Tuomala (2008) adopts a more flexible specification of the utility function he gets results that are qualitatively close to those found in this paper.

b) The tables show that the more egalitarian the criterion is, the more progressive is the optimal tax rule. For example the optimal rule according to Bonferroni is more progressive than the optimal rule according to Gini, which in turn is more progressive than the optimal utilitarian rule. The optimal rule according to the utilitarian criterion turns out to be the closest one to the current (1994) rule.

c) All the optimal rules imply a higher income after tax for most levels of gross income. In other words, the optimal rules are able to extract the same total tax revenue from a larger total gross income (i.e. applying a lower average tax rate). The result is due to a sufficiently high labour supply response estimated and accounted for by the model. The optimal rules induce (some of) the households to move to alternatives with longer hours and/or higher wages. Second, the optimal marginal tax rates applied to average or low-average income brackets are markedly lower than the ones implied by the current tax rule. This result provides a controversial perspective in view of the tax reforms implemented in many developed countries during the last decades. In most cases those reforms embodied the idea of improving efficiency and labour supply incentives through a lower average tax rate and lower marginal tax rates on higher incomes.[11] Our optimal tax computations give support to the first part (lowering the average tax rate), much less to the second; on the contrary our results suggest that a lower average tax rate should be obtained by lowering the marginal tax rates particularly on low and

[10] The numerical simulations reported in Saez (2001) produce also an optimal tax-transfer rule envisaging a negative income tax mechanism coupled with more or less constant marginal tax rates. Another contribution by Saez (2002) – that attributes a crucial role to the relative magnitude of the elasticities at the extensive margin and at the intensive margin – has stimulated applications where mechanisms like in-work benefits turn out to be superior to the negative income tax (e.g. Immervoll et al. 2007).

[11] For example Blundell (1996) reports that during the 80's and early 90's in some countries the top marginal tax rates were cut from 70-80% down to about 40-50%. On these issues the discussion in Røed and Strøm (2001) is especially relevant.

average income brackets[12]. Clearly the pattern of elasticities – sharply decreasing with respect to income – illustrated in Table 3.1 contribute to the profile of the optimal marginal tax rates.

d) Table 5.5 shows that the strongest labour supply response comes from households in the lower income deciles, who are those who show a more elastic labour supply (Section 3).

e) Table 5.6 shows the percentage of winners under the optimal rules, by marital status, gender and household income decile under the current 1994 rule. An individual is defined as a winner if her/his welfare is higher under the new tax rule than under the current 1994 rule. All the optimal rules would largely “win the referendum” against the current rule, since they all imply a strong majority of winners. The percentage of winners, however, varies substantially across the different subgroups and especially across income deciles. Singles women in the IX and X income deciles are the only ones who would “vote against” all the optimal tax rules. The current (1994) tax system provides important deductions for children. It appears that these deductions favour in particular the group of relatively well-off single women with children. The deductions are removed in the class of tax-transfer rule we optimize upon. As a consequence, a majority of those women loose under the optimal rules.

f) The lump-sum transfer T turns out to be a tax. The amount is relatively modest for $i > 2$, but more significant for the Gini and Bonferroni welfare criteria. This result can be explained by the fact that individuals/couples with small and medium high incomes are particularly sensitive to changes in marginal taxes (see Table 3.1). Thus marginal tax rates on low and average incomes are kept low both for minimizing distortions and for fulfilling distributive goals. However, since the total net tax revenue must be kept unchanged and the top marginal tax rate must not exceed 75%, the optimal tax rule envisages a universal lump-sum tax.

[12] A second important difference between our exercise and the implemented reforms referred to in the main text, is that those reforms typically envisaged a reduction of the total tax revenue together with the reduction in the average tax rate, while in our simulations we keep the total tax revenue unchanged.

Table 5.3 Proportion of individuals by income intervals(*) under different tax systems. Per cent

	Proportions located in various gross income segments			
Income intervals	**1994 tax system**			
	Couples (Males)	Couples (Females)	Single Males	Single Females
0 –30 000	4.7	16.2	0.0	0.0
30 000 – 130 000	11.0	33.2	25.8	24.4
130 000 – 230 000	30.8	34.9	40.9	51.2
230 000 – 730 000	51.6	15.6	33.0	24.4
730 000 ->	1.9	0.1	0.3	0.0
	W_1 – optimal tax system			
0 –30 000	2.6	10.6	0.0	0.0
30 000 – 130 000	9.7	32.8	22.0	20.3
130 000 – 230 000	30.2	38.8	40.9	52.6
230 000 – 730 000	55.9	17.7	36.8	27.1
730 000 ->	1.6	0.1	0.3	0.0
	W_2 – optimal tax system			
0 –30 000	3.1	12.1	0.0	0.0
30 000 – 130 000	8.8	31.9	21.6	18.6
130 000 – 230 000	28.9	38.0	41.2	54.0
230 000 – 730 000	57.5	17.9	36.8	27.5
730 000 ->	1.7	0.1	0.3	0.0
	W_3 – optimal tax system			
0 –30 000	3.4	13.3	0.0	0.0
30 000 – 130 000	8.7	31.9	21.3	18.9
130 000 – 230 000	28.0	36.8	41.2	53.6
230 000 – 730 000	58.2	17.9	37.1	27.5
730 000 ->	1.7	0.1	0.3	0.0
	W_∞ - optimal tax system			
0 –30 000	3.3	14.0	0.0	0.0
30 000 – 130 000	8.0	31.6	21.0	182
130 000 – 230 000	26.0	36.2	39.9	51.9
230 000 – 730 000	60.9	18.0	38.8	29.9
730 000 ->	1.8	0.2	0.3	0.0

(*) The income intervals are the optimal income brackets in the W_2 - optimal tax rule

Table 5.4 Percentage changes in participation rates, annual hours of work and disposable income under the optimal tax rules

		Social welfare function			
		W_1 (Bonferroni)	W_2 (Gini)	W_3	W_∞ (Utilitarian)
Single males	Participation rates	2.3	2.3	2.3	2.3
	Annual hours	4.8	5.0	5.0	6.2
	Disposable income	10.0	10.2	10.2	12.4
Single females	Participation rates	4.4	5.2	4.8	5.2
	Annual hours	6.3	7.9	7.9	9.7
	Disposable income	4.5	5.3	4.9	7.1
Couples	Participation rates, M	2.7	2.3	2.1	2.9
	Participation rates, F	5.4	4.1	2.8	2.6
	Annual hours, M	6.2	6.7	6.8	9.9
	Annual hours, F	10.3	8.9	6.9	6.5
	Disposable income	9.5	10.3	10.2	13.7

Table 5.5 Percentage changes in labour supply (total hours) by household income decile under the optimal tax rules

		Social welfare function							
		W_1 (Bonferroni)		W_2 (Gini)		W_3		W_∞ (Utilitarian)	
	Income decile under the 1994 system	Male	Female	Male	Female	Male	Female	Male	Female
Singles	I	60.5	71.7	57.3	71.7	57.3	64.7	62.8	76.1
	II	18.6	17.9	18.6	29.3	20.3	29.3	24.0	29.3
	III-VIII	0.7	3.0	1.2	4.5	1.1	4.9	1.7	7.0
	IX	0.0	0.0	0.0	0.0	0.0	0.0	2.6	0.7
	X	1.3	0.0	1.3	0.0	1.3	0.0	1.3	0.0
	All	*4.8*	*6.3*	*5.0*	*7.9*	*5.0*	*7.9*	*6.2*	*9.7*
Couples	I	50.6	72.6	45.0	61.9	40.5	51.9	49.6	59.7
	II	23.6	22.7	24.7	22.3	24.2	22.2	34.7	23.1
	III-VIII	2.7	7.7	3.8	6.3	4.5	3.9	7.1	2.7
	IX	0.7	0.5	0.7	1.0	0.7	0.5	1.2	-0.3
	X	-2.5	-1.3	-1.8	-1.6	-1.8	-0.8	-0.9	-0.4
	All	*6.2*	*10.3*	*6.7*	*8.9*	*6.8*	*6.9*	*9.9*	*6.5*

Table 5.6. Percentage of winners under optimal tax rules

		Social welfare function							
		W_1 (Bonferroni)		W_2 (Gini)		W_3		W_∞ (Utilitarian)	
	Income decile under the 1994 system	Male	Female	Male	Female	Male	Female	Male	Female
Singles	**I**	79	79	66	79	79	76	62	72
	II	66	59	59	59	55	59	52	48
	III-VIII	85	67	85	68	80	68	75	66
	IX	79	45	83	45	83	45	83	48
	X	72	34	79	38	79	38	86	41
	All	*80*	*62*	*79*	*63*	*78*	*63*	*74*	*60*
Couples	**I**	61	63	61	63	60	62	56	60
	II	70	71	68	68	70	70	68	70
	III-VIII	82	83	83	85	83	86	82	86
	IX	82	83	86	88	87	88	88	91
	X	74	72	75	74	75	74	78	77
	All	*78*	*79*	*79*	*80*	*79*	*81*	*78*	*82*

6. Conclusions

We have performed an exercise in designing optimal income taxes that – differently from what is typically done in the literature – does not rely on *a priori* theoretical optimal taxation results, but instead employs a microeconometric model of labour supply in order to maximize a social welfare function with respect to a parametrically defined income tax rule. The microeconometric model can be considered as an extension of the standard multinomial logit model, and is designed to allow for a detailed description of complex choice sets and budget constraints. This model differs from the traditional models of labor supply in several respects. First, it accounts for observed as well as unobserved heterogeneity in tastes and allows for constraints in the choice of hours of work. Second, it includes both single person households and married/cohabiting couples and allows for simultaneous treatment of both spouses choices. Third, the model allows for an exact representation of income taxes. The model, which contains 78 parameters that capture the heterogeneity in preferences as well as in opportunities among households and individuals, is estimated with Norwegian micro data from 1995. The estimated model is used to simulate the choices made by single individuals and couples for any given tax-transfer rule. Those choices are therefore generated by preferences and opportunities that vary across the decision units. We identify optimal tax rules – within a class of 9-parameter piece-wise linear rules - by iteratively running the model until the social welfare function is maximized under the constraint of keeping constant the total net tax revenue.

We focus on the profile of the marginal tax rates and keep fixed the current (1994) system of transfers, income support and social assistance policies, but allow for a lump-sum that can be positive (i.e. a transfer) or negative (i.e. a tax). We explore a variety of different social welfare criteria. The marginal tax rates always turn out to be monotonically increasing with income. More egalitarian social welfare functions tend to imply more progressive tax rules. Irrespective of the social welfare criterion used, the top optimal marginal tax rate always turns out to be 75 per cent for sufficiently high gross income levels (approximately above 700 000 Norwegian Kroner (1994) $\approx$ 87 000 Euros), which concerns 1.8 per cent of the tax payers. All the optimal tax rules imply an average tax rate lower than the current 1994 one and imply – with respect to the current rule – lower marginal rates on low and/or average income levels and a higher marginal rate on sufficiently high income levels. The pattern of labour supply elasticities illustrated in Section 3 contributes to explaining the profile of the optimal tax rules. Our results are partially at odds with the tax reforms that took place in many countries during the last decades. While those reforms embodied the idea of lowering average tax rates, the way to implement it has typically consisted in reducing the top marginal rates. Our results instead suggest lowering average tax rates by reducing marginal rates on low and average income levels and increasing marginal rates on very high income levels.

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Appendix A

The microeconometric model - Empirical specification and estimation results

The modelling approach of this paper differs from the traditional textbook model by treating the utility function as a random variable and analyzing labour supply as a random utility maximization problem. This framework can be considered as an extension of the standard multinomial logit model; see Dagsvik (1994) and Aaberge et al. (1999) for further details. To account for the fact that single individuals and married couples may face different choice sets and exhibit different preferences over income and leisure we estimate separate models for single females and males and married couples.

A.1. Single females and males

The utility functions for single females and males is assumed to be of the following form

(A1) $$U\left(f\,(hw,I),h,\,s,\,j\right) = v(h,w,\,s)\,e(\,j)$$

where

w = wage rate

h = hours of work

I = exogenous income

s = 1 if the job belongs to the public sector (= 0 if the job belong to the private sector),

j = unobserved (by the analyst) job and/or household characteristics,

$f\left(wh,I\right)$ is disposable income (income after tax) measured in 100 000 NOK

and e follows a Type III extreme value distribution.

The systematic part is specified as follows

(A2)
$$\log\left(v(h,w,\,s)\right) = a_2\left(\frac{f\,(hw,I)^{a_1}-1}{a_1}\right)$$
$$+\left(a_4 + a_5\log A + a_6\left(\log A\right)^2 + a_7 s + a_8 C_1 + a_9 C_2 + a_{10} C_3 + a_{11} sC_1 + a_{12} sC_2 + a_{13} sC_3\right)\left(\frac{L^{a_3}-1}{a_3}\right)$$

where

L is leisure, defined as $L = 1 - (h/8736)$,

A is age,

C_1, C_2, and C_3 are number of children below 3, between 3 and 6 and between 7 and 14 years old, respectively.

The α–parameters are gender-specific.

The children terms are dropped in the utility function for single males since we observe very few children living with single males.

The stochastic component e is assumed to be independently drawn from a Type IIII extreme value distribution.

The individuals maximize their utility by choosing among opportunities defined by hours of work, hourly wage and sector of employment. Opportunities with $h = 0$ (and $w = 0$) are non-market opportunities (i.e. alternative allocations of "leisure").

We write the density of opportunities in sector s requiring h hours of work and paying hourly wage w

(A3) $$p(h,w,s) = \begin{cases} p_0 g_{1s}(h) g_{2s}(w) g_3(s) & \text{if} \quad h > 0 \\ 1 - p_0 & \text{if} \quad h = 0 \end{cases},$$

where p_0 is the proportion of market opportunities in the opportunity set, g_{1s}, g_{2s} and g_3 are respectively the densities of hours, wages, and opportunities in sector s, conditional upon the opportunity being a market job.

Given the above assumption upon the stochastic component and upon the density of opportunities, it turns out that the probability (density) that an opportunity (h,w,s) is chosen is

(A4) $$j(h,w,s) = \frac{v(h,w,s)\,p(h,w,s)}{\sum_{s=0,1} \int\int v(x,y,s)\,p(x,y,s)\,dxdy}.$$

In view of the empirical specification it is convenient to divide both numerator and denominator by $1 - p_0$ and define $g_0 = \frac{p_0}{1 - p_0}$. We can then rewrite the choice density as follows:

(A5) $$j(h,w,s) = \frac{v(h,w,s)\,g_0 g_{1s}(h) g_{2s}(w) g_3(s)}{v(0,0,\cdot) + \sum_{s=0,1} \int_{x>0} \int_{y>0} v(x,y,s)\,g_0 g_{1S}(x) g_{2S}(y) g_3(s) dxdy}$$

for $\{h,w\} > 0$ and

(A6) $$j(0,0,\cdot) = \frac{v(0,0;)}{v(0,0,\cdot) + \sum_{s=0,1} \int_{x>0} \int_{y>0} v(x,y,s)\, g_0 g_{1s}(x) g_{2s}(y) g_3(s) dxdy}$$

for $\{h,w\} = 0$. Note that the sector variable s vanishes and is replaced by the symbol $\cdot$ for the non-market alternatives ($\{h,w\} = 0$).

Except for possible peaks corresponding to part time (*pt*, 18-20 weekly hours) and to full time (*ft*, 37-40 weekly hours) we assume that the distribution of offered hours is uniformly distributed. Thus, g_1 is given by

(A7) $$g_{1s}(h) = \begin{cases} g_s & \text{if} \quad h \in (52,910] \\ g_s \exp(p_1 + p_2 s) & \text{if} \quad h \in (910,1066] \\ g_s & \text{if} \quad h \in (1066,1898] \\ g_3 \exp(p_3 + p_4 s) & \text{if} \quad h \in (1898,2106] \\ g_s & \text{if} \quad h \in (2106,3640] \end{cases}.$$

Since the density values must add up to 1, we can also compute g_s according to:

$$g_s\left((910-52) + (1066-52))\exp(p_1 + p_2 s) + (1898-1066) + (2106-1898)\exp(p_3 + p_4 s) + (3640-2106)\right) = 1 .$$

We also specify:

(A8) $$g_0 g_3(s) = \exp\left(m_1 s + m_2 (1-s)\right).$$

The above parameters p and m vary by gender. In the tables we refer to p and m as the parameters of the *job opportunity density*.

The density of offered wages is assumed to be lognormal with mean that depends on length of schooling (*Ed*) and on past potential working experience (*Exp*), where experience is defined to be equal to age minus length of schooling minus five, i.e.

(A9) $$\log w = b_0 + b_1 Exp + b_2 Exp^2 + b_3 Ed + s h$$

where h is standard normally distributed. The parameters b vary by gender and sector of employment.

A2. Married couples

The labour supply model for married couples accounts for both spouses' decisions through the following specification of the structural part of the utility function for couples

(A10)

$$\log v(h_M, h_F, w_M, w_F, s_M, s_F) = a_2\left(\frac{f(h_F w_F, h_M w_F, I)^{a_1} - 1}{a_1}\right)$$

$$+\left(a_4 + a_5 \log A_F + a_6 (\log A_F)^2 + a_7 s_F + a_8 C_1 + a_9 C_2 + a_{10} C_3 + a_{11} s_F C_1 + a_{12} s_F C_2 + a_{13} s_F C_3\right)\left(\frac{L_F^{a_{14}} - 1}{a_{14}}\right)$$

$$+\left(a_{15} + a_{16} \log A_M + a_{17} (\log A_M)^2 + a_{18} s_M + a_{19} C_1 + a_{20} C_2 + a_{21} C_3 + a_{22} s_M C_1 + a_{23} s_M C_2 + a_{24} s_M C_3\right)\left(\frac{L_M^{a_3} - 1}{a_3}\right)$$

$$+ a_{25}\left(\frac{L_M^{a_3} - 1}{a_3}\right)\left(\frac{L_F^{a_{14}} - 1}{a_{14}}\right).$$

where the leisure L_i is defined as $L_i = 1 - (h_i / 8736),\ i = F, M$. We allow for sector- and gender-specific job opportunities in accordance with the functional forms ((A2)-(A6)) that were used for single females and males.

In this case the households choose among opportunities defined by a vector $(h_M, h_F, w_M, w_F, s_M, s_F)$. Here $s_k = 1$ if the partner of gender k is employed in the public sector, with k = M, F. Analogously to what we have done with singles, we specify the corresponding density function as

(A11)

$$p(h_M, h_F, w_M, w_F, s_M, s_F) = \begin{cases} p_{0M}\, g_{1s_M}(h_M)\, g_{2s_M}(w_M)\, g_3(s_M)\, p_{0F}\, g_{1s_F}(h_F)\, g_{2s_F}(w_F)\, g_3(s_F) & \text{if } h_M > 0, h_F > 0 \\ p_{0M}\, g_{1s_M}(h_M)\, g_{2s_M}(w_M)\, g_3(s_M)(1 - p_{0F}) & \text{if } h_M > 0, h_F = 0 \\ (1 - p_{0M})\, p_{0F}\, g_{1s_F}(h_F)\, g_{2s_F}(w_F)\, g_3(s_F) & \text{if } h_M = 0, h_F > 0 \\ (1 - p_{0M})(1 - p_{0F}) & \text{if } h_M = 0, h_F = 0 \end{cases}$$

The choice density of an opportunity $(h_M, h_F, w_M, w_F, s_M, s_F)$ is:

(A12)

$$j\left(h_M,h_F,w_M,w_F,s_M,s_F\right)=\frac{v\left(h_M,h_F,w_M,w_F,s_M,s_F\right)p\left(h_M,h_F,w_M,w_F,s_M,s_F\right)}{\sum_{s_M=0,1}\sum_{s_F=0,1}\iiiint v\left(x_M,x_F,y_M,y_F,s_M,s_F\right)p\left(x_M,x_F,y_M,y_F,s_M,s_F\right)dx_M\,dy_F\,dx_M\,dy_M}$$

For the purpose of empirical specification and estimation it is convenient to divide the density $p(\cdot)$ by $\left(1-p_{0M}\right)\left(1-p_{0F}\right)$ and define

(A13)

$$g_{0M}=\frac{p_{0M}}{\left(1-p_{0M}\right)}$$

$$g_{0F}=\frac{p_{0F}}{\left(1-p_{0F}\right)}$$

$$g_{0MF}=\frac{p_{0M}\,p_{0F}}{\left(1-p_{0M}\right)\left(1-p_{0F}\right)}$$

Now the choice density can be written as follows:

(A14)

$$j\left(h_M,h_F,w_M,w_F,s_M,s_F\right)=\frac{v\left(h_M,h_F,w_M,w_F,s_M,s_F\right)g_{0MF}\,g_{1s_M}(h_M)\,g_{2s_M}(w_M)\,g_3(s_M)\,g_{1s_F}(h_F)\,g_{2s_F}(w_F)\,g_3(s_F)}{D}$$

if both work;

(A15) $$j\left(h_M,0,w_M,0,s_M,\cdot\right)=\frac{v\left(h_M,0,w_M,0,s_M,\cdot\right)g_{0M}\,g_{1s_M}(h_M)\,g_{2s_M}(w_M)\,g_3(s_M)}{D}$$

if only the husband works;

(A16) $$j\left(0,h_F,0,w_F,\cdot,s_F\right)=\frac{v\left(0,h_F,0,w_F,\cdot,s_F\right)g_{0F}\,g_{1s_F}(h_F)\,g_{2s_F}(w_F)\,g_3(s_F)}{D}$$

if only the wife works;

(A17) $$j\left(0,0,0,0;\cdot,\cdot\right)=\frac{v\left(0,0,0,0;\cdot,\cdot\right)}{D}$$

if none of them work, where we have defined

(A18)

$$D = v(0,0,0,0,\cdot,\cdot)$$
$$+\sum_{s_M=0,1} \iint_{\substack{x>0\\y>0}} v(x_M,0,y_M,0,s_M,\cdot)\, g_{0M}\, g_{1s_M}(x_M)\, g_{2s_M}(y_M)\, g_3(s_M)\, dx_M\, dy_M$$
$$+\sum_{s_M=0,1} \iint_{\substack{x>0\\y>0}} v(0,x_F,0,y_F,\cdot,s_F)\, g_{0F}\, g_{1s_F}(x_F)\, g_{2s_F}(y_F)\, g_3(s_F)\, dx_F\, dy_F$$
$$+\sum_{s_M=0,1} \sum_{s_F=0,1} \iiint\int_{\substack{x>0\\y>0}} v(x_M,x_F,y_M,y_F,s_M,s_F)\, g_{0MF}\, g_{1s_M}(x_M)\, g_{2s_M}(y_M)\, g_3(s_M)\, g_{1s_F}(x_F)\, g_{2s_F}(y_F)\, g_3(s_F)\, dx_M\, dy_F\, dx_M\, dy_M$$

The hour densities and the wage densities are the same as specified for singles. The same applies to $g_{0M}\, g_3(s_M)$ and $g_{0F}\, g_3(s_F)$. Moreover:

(A19) $$g_{0MF}\, g_3(s_M)\, g_3(s_F) = \exp\left(m_0 + m_{1M}(s_M) + m_{2M}(1-s_M) + m_{1F}(s_F) + m_{2F}(1-s_F)\right).$$

The estimates of the preference parameters for couples are reported in Table A2.

A.3. Estimates

The estimation of the model is based on data from the 1995 Norwegian Survey of Level of Living, which includes detailed income data from tax reported records. We have restricted the ages of the individuals to be between 20 and 62 in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since analysis of retirement decisions is beyond the scope of this study. Moreover, self-employed as well as individuals receiving permanent disability benefits are excluded from the sample. Table A4 reports incomes, participation rates and hours of work observed for the sample based on data for 1842 couples, 309 single females and 312 single males.

All the parameters of the utility function and of the opportunity densities are estimated simultaneously by the method of maximum likelihood. The likelihood function is equal to the products of the choice densities for single females, single males and couples. Notice that the utility function parameters are different depending on the individual living as single or living in a couple, while the opportunity density parameters are common. The estimates of the opportunity density parameters are reported in Table A3, whilst the preference parameters for single females and males and for couples are reported in Tables A1 and A2, respectively.

Table A1. Estimates of the parameters of the utility functions for single females and males. Norway 1994

Variable	Parameter	Single females		Single males	
		Estimate	Std. Dev.	Estimate	Std. Dev.
Consumption					
	α_1	-0.59	0.28	0.24	0.33
	α_2	4.37	0.52	2.27	0.44
Leisure					
	α_3	0.65	0.92	0.76	0.99
	α_4	498.50	145.18	337.40	128.84
Log age	α_5	-265.77	79.22	-180.89	70.63
Log age squared	α_6	36.36	10.89	24.81	9.75
# children, 0 – 2 years old	α_7	3.62	2.43		
# children, 3 – 6 years old	α_8	-0.36	7.87		
# children, 7 – 14 years old	α_9	-2.24	1.42		
Employed in public sector	α_{10}	-2.97	0.87	-2.20	0.90
(Empl. in pub. sec.)(# child., 0 – 2 years old)	α_{11}	-7.29	7.46		
(Empl. in pub. sec.)(# child., 3 – 6 years old)	α_{12}	-1.02	2.10		
(Empl. in pub. sec.)(# child., 7 – 14 years old)	α_{13}	1.15	1.10		

Table A2. Estimates of the parameters of the utility function for married/cohabitating couples. Norway 1994

Variable	Parameter	Estimate	Std. Dev.
Consumption			
	α_1	0.14	(0.09)
	α_2	6.49	(0.43)
Wife's leisure			
	α_3	-3.81	(0.43)
	α_4	194.89	(28.53)
Log age	α_5	-107.09	(15.88)
Log age squared	α_6	15.14	(2.23)
# children, 0 – 2 years old	α_7	0.34	(0.31)
# children, 3 – 6 years old	α_8	1.31	(0.31)
# children, 7 – 14 years old	α_9	1.70	(0.26)
Employed in public sector	α_{10}	-0.95	(0.30)
(Empl. in pub. sec.)(# child., 0 – 2 years old)	α_{11}	0.40	(0.33)
(Empl. in pub. sec.)(# child., 3 – 6 years old)	α_{12}	0.39	(0.32)
(Empl. in pub. sec.)(# child., 7 – 14 years old)	α_{13}	-0.97	(0.24)
Husband's leisure			
	α_{14}	-1.01	(039)
	α_{15}	222.99	(41.03)
Log age	α_{16}	-116.55	(22.34)
Log age squared	α_{17}	15.85	(3.06)
# children, 0 – 2 years old	α_{18}	-0.08	(0.40)
# children, 3 – 6 years old	α_{19}	-0.30	(0.35)
# children, 7 – 14 years old	α_{20}	-0.15	(0.25)
Employed in public sector	α_{21}	-0.60	(0.51)
(Empl. in pub. sec.)(# child., 0 – 2 years old)	α_{22}	-0.16	(0.39)
(Empl. in pub. sec.)(# child., 3 – 6 years old)	α_{23}	-0.93	(0.31)
(Empl. in pub. sec.)(# child., 7 – 14 years old)	α_{24}	-0.16	(0.25)
Leisure interaction between spouses	α_{25}	4.84	(1.12)

[*)] Standard deviations in parentheses.

Table A3. Job, Hours and Wage densities, Norway 1994

	Parameter	Females		Males	
		Estimate	Std. Dev.	Estimate	Std. Dev.
Job opportunity	μ_0	-2.10	(0.18)	-3.17	(0.23)
	μ_1	-1.51	(0.18)	-2.68	(0.20)
	μ_2	1.39	(0.17)	1.39	(0.17)
Hours	p_1	0.49	(0.13)	-0.50	(0.22)
	p_2	-0.23	(0.23)	0.09	(0.51)
	p_3	1.47	(0.09)	1.81	(0.07)
	p_4	0.03	(0.14)	0.06	(0.13)
Wage – Private sector	b_0	3.62	(0.07)	3.50	(0.06)
	b_1	2.60	(0.30)	2.83	(0.31)
	b_2	-4.04	(0.64)	-4.41	(0.64)
	b_3	3.93	(0.50)	5.38	(0.41)
	s	0.24	(0.00)	0.28	(0.01)
Wage - Public sector	b_0	3.71	(0.08)	3.62	(0.09)
	b_1	2.14	(0.33)	2.46	(0.44)
	b_2	-3.37	(0.71)	-3.82	(0.91)
	b_3	3.59	(0.46)	4.95	(0.47)
	s	0.18	(0.01)	0.22	0.01

Table A4. Incomes and labour supply under the current tax rule, Norway 1994

Family status	Household income decile	Participation rates (Per cent)		Annual hours				Household income, NOK 1994		
				Given participation		In the total population		Gross income	Taxes	Disposable income
		M	F	M	F	M	F			
Single males (M)	I	69		1285		886		85922	14144	71778
	II	86		1343		1157		105799	18281	87518
	III-VIII	95		2041		1936		189772	46930	142842
	IX	97		2304		2225		309909	94218	215691
	X	76		2684		2036		466720	159738	306982
	All	90		1999		1793		210626	56762	153864
Single females (F)	I		66		1128		739	85309	11099	74210
	II		76		1362		1033	107709	14877	92832
	III-VIII		87		1801		1564	179199	38759	140441
	IX		93		2118		1972	265653	63411	202243
	X		97		2743		2649	324394	78749	245645
	All		85		1851		1578	185803	40064	145739
Couples	I	75	59	1459	1111	1090	655	191006	33005	158001
	II	79	79	1641	1245	1293	988	259226	51660	207566
	III-VIII	92	86	2029	1524	1870	1316	400954	103150	297804
	IX	95	92	2406	1751	2285	1604	584018	176183	407835
	X	86	81	2583	1737	2220	1415	833657	260049	573608
	All	89	83	2041	1514	1811	1256	427342	113973	313368

Appendix B

Specification of the individual welfare function

The individual welfare function (V) is specified as follows,

(B.1) $$\log V(y,h) = g_2\left(\frac{y^{g_1}-1}{g_1}\right)+g_4\left(\frac{L^{g_3}-1}{g_3}\right)$$

where L is leisure, defined as $L=1-(h/8736)$, and y is the individual's income after tax defined by

(B.2) $$y=\begin{cases} c=f(wh,I) & \text{forsingles} \\ \dfrac{c}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}f(w_F h_F, w_M h_M, I) & \text{for married/cohab. individuals.} \end{cases}$$

The parameters of V are estimated in a similar way as the parameters of the systematic utility functions v that appear in expressions (2.4). Since the observed chosen combinations of leisure and disposable income depend on the availability of various job opportunities, we use expression (2.4), where the systematic part of the utility function (v) is replaced by the individual welfare function (V) defined by (B.1). Table B.1 displays the parameter estimates.

Table B.1. Estimates of the parameters of the welfare function for individuals 20 – 62 years old, Norway 1994

Variable	Parameter	Estimate	Stand.dev.
Income after tax (y)			
	g_1	-0.649	0.086
	g_2	3.026	0.138
Leisure (L)			
	g_3	-12.262	0.556
	g_4	0.045	0.011

Appendix C. Prediction performance of the microeconometric model

This appendix illustrates the prediction performance of the model used for identifying the optimal tax rules. We present two exercises: prediction ("within-sample") of the outcomes under the current (1994) tax regime and prediction ("out-of-sample") of outcomes under the 2001 tax regime.

Tables 5.1 and C3 describe some of the characteristics of the 1994 and 2001 tax regimes.

Disposable income is the variable used for comparing predicted outcomes to observed outcomes.

The predictions are obtained individual by individual, evaluating the utility function – including the stochastic component drawn from the Type I extreme value distribution – at each alternative and identifying the selected alternative as the one with the highest utility level. The individual predictions are then aggregated into the 10 means of the 10 income deciles.

Table C1 provides the results of the exercise under the 1994 tax regime. For each of the 10 income deciles, we report the observed and the simulated average values of disposable income relative to the sample average. For example "90" means 90% of the sample average. This is just a "test" of the ability to reproduce the observed income distribution. Instead Table C2 reports the results of the more requiring out-of-sample prediction exercise. In this second exercise we use the model estimated on 1994 data (i.e. the parameters of Tables A.1 and A.2 of Appendix A) and the data (exogenous variables) from the Norwegian Survey of Level of Living in 2002, in order to predict the choices made in 2002 under the new tax rules introduced in 2001. In both exercises the model turns out to be rather successful in reproducing the income distributions.

Table C1. Observed and predicted *relative* distributions of disposable income in 1994. Mean decile incomes in percent of mean income

Deciles	Couples		Single females		Single males	
	Observed	*Simulated*	*Observed*	*Simulated*	*Observed*	*Simulated*
1	52	51	49	51	46	47
2	69	66	64	63	59	57
3	77	75	76	73	69	68
4	84	84	85	81	79	76
5	90	91	94	92	86	86
6	96	98	101	100	95	96
7	104	106	111	110	104	109
8	112	116	122	122	115	121
9	125	129	134	139	138	141
10	199	184	163	169	208	200

Table C2. Observed and predicted *relative* distributions of disposable income in 2001. Mean decile income in percent of mean income

Deciles	Couples		Single females		Single males	
	Observed	*Simulated*	*Observed*	*Simulated*	*Observed*	*Simulated*
1	50	49	45	47	41	42
2	68	64	56	61	54	55
3	77	74	68	71	65	67
4	83	83	79	79	76	76
5	89	90	90	88	87	86
6	95	98	101	98	97	97
7	102	107	111	108	107	108
8	111	117	123	121	119	121
9	125	131	139	138	137	141
10	199	187	189	188	218	207
9	129	128	142	136	150	135
10	159	151	177	166	178	161

Table C3. The 2001 tax function for singles without children and couples without children and with two wage earners. NOK 2001

Earnings(Y)	Tax
[0 – 22200)	0
[22200 – 32267)	0.25Y – 5550
[32267 – 60600)	0.078Y
[60600 – 144545)	0.358Y – 16968
[144545 – 183182)	0.296Y – 8064
[183182 – 289000)	0.358·Y – 19 348
[289000 – 793200)	0.493·Y – 58 363
[793200 –)	0.553·Y – 105 955