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A Multi-Product Framework Generating Waves of Mergers and Divestitures*

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Recent waves of corporate mergers followed by divestitures have sparked new interest in economic analyses of these issues. We take the merger paradox from the standard oligopoly literature as a starting point and show that in the absence of any cost-synergies of merger activities, firms do have an incentive to divest further instead of joining mergers. We then analyze conditions where mergers may emerge endogenously as a result of a market game. Due to the nature of the interaction of market-share and market-concentration effects in Cournot oligopolies, a stable internal equilibrium where mergers arise endogenously and simultaneously requires both cost synergies and cost dis-synergies. Endogenous merger size is then a function of market parameters as well as cost synergy parameters. Hence anticipated changes in market size or cost synergies attainable through mergers lead to reconfigurations of merger sizes. If ex-ante expectations about merger-promoting changes are not fully realized ex-post, merger waves will be followed by divestiture waves. Firm valuation - based on ex-ante expectation - may increase while actual profits and efficiency of the merged entity - according to the ex-post realization - may fall.

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1. Introduction

The waves of corporate mergers during the nineties and at the beginning of the new millennium have sparked new interest in economic analyses of the preconditions and consequences of increasing market concentration. In the EU alone, cross-border mergers reached almost USD 600 Billion in the year 2000 after rising steadily since 1988.[1] Since then, European merger activity has slowed down again considerably. The merger boom of the early 2000s is only the last of maybe five major merger waves documented.[2] In contrast to mergers earlier in the last century, recent mergers are mostly horizontal[3] and increasingly cross-border.[4] There is also mounting evidence that high merger activity is followed by subsequent divestitures by some of these mergers. For example, one of the biggest mergers of this recent wave was the purchase of Time Warner by AOL[5]; in 2005 news reports appeared about plans to divest Time Warner Inc. in order to increase its market valuation.[6] In general, the number of divestitures may be said to have increased in tandem with the number of mergers since the mid-1980s and during the 1990s.[7] In addition, for the 1990s, there is evidence that maybe half of all merger activities in the U.S. were divestitures.[8] When mergers occur, they mostly decrease profits and efficiency while increasing market value. The results are similar across different countries and sectors, and also between domestic and cross-border mergers.[9] Nevertheless,

[1] UNCTAD (2000) and UNCTAD (World Investment Report, reported in CESifo Forum, 5(4), Winter 2004). For a general overview of recent developments, see, e.g. Markusen (2002).

[2] See, e.g., Golbe/White (1993) for a documentation of the last four waves.

[3] Using Thompson Financial Security data from the mid-1980s to late 1990s, Pryor (2001) reports that roughly two-thirds to three-quarters of all mergers in value terms were within the same SIC industry. Measured in numbers, these shares are around half. Furthermore, about 60 percent of mergers at the end of the 1990s were in manufacturing, communication, and financial services. Similar results are reported by Gugler, et al. (2003).

[4] According to Gugler, et al. (2003), the share of cross-border mergers among European mergers in the 1990 rose from about 24 to close to 40 percent.

[5] This acquisition had a planned value of USD 165 billion. In the same year, there was Mannesmann and Vodafone Airtouch at USD 183 billion, SmithKline Beecham and Glaxo Wellcome at USD 76 billion, Pfizer and Warner Lambert at USD 92 billion, and Warner Records with EMI's music division at USD 20 billion. (Quoted from Pryor, 2001.)

[6] Süddeutsche Zeitung, 17 August 2005, p. 21. The same issue also contained reports about the German Telekom's failed plans to take over O2, Springer's takeover of the German television station Pro Sieben Sat 1, and Herlitz's takeover by Stationary Products, a Luxembourg-based company owned in turn by the US investment fund Advent International...

[7] Compare the data reported in Gugler et al. (2001), Table 1, based on SDC Thompson Financial Securities data.

[8] See, e.g., Mulherin/Boone (2000).

[9] See Berry/Pakes (1993) and Gugler et al. (2003) for summaries. The latter, in particular, use a large panel data set in order to analyze the effects of mergers internationally and over time. For this purpose, merging firms' profits and sales before and after the merger are compared.

industrial concentration has apparently increased during the last two decades.[10] Increased concentration may also have lead to increased product variety as well as to increased entry and exit activities.[11]

We aim to incorporate these stylized facts about mergers into the behavior of a standard oligopoly model. In the traditional industrial-organization literature, oligopoly models are used to analyze mergers, their effects on market concentration, and firms' incentives to participate in them.[12] Our study methodologically extends these earlier approaches while also referring to some newer literature on mergers with product differentiation and trade[13]. However, earlier literature already emphasized that this approach may lead to paradoxical results. The original merger paradox in a homogeneous Cournot oligopoly without cost effects[14] emphasizes several points: 1.) For a single firm, the marginal incentive to form a merger with another firm – i.e. a pair-wise merger – is negative. The marginal incentive to join a merger of size $m < n$ where n is the number of firms in the market is also negative for small n and only becomes positive when at least about half the market participates. This is due to

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Event studies looking at changes in market valuation typically find that acquiring firms break even while acquired firms gain. See, e.g. Banerjee/Eckard (1998).

Models analyzing competition and market concentration in differentiated-product markets with exogenous market size have been forwarded by e.g. Trajtenberg (1985), Bresnahan (1987), Anderson et al. (1992). Analyses of mergers' effects on prices and profits are contained in e.g. Nevo (2000) and Ivaldi/Verboven (2001), whereas Berry et al. (1995), Petrin (2000), and Nevo (2000) present the effects on substitution behavior of consumers.

Studies investigating particular industries have been forwarded recently, e.g. by Gugler/Siebert (2004) for the semiconductor industry, by Berry/Waldfogel (2001) for radio broadcast markets, and by Park, et al. (2001) for railroad mergers. Earlier studies on railroad mergers were conducted by MacDonald (1987 and 1989), Lemke/Babcock (1987).

10 Effects of mergers on market shares have been investigated by Mueller (1985). Using panel data for the largest 1000 firms between 1950 and 1972, and controlling for non-relevant factors, the market shares of acquired firms are compared to those of non-acquired firms. The results indicate that acquired firms perform rather worse than non-acquired firms, which is interpreted as evidence against the hypothesis that mergers improve efficiency. Pryor (2001) analyzes data from Thompson Financial Securities for 1985-1999 and finds that enterprise size and industrial concentration have both increased. Similar conclusions were drawn by Golbe/White (1993).

11 Effects on product variety were investigated by Berry/Waldfogel (2001) using a panel data set of 243 U.S. radio broadcast markets in 1993 and 1997. Since the Telecommunications Act of 1996 substantially relaxed ownership restrictions in these markets, increased concentration has reduced station entry while increasing variety.

The long-term effects of mergers are taken into account by studies using dynamic merger models (see e.g. Berry/Pakes, 1993; Pakes/McGuire, 1994). One major aspect of long-term behavior to be taken into account here is entry and exit of firms adjusting to the new market structure resulting from mergers. Furthermore, the relevance of innovative activities and product innovations for merger behavior had already been noticed early on (Stigler, 1965).

12 See, e.g., Jacquemin/Slade (1988), Salant/Switzer/Reynolds (1983), Perry/Porter (1985), Farell/Shapiro (1990), Berry/Pakes (1993).

13 See, e.g. Long/Vousden (1995), Falvey (1998, 2003), Horn/Levinsohn (2001), Ryan/Kendall (2001), Neary (2003), Tombak (2003).

14 Salant/Switzer/Reynolds (1983).

two counter-acting effects, the negative market-share effect[15] and the positive market-concentration effect. For small merger sizes, the market-share effect dominates, while for mergers approaching all firms, the market-concentration effect dominates. 2.) A single firm will only improve its profits - compared to non-merger oligopoly profits – when it participates in a merger covering at least 80 percent of the market. 3.) For a firm staying single while another merger forms in the market, both the market-share effect and the market-concentration effect are positive. An outside firm always gains more than any merger participant.

We take the merger paradox as a starting point and show that in the absence of any cost-synergies or similar effects of merger activities, firms do even have a strong incentive to divest further instead of joining mergers. We then analyze conditions where mergers may emerge endogenously as a result of a market game.[16] Due to the nature of the interaction of market-share and market-concentration effects in Cournot oligopolies, a stable internal equilibrium where mergers arise endogenously and simultaneously requires both cost synergies and cost dis-synergies. We derive the conditions for the existence of such an equilibrium and describe its properties. In the resulting equilibrium, endogenous merger size is a function of market parameters as well as cost synergy parameters. Hence anticipated changes in market size and/or cost synergies attainable through mergers will lead to reconfigurations of merger sizes.

If ex-ante expectations about merger-promoting changes are not fully realized ex-post, merger waves will be followed by divestiture waves in our model. In addition, firm valuation - based on ex-ante expectation - may increase while actual profits and efficiency of the merged entity - according to the ex-post realization - may fall. Our model is also adaptable to an international trade context.[17]

The remainder of the paper is organized as follows. Section 2 surveys previous theoretical work on mergers and acquisitions. Our model of mergers with differentiated products and Cournot competition is presented in section 3. Section 4 analyzes incentives to merge or to divest when costs are not changed by mergers. In section 5, the optimal merger size in a model of monopolistic

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15 Since the market-share effect also leads to the reduction of market shares of other firms in the own merger, it is also called cannibalization effect. See, e.g., Huck/Konrad (2004) or in the context of multi-product firms Eckel/Neary (2005).

16 Lindqvist/Stennek (2005) provide experimental evidence on the persistence of the insider's dilemma in merger formation and conclude that this provides support for endogenous merger theory.

17 Compare our model with the setup in Huck/Konrad (2004). In addition, if cost effects arise only for the case of cross-border mergers, our model can give rise to joint cross-border mergers and domestic divestitures.

competition with variable-cost synergies is derived. The last section summarizes the findings and concludes.

2. Previous Models of Mergers and Acquisitions

This section discusses some of the theoretical literature on mergers[18] used as a basis for our own modeling. Traditional methods analyze horizontal mergers assuming that products in the relevant market are homogenous (Jacquemin/Slade, 1988). Thus, Salant/Switzer/Reynolds (1983), modeling mergers as an exogenous change in market structure starting from an initial Cournot equilibrium and doing comparative statics, show that most exogenous mergers are unprofitable for the merging firms, while outsiders not participating in the merger gain. This is the "Merger Paradox". In the standard Cournot setting without cost-altering effects of mergers, profitable mergers would have to involve more than 80 percent of the market while outsiders would always gain more than insiders. Other authors such as Farell/Shapiro (1990) find that mergers in a Cournot oligopoly not creating cost synergies will raise prices and decrease market share. On the other hand, in the presence of cost synergies, output-reducing mergers can improve welfare by closing down inefficient firms. Allowing for pair-wise mergers significantly reduces the minimum share of the market necessary to participate in mergers in order to make them profitable. However, the outsider advantage from mergers remains. See Böckem (2002), Neus (2002).

Other authors, such as Kamien/Zang (1993) or Rothschild (1999), Tombak (2002), Böckem (2002), Ulukut (2003) study horizontal mergers between firms with heterogeneous costs. Different but fixed unit production costs lead here to two effects, one encouraging mergers while the other discourages them. Mergers between high and low-cost firms decrease average cost of the merging firms; this is a positive effect similar to cost synergies. Mergers with high-cost rivals tend to decrease market price, since the high-cost, high-price rival is removed from the market; this tends to make mergers of that kind less profitable. As a result, there is only a particular window of cost asymmetries where mergers of this kind are profitable. With both very high and very low cost asymmetries, mergers will not take place. In this context, note that a large part of the mergers and trade literature also operates with explicitly different costs or cost asymmetries due to trade costs. Perry/Porter (1985) introduce strong cost economies due to a tangible asset available in fixed quantity that

[18] A well structured review of the theoretical literature is also provided by Ulukut (2003).

reduces marginal costs. Each firm has a fixed fraction of this asset. Depending on the degree of cost synergies, this significantly increases merger incentives. Due to cost reductions resulting from a merger, insiders now have an advantage over outsiders and may gain more than the latter. The sources of such cost-synergies are explicitly taken into account in some newer literature; e.g., Davidson/Ferrett (2005) introduce spillovers from R&D between merging firms only.

In order to analyze price competition[19] fully, some degree of product differentiation needs to be introduced. For this case, Deneckere/Davidson (1985) show that mergers are always profitable for insiders, even without resulting synergies, since prices act as strategic complements. However, outsiders still gain more by such mergers. Consequently, other literature such as McAfee/Simons/Williams (1992) or Lommerud/Sørgard (1997), Rothschild (2000), Rothschild et. al. (2000) explores different conditions such as locational differences and multi-product operations. Werden/Froeb (1994) show how such a model can be estimated and simulated combining logit demands with constant marginal costs. For the purpose of estimating mergers empirically for the semiconductor industry, Gugler/Siebert (2004) introduce a model with heterogeneous products and quantity competition.[20] We will use a similar framework for our analysis below.

3. A Basic Model of Mergers with Differentiated Products and Cournot Competition

For a model with differentiated products and Cournot oligopoly, we present an analysis of endogenous merger formation. After introducing the model, we solve the second Cournot-stage of the game. Since we assume symmetric utility and ex-ante identical firms, we can make use of the resulting aggregative properties of the game in the second stage. This is followed by deriving the first stage of the game: a non-cooperative Nash-equilibrium in forming cooperative mergers. While this will result in multiple symmetric Nash-equilibria, there will be a unique equilibrium that is not Pareto-dominated by any alternative. This constitutes a form of coalition formation where participating firms

[19] Zachau (1987) introduces mergers into the standard model of price competition with vertically differentiated products (Gabszewicz/Thisse, 1979, 1980; Shaked/Sutton, 1982, 1983). He finds that both the merging and the outside firms always gain by a merger. Furthermore, mergers are sometimes more profitable for the merging firms than for the outside firms. However, when merger decisions are fully endogenous, the monopoly created by a merger of all firms is the unique stable industry structure in a very strong sense. For newer related literature on vertical product differentiation, see also Wauthy (1996), Greenstein/Ramey (1998), Johnson/Myatt (2003).

[20] Apparently, there is no other lterature using this approach, even though many industries besides the semiconductor industry could be characterized by this kind of competition.

only care about the resulting size of the coalition, but not about individual firm identities. We also make use of the property that any merger configuration forms a partition of the initial market.

3.1. Model Setup

There are *n* identical firms or plants (and their respective products) in a single (domestic) market. Firms may collude with other firms to form a merger. Otherwise each single-product firm is identical.

Costs may depend on the number of single-product plants (or firms) *m* being part of an existing merger. Marginal production cost is denoted as n_m. Setup ("fixed") cost per single-product plant are f_m . Without cost economies of scope we will drop the suffix *m* from these variables. Marginal costs may exhibit economies of scope; in this case $n_m' < 0$, n_m'' ³ 0. Setup cost per single-product plant may exhibit diseconomies of scope *with* $f_m' > 0$, f_m'' ³ 0 .

Utility is quadratic with potentially imperfect substitutability[21], resulting in linear demands. Denote the aggregate quantity demanded of good *i* as q_i. Let *a* > 0, *b* > 0, 0 = ? = *b*. With *L* consumers (equal to aggregate labor supply), inverse demand functions per consumer will be:

$$p_i = a - (b - g)\frac{q_i}{L} - g\frac{\sum_{j=1}^{n} q_j}{L}, i = 1,..., n \quad (3.1)$$

Note that utility and demand are completely symmetric with respect to products.

Firms play a two-stage industry game:

1) Firms simultaneously choose their optimal merger size and join a merger of that size;

2) Firms simultaneously compete in quantities (Cournot).

Our model is quite flexible and can capture a number of different constellations presented already in the literature. These include single-merger models, homogenous-good models, and models including pair-wise mergers.[22]

While the second stage of this game is a standard industry-oligopoly game, the first stage is a bit more subtle. In that first stage, any of the *n* firms non-cooperatively chooses an optimal desired

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[21] Quasi-linear generalized quadratic utility function of a single consumer is given by U = $\alpha\Sigma c_i - 0.5\gamma\Sigma c_i\Sigma c_j - 0.5(\beta-\gamma)\Sigma c_i^2$. For large n, we can approximate this relation by using integrals rather than sums: U = $\alpha\int c_i - 0.5\gamma\int c_i\int c_j - 0.5(\beta-\gamma)\int c_i^2$.

merger size and then will form a merger, i.e. a coalition, with the appropriate number of like-minded other firms. Since in equilibrium, this will result in all firms being part of some merger, these mergers form a partition of the original space of n firms.[23] In addition, due to the nature of coalition formation, merger choices of any firms actually change available strategy spaces of all other firms.

Nevertheless, since the individual decision to join a coalition is non-cooperative, we still can obtain Nash-equilibria in mergers under certain conditions. In particular, we can formulate conditions for the existence of a symmetric equilibrium and then describe its properties. The game is solved, as usual, by backwards induction, so we present the Cournot stage next.

3.2. Cournot Competition

Given that firms are identical, we can invoke symmetry within a particular merger m. The profit function per m-firm merger is then given by:

$$\Pi_m = m(q_i((a - b\frac{q_i}{L} - g(m-1)\frac{q_i}{L} - g\sum_{h=m+1}^{n}\frac{q_h}{L}) - n_m) - f_m) \qquad (3.2)$$

Taking the partial derivative with respect to q_i, we get:

$$\partial\Pi_m / \partial q_i = m((b - (m-1)g)\frac{q_i}{L} + ((a - b\frac{q_i}{L} - g(m-1)\frac{q_i}{L} - g\sum_{h=m+1}^{n}\frac{q_h}{L}) - n_m)) \qquad (3.3)$$

Solving the first-order condition for production quantity per firm q_{im} then yields the Cournot quantity reaction function:

$$q_{im}^{Q-m} = \frac{L(a - n_m)}{2(b + (m-1)g)} - \frac{gQ^{-m}}{2(b + (m-1)g)} \qquad (3.4)$$

where $Q^{-m} = \sum q_h = Q - m\, q_{im}$.[24] Replacing Q^{-m} with Q, this can also be written as a quantity replacement function:

$$q_{im}^{Q} = \frac{L(a - n_m)}{2b + (m-2)g} - \frac{g}{2b + (m-2)g}Q \qquad (3.5)$$

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[22] Compare Perry/Porter (1985), Salant/Switzer/Reynolds (1983), Böckem (2002), and others.

[23] On coalition formation, see Ray/Vohra (1999) and, e.g. the literature discussion in Levy (2004).

[24] The Cournot stage of this game is an aggregative game since the underlying utility is completely symmetric. On aggregative games and their properties, see e.g. Cornes/Hartley (2005).

Since the reaction function (3.4) is continuously differentiable and monotonously decreasing with a slope between zero and minus one half for all parameter constellations, a Cournot equilibrium exists for any possible merger partition among the *n* firms in the market.

Let a merger partition *M* be given by:

$$M = \left\{ \{m_1,\dots,m_k\} \middle| \sum_{j=1}^{k} m_j = n \right\} \tag{3.6}$$

For any merger partition *M*, the Cournot equilibrium aggregate market quantity can then be expressed as:

$$Q^M = \frac{\sum_{m_j \in M} m_j \frac{L(a - n_{m_j})}{2b + (m_j - 2)g}}{1 - \sum_{m_j \in M} m_j \frac{g}{2b + (m_j - 2)g}} \tag{3.7}$$

Combining equations (3.5) and (3.7), we then get the Cournot equilibrium quantity per firm *i* in merger *j*:

$$q_i^j = \frac{L(a - n_{m_j})}{2b + (m_j - 2)g} - \frac{g \sum_{m_k \in M} m_k \frac{L(a - n_{m_k})}{2b + (m_k - 2)g}}{2b + (m_j - 2)g + \sum_{m_k \in M} m_k g \frac{2b + (m_j - 2)g}{2b + (m_k - 2)g}} \tag{3.8}$$

3.3. Cournot Competition with Symmetric Mergers

Since the demand structure is completely symmetric and firms are a-priori identical, individual firms will only care about the number of firms in the merger they prefer to join as well as about all other mergers' sizes, but not about individual identities. As we will show further below, this implies the existence of equilibria that are symmetric in merger sizes under certain conditions. While symmetric merger equilibria are not possible for any $n > 4$ in the standard Cournot case without cost effects of mergers[25], they will result if mergers reduce variable costs of merger-insiders sufficiently.

Therefore, it is useful to characterize Cournot equilibria for the case of *n/m* symmetric mergers of size *m* each. Such a Cournot equilibrium implies that $Q = n\, q_{im}^{Q}$; using this relationship to solve equation (3.5) yields then the following symmetric equilibrium solution:

$$q_i^m = \frac{L(a - n_m)}{(2b + (n + m - 2)g)} \tag{3.9}$$

with these equilibrium quantities, prices and single-firm profits will then be:

$$p_i^m = \frac{(a + n_m)(b + mg)}{(2b + (n + m - 2)g)} \tag{3.10}$$

$$\Pi_i^m = \frac{L(a - n_m)^2 (b + mg)}{(2b + (n + m - 2)g)^2} - f_m \tag{3.11}$$

Note that q_i^m is the quantity produced and $\tilde{O}_i^m$ is the profit gained by a single firm (within each merger m).

3.4. Individual Merger Choice and Merger Partitions

Define a residual partition to the (potential) merger *j* as:

$$M^{-j} = \left\{ \{m_1, \ldots, m_{k-1}\} \middle| \sum_{l=1}^{k-1} m_l = n - m_j \right\} \tag{3.12}$$

For any merger partition $M = \{M^{-j}, m_j\}$ involving *k* mergers, we can write *k* representative single-product profit functions of the form:

$$\Pi_i^j(M) = q_i^j(M)(p_i(q_i^j(M), Q^M(M)) - n_{m_j}(m_j)) - f_{m_j}(m_j)),\ j = 1, \ldots k \tag{3.13}$$

The type of equilibria we will receive will also depend on the equilibrium concept applied and the behavioral assumptions used. In the simplest case, we can assume that all firms act non-cooperatively, even within potential mergers, and that they chose mergers to join simultaneously such that a subgame-perfect Nash equilibrium results. A sufficient condition for any such Nash equilibrium between non-cooperative firms, where size of the merger to be joined is the individual strategic variable, is then given by:

$$\begin{gathered} \Pi_i^j(\{M^{-j*}, m_j^*\}) \geq \Pi_i^j(\{\{M^{-j*}, (m_j^* - 1)\}, 1\}) \\ \forall m_j^* \in M \end{gathered} \tag{3.14}$$

[25] Ray/Vohra (1999), section 3.3 and Theorem 3.6.

Proposition 1

If (3.14) holds for all $m_j^* \in M^*$, then M^* is a Nash-equilibrium merger partition.

Proof: Due to the definition of Nash-equilibrium, M^{-j^*} will be unchanged regardless of the action of any firm within the merger m_{j^*}. If one firm leaves merger m_{j^*}, then that firm becomes a singleton and merger m_{j^*} loses one member. As a singleton, the leaving firm will make less profit than if it remained in the merger. QED

Note that Proposition 1 holds for symmetric and asymmetric merger partitions alike. Since any (candidate) equilibrium is a partition, the strategy space of each firm is bounded upwards at the equilibrium. However, this implies that firms do not coordinate even within an existing merger coalition if breaking up that coalition would benefit all of them. This assumption is arguably too strong. Allowing for coordination to dissolve or reconfigure within existing mergers then requires in addition to (3.14) that each merger within an equilibrium merger partition M^* is coalition-proof. A sufficient condition for this is:

$$\begin{aligned} &\Pi_i^j(\{M^{-j*}, m_j^*\}) \geq \Pi_i^j(\{M^{'-k*}, m_k^*\}) \\ &\forall m_k^* \leq m_j^*, \\ &\forall m_j^* \in M, \\ &\forall M' s.t. M^{-j*} \cap M^{'-k^*} = M^{-j*} \end{aligned} \tag{3.15}$$

Proposition 2

If (3.15) holds for all $m_j^* \in M^*$, then M^* is a Nash-equilibrium merger partition with individually coalition-proof mergers.

Proof: The equilibrium property follows from Proposition 1, since condition (3.15) fully contains condition (3.14). Coalition-proofness follows since the right-hand side of (3.15) covers all possible sub-coalitions of any merger m_{j^*}. $\in M^*$. QED

Condition (3.15) does not guarantee that any individually coalition-proof Nash equilibrium in mergers is also a stable coalition structure if renegotiation between equilibrium mergers were allowed. It is not ruled out that any such equilibrium may be Pareto-dominated by other existing Nash-equilibria. Still condition (3.15) is too general for our purposes, since the number of sub-coalitions to be checked becomes large very quickly with increasing merger size.

If (3.14) holds, then (3.15) will be satisfied if the following holds:

$$
\begin{aligned}
&(a)\frac{\partial\Pi_i^j(\{M^{-j*}, m_j\})}{\partial m_j} > 0 \forall m_j : 1 < m_j \le m_j^*, \\
&(b)\frac{\partial\Pi_i(\{M^{-j*}, m_j\})}{\partial m_j} > \frac{\partial\Pi_i^j(\{M^{-j*}, m_j\})}{\partial m_l} \forall m_j, \le m_j^*, m_l \le n, l \ne j \qquad (3.16) \\
&\forall m_j^* \in M
\end{aligned}
$$

In (3.16), condition (a) requires that profits are strictly increasing in merger size, while condition (b) requires that the profit change resulting from a change of own-merger size is always larger than the effect resulting from compensating changes in any other merger.

Proposition 3

Assume that (3.14) and (3.16) hold for all $m_j^* \in M^*$ and M^* is a symmetric merger partition with k mergers. Then there exists no other merger partition with $k' \geq k$ mergers that Pareto-dominates M^*.

Proof: Any alternative merger partition M' with $k' \geq k$ mergers would need to include at least one merger with $m_j' < m_j^*$, resulting in lower profits for the firms in this merger. QED

Any merger coalition satisfying (3.14) and (3.16) cannot be Pareto-dominated by some other coalition involving one or more smaller mergers. It follows, that symmetric merger coalitions, if at all, can only be dominated by coalitions that weakly increase merger sizes for all firms involved.

Corollary 1

(a) Assume that (3.14) and (3.16) holds for full cartelization, i.e. all M^* contains one merger only with m_j^* = n. Then M^* is Pareto-efficient, i.e. M^* is a stable coalition structure. Furthermore, any other symmetric merger partition, is Pareto-dominated, i.e. it is not a stable coalition structure.[26]

[26] See also Ray/Vohra (1999), section 3.

Proof: Result (a) follows directly from Proposition 3. For result (b), Any alternative symmetric merger partition *M'* with *k' > 1* would involve strictly smaller mergers. Moving to full cartelization therefore strictly increases profits of all firms. QED

However, asymmetric merger coalitions are still possible, even with identical firms. If firms are identical, asymmetric equilibria require that mergers are "corner solutions" for at least *(k-1)* of the *k* mergers in equilibrium. Corner solutions here mean that condition (a) in (3.16) potentially continues to be satisfied for merger sizes larger than the equilibrium merger size. This is true, in particular, when condition (a) in (3.16) is satisfied for all merger sizes less than or equal to market size n (the total number of firms in the market).

$$\begin{aligned}&(a)\frac{\partial \Pi_i^j(\{M^{-j*},m_j\})}{\partial m_j} > 0 \forall m_j \le m_j^*, m_j^* \le n,\\&(b)\frac{\partial \Pi_i(\{M^{-j*},m_j\})}{\partial m_j} > \frac{\partial \Pi_i^j(\{M^{-j*},m_j\})}{\partial m_l} \forall\forall m_j \le m_j^*, m_j^* \le n, m_l \le n, l \ne j\end{aligned} \tag{3.17}$$

On the other hand, we can have "internal merger equilibria" if the payoff functions of individual firms are globally maximized at the equilibrium merger size. In that case, larger mergers attainable through fusions with additional firms from other mergers can only lead to less profits per firm. Hence, reducing the number of mergers does not lead to an increase in merger size, since members of former mergers would not be welcome to join any other remaining merger.

$$\begin{aligned}&(a)\frac{\partial \Pi_i^j(\{M^{-j*},m_j^*\})}{\partial m_j} = 0 \forall m_j \le m_j^*, m_j^* \le n,\\&(b)\frac{\partial^2 \Pi_i^j(\{M^{-j*},m_j\})}{\partial {m_j}^2} < 0 \forall m_j \le m_j^*, m_j^* \le n,\\&(c)\frac{\partial \Pi_i(\{M^{-j*},m_j\})}{\partial m_j} > \frac{\partial \Pi_i^j(\{M^{-j*},m_j\})}{\partial m_l} \forall\forall m_j \le m_j^*, m_j^* \le n, m_l \le n, l \ne j\end{aligned} \tag{3.18}$$

Proposition 4

Assume that (3.14) and (3.18) hold for all $m_j^* \in M^*$ with k symmetric mergers. Then M^* is a (symmetric) Nash equilibrium merger partition. It is also Pareto-optimal, hence a stable coalition structure.

Proof: From (3.14) and (3.18) (a) and (b), profit is globally concave in own merger size for the whole range of possible merger sizes in an *n*-firm market. Given that, it follows from (3.18) (c) that profit is also single-peaked across all possible residual merger configurations. QED

Below in Section 5, we present a simplified model with independent demands, i.e. without cross-price effects, satisfying conditions (3.14) and (3.18) and leading to a Pareto-optimal, internal symmetric Nash equilibrium merger partition, i.e. a symmetric stable coalition structure.

4. Mergers or Divestitures in a Heterogeneous-Goods Demand System without Cost Effects of Mergers

Before further analyzing merger equilibria, it is instructive to investigate what incentives firms within an n-firm oligopoly have with respect to a possible divestiture rather than joining a merger. Below we present this analysis for our demand system, i.e. for heterogeneous goods and linear demand. Note that for a Cournot oligopoly in homogeneous goods, this problem has already been addressed by Baye/Crocker/Ju (1996); they also present a generalization for demand functions satisfying Novshek's (1985) conditions for the existence of a Cournot equilibrium.

4.1. Incentive for a Single Firm to Join a Merger or to Divest

In the simplest possible setup with constant unit cost (i.e. constant marginal costs and no fixed costs), "divesting" for a single firm could be the decision to split up into a number *j* of additional independent firms rather than staying single or joining a merger. When this is done, the new divested entity owns *(j+1)* independent firms and their profits and the market has increased to size *(n+j)*. If the firm hypothetically joined a merger of all n firms, then it would receive an n^{th} share of that markets monopoly profits.

As we know already, joining a monopoly merger always increases profits in comparison to the initial n-firm oligopoly. However, for a single firm, marginal profits from merging are negative since the (negative) market-share effect dominates the (positive) market-concentration effect. Since divesting into additional firms reverses direction, the signs of these effects are reversed and establishing additional firms marginally increases profits. Eventually, the (now negative) market concentration effect will dominate again, so that there is another local maximizer at some number $j*$ of divested additional firms. Below, we compare profits at these two local maximizers after calculating the optimizing number of divested firms.

An individual firm i's profits in an n-firm market is given as:

$$\Pi_i^n = \frac{L(a-n)^2 b}{(2b+(n-1)g)^2} - f \tag{4.1}$$

If a single firm divests into j additional firms, the sum of the resulting profits, the divestiture profits, are:

$$\Pi_j^n = (1+j)\Pi_i^{(n+j)} = \frac{(1+j)L(a-n)^2 b}{(2b+(n+j-1)g)^2} - f \tag{4.2}$$

Maximal divestiture profits are given by choosing the number of additional firms j such that:

$$j^* = (n-3) + 2\frac{b}{g} \tag{4.3}$$

Hence optimal divestiture profits will be:

$$\Pi_{j^*}^n = \frac{L(a-n)^2 b}{(2b+(n-2)g)^2} - f \tag{4.4}$$

In contrast, profits gained from participating in a merger of all market firms would be given by:

$$\Pi_m^n = \frac{L(a-n)^2}{4b+(n-1)g} - f \tag{4.5}$$

For $f = 0$, the ratio of divestiture profits to merger participation profits is then:

$$\left(\frac{\Pi_{j^*}^n}{\Pi_m^n}\right) = \left(\frac{b}{g}\right)\frac{\left(b+(n-1)g\right)}{\left(2b+(n-2)g\right)} \tag{4.6}$$

Note that for all $0 < ? = \beta$, this ratio will be greater than or equal to 1; it will be exactly equal to one for $? = \beta$. Consequently, for any heterogeneous-goods demand system with constant unit costs, the following will hold:

1.) A single firm will always be better off divesting than staying single, ceteris paribus.

2.) The optimal number of divested firms is (n-3) +2 ß/?.

3.) When choosing to divest into the optimal number of new firms, the owner of these new firms is (collectively) at least as well off as if he/she had participated in an n-firm (monopoly) merger!

4.) Given that firms always prefer divesting to a single monopoly-merger, they also always prefer divestiture to any (local) symmetric merger equilibrium with (n/m_1) mergers merging m_1 firms each.

4.2. Joint Decisions of all Firms: A Nash-Equilibrium in Divestitures

Since $\partial j^*/\partial n = 1$ and $\partial n/\partial j^* = 1$, there can be no finite Nash equilibrium in divestitures without cost increasing in j*.[27]

However, with a fixed cost of f per new single-product firm, an internal divestiture equilibrium exists. Let n_j be the total number of resulting firms in the divestiture equilibrium, while n is the number of divesting initial firms, i.e. the number of owners. Let the operating profits of a single firm in equilibrium be given by:

$$\Pi_j^{n_j} = \Pi_i^{(n+j)} = (1+j)\frac{L(a-n)^2 b}{(2b+(n_j+j-1)g)^2} \tag{4.7}$$

Setting the operating profits equal to the fixed setup cost of the marginal firm and solving for n_j will yield the Nash-equilibrium number of firms to divest as $(n_j^*/n)-1$ where n_j^* is given by:

$$n_j^* = \frac{-2b + g^2 f + \sqrt{a^2bg^2f - 2abg^2nf + bg^2n^2f}}{g^2 f} \tag{4.8}$$

Constant f requires zero-profits in equilibrium, since equilibrium is reached when each owner sets market operating profits of the marginal new firm equal to marginal f. With f increasing in n, positive overall profits of the multi-firm owners are possible! However, any divestiture equilibrium of this kind will now be Pareto-dominated by a single monopoly merger. Proposition 1 in Baye/Crocker/Ju (1996) already established existence of the equilibrium result in equation (4.8) above for the homogeneous-goods case, where the solution of equation (7), d^*, corresponds to n_j^* in our equation (4.8).

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[27] This result corresponds to Baye/Crocker/Ju (1996), Proposition 2, for homogeneous goods.

5. Mergers and Divestitures under Monopolistic Competition

This section presents internal solutions with several symmetric mergers in a monopolistic market, where variable costs fall with merger size while fixed costs per single-product unit rise with merger size. Explicit analytical solutions for this model are available, because we set *? = 0* and therefore individual single-product inverse demand collapses to $p_i = a - b\ q_i$. Therefore, this model can serve as a benchmark.

Mergers are always profitable in comparison to staying an outsider because of the resulting reduction of variable costs. Since the products exhibit zero-substitutability, individual merger choices are independent of the size of other mergers in the market, since own market demands are not affected. Likewise, outsiders' market shares are not affected so they cannot gain from any mergers among other firms.

Cournot quantity reaction functions and equilibrium quantities are identical since they do not depend on other mergers' size:

$$q_i^{rcm} = \frac{L(a - n_m)}{2b} = q_i^{cm} = \frac{L(a - n_m)}{2b} \tag{5.1}$$

Cournot equilibrium prices and profits are then:

$$p_i^{cm} = \frac{a + n_m}{2} \tag{5.2}$$

$$\Pi_i^{cm} = \frac{L(a - n_m)^2}{4b} - f_m \tag{5.3}$$

Assuming L=1 and setting cost functions equal to $n_m = n/m$ and $f_m. = (f\ \ m)$, the Cournot equilibrium then becomes:

$$q_i^{cm} = \frac{ma - n}{2mb},\ p_i^{cm} = \frac{ma + n}{2m} \tag{5.4}$$

$$\Pi_i^{cm} = \frac{(ma - n)^2}{4b\,(m)^2} - fm \tag{5.5}$$

In order to find the optimal merger choice, we solve for the first-order condition and check the second-order condition:

$$\frac{\partial}{\partial m}\Pi_i^{cm} = \frac{a\,(ma - n)}{2b\,(m)^2} - \frac{(ma - n)^2}{2b\,(m)^3} - f = 0 \tag{5.6}$$

$$\frac{\partial^2}{\partial m^2}\Pi_i^{cm} = \frac{a^2}{2b\,(m)^2} - \frac{2a\,(ma - n)}{b\,(m)^3} + \frac{3(ma - n)^2}{2b\,(m)^4} \leq 0 \text{ for } a \text{ ³ } 1.5\, n \tag{5.7}$$

The second-order condition is satisfied, given that market size is large relative to marginal costs. Then optimal merger size chosen by any merger will be given by the solution to the first-order condition in m:

$$m^* = \frac{an}{2(6)^{1/3}\left(9b^2n^2f^2 + \sqrt{3}\sqrt{27b^4n^4f^4 - 2a^3b^3n^3f^3}\right)^{1/3}} + \frac{\left(9b^2n^2f^2 + \sqrt{3}\sqrt{27b^4n^4f^4 - 2a^3b^3n^3f^3}\right)^{1/3}}{2(6)^{2/3}\,bf} \tag{5.8}$$

When the parameter restriction for the second-order condition is satisfied, we also have:

$$\frac{\partial}{\partial a}m^* > 0\,,\ \frac{\partial}{\partial n}m^* > 0\,,\ \frac{\partial}{\partial f}m^* < 0 \tag{5.9}$$

In the model presented in this chapter, endogenous merger size: 1.) increases with market size (α); 2.) increases with variable costs (ν), since that also increases economies of scope; 3.) decreases with fixed costs (F), since that also increases diseconomies of scope.

Hence, a merger wave followed by a divestiture wave can happen in each of the following three ways:

A.) An increase in market size (α) where the ex-post realization is smaller than the ex-ante expected increase.

B.) An increase in variable cost (ν) where the ex-post realization is smaller than the ex-ante expected increase.

C.) A decrease in fixed cost (F) where the ex-post reduction is smaller than the ex-ante expected decrease.

6. Conclusions

In this section, we recapitulate our findings obtained so far; these results can be summarized as follows.

If decisions to join a merger are Nash, i.e. non-cooperative decisions to cooperate are made by individual firms, then there may exist an internal Pareto-optimal multi-merger equilibrium under certain conditions. This solution requires both strong economies of scope for small merger sizes (mergers reduce cost per single-product firm) and increasing diseconomies of scope for large merger sizes (then cost per firm must eventually increase). These conditions are sufficient to have mergers being more profitable than staying outside a merger as a single-product firm. However, in order to have a result where mergers reduce profits of outsiders, cost economies of scope must decrease variable costs sufficiently. Hence some cost synergies are necessary to obtain stability of equilibria in the sense that no firm has an incentive to wait for others to form mergers rather than joining.

In the resulting equilibrium, endogenous merger size is a function of market parameters as well as cost synergy parameters. Hence anticipated changes in market size and/or cost synergies attainable through mergers will lead to reconfigurations of merger sizes. If ex-ante expectations about merger-promoting changes are not fully realized ex-post, merger waves will be followed by divestiture waves in our model. In addition, firm valuation - based on ex-ante expectation - may increase while actual profits and efficiency of the merged entity - according to the ex-post realization - may fall.

As future research will show, these results hold in principle no matter what the degree of substitutability between products: from monopolistic to homogenous products. However, the higher the substitutability, the higher need to be both the scope economies (to make a merger profitable at all) and the scope diseconomies (to yield an internal solution) – internal here means more than just one merger in equilibrium! With zero-substitutability monopolistic competition, results will be totally driven by cost effects of mergers.

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Appendix A: A Continuous Interval of Firms

Our analysis remains virtually unchanged, if we assume the range of single-product firms to be an interval [1, n] rather than a series of countable products. To see this, we introduce integrals into the demand and profit functions. We then have:

$$\Pi_i = (q_1((a - b\frac{q_1}{L} - g(m)\frac{q_1}{L} - g\int_{m_{12}}^{n}\frac{q_h}{L}dh) - n_m) - f_m) \tag{A.1}$$

Replacing $\int q_h$ with $(Q - m\, q_i)$ and noting that $Q = n\, q_i$, we obtain the quantity replacement function:

$$q_i^{cr} = \frac{L(a - n_m)}{2b + mg} - \frac{g}{2b + mg}Q \tag{A.2}$$

Cournot equilibrium with symmetric merger size would then be given by:

$$q_i^c = \frac{L(a - n_m)}{(2b + (n+m)g)} \tag{A.3}$$

$$p_i^c = \frac{(a + n_m)(b + mg)}{(2b + (n+m)g)} \tag{A.4}$$

$$\Pi_i^c = \frac{L(a - n_m)^2(b + mg)}{(2b + (n+m)g)^2} - f_m \tag{A.5}$$

Note that q_i^c is the quantity produced and $\prod_i^c$ is the profit gained by a single firm (within each merger m).

Appendix B: Demands in the Presence of Other Mergers and a Single-Firm Fringe

With one merger of size m_1 and n_2 mergers of size m_2, that the inverse demand functions can be written as:

$$p_1 = a - b\,\frac{q_1}{L} - g\,(m_1 - 1)\frac{q_1}{L} - g\,(n_2 m_2)\frac{q_2}{L} - g \sum_{\substack{h=m_1+\\ n_2 m_2+1}}^{n} \frac{q_h}{L} \tag{B.1}$$

$$p_2 = a - b\,\frac{q_2}{L} - g\,(m_1)\frac{q_1}{L} - g\,(n_2 m_2 - 1)\frac{q_2}{L} - g \sum_{\substack{h=m_1+\\ n_2 m_2+1}}^{n} \frac{q_h}{L} \tag{B.2}$$

$$p_j = a - b\,\frac{q_j}{L} - g\,(m_1)\frac{q_1}{L} - g\,(n_2 m_2)\frac{q_2}{L} - g \sum_{\substack{h=m_1+\\ n_2 m_2+2}}^{n} \frac{q_h}{L} \tag{B.3}$$

or

$$p_k = a - b\,\frac{q_k}{L} - g\,(m_1)\frac{q_1}{L} - g\,(n_2 m_2)\frac{q_2}{L} - g \int_{\substack{m_1+\\ n_2 m_2}}^{n} \frac{q_h}{L}\,dh\,,\ k = 1, 2, j \tag{B.4}$$

where subscript 1 denotes merger 1, subscript 2 denotes each of the n_2 other mergers, and subscript h denotes each of the remaining single-product firms.

Appendix C: Merger Partial Derivatives of the Profit Function

Partially differentiating single-firm profits in merger m_j with respect to some merger size m_l yields:

$$\begin{aligned}
&\frac{\partial \Pi_i^j(M)}{\partial m_l} = \frac{\partial q_i^j(M)}{\partial m_l}\left(p_i\left(q_i^j(M), Q^M(M)\right) - n_{m_j}(m_j)\right) \\
&+ q_i^j(M)\,\frac{\partial p_i\left(q_i^j(M), Q^M(M)\right)}{\partial m_l} \\
&- q_i^j(M)\,\frac{\partial n_{m_j}(m_j)}{\partial m_l} - \frac{\partial f_{m_j}(m_j)}{\partial m_l}, \\
&\forall m_j, m_l \in M
\end{aligned} \tag{C.1}$$

For homogeneous goods ,i.e. $g = b$, we also have:

$$\frac{\partial p_i}{\partial m_l} = -b\,\frac{\partial q_i^j}{\partial m_l}\,\forall m_l \in M,$$

$$\frac{\partial q_i^j}{\partial m_j} = -\frac{\partial n_{m_j}/\partial m_j}{b\,m_j} - \frac{a - n_{m_j}}{b\,m_j^2} - \frac{\partial n_{m_j}/\partial m_j}{(k+1)b\,m_j} + \frac{ka - \sum_{m_k \in M} n_{m_k}}{b\,(k+1)m_j^2} \tag{C.2}$$

$\forall m_j \in M,$

$$\frac{\partial q_i^j}{\partial m_l} = -\frac{\partial n_{m_j}/\partial m_l}{(k+1)b\,m_j}\,\forall m_l \in M, l \neq j$$

Appendix D: No Cost Synergies – No Complete Merger Partition: Example 1

In a Cournot oligopoly with linear demand, homogeneous goods and constant cost, let $\{L = 1, (a\text{-}n) = 1, b = 1, f = 0\}$, then profits with n distinct firms are:

$$\Pi_n = \frac{1}{(n+1)^2} \tag{D.1}$$

Let P_n^m be profits of a single firm in a merger with m firms that competes in a market with a total of n competitors (including the merger). Then:

$$\begin{aligned}
&(a)\Pi_n^m = \frac{1}{m}\frac{1}{(n+1)^2}, \Pi_{10}^1 = \frac{1}{1}\frac{1}{121} = \frac{1}{121},\\
&(b)\Pi_2^1 = \frac{1}{9}, \Pi_2^9 = \frac{1}{9}\frac{1}{9} = \frac{1}{81} > \Pi_{10}^1,\\
&(c)\Pi_3^1 = \frac{1}{16}, \Pi_3^8 = \frac{1}{8}\frac{1}{16} = \frac{1}{128} < \Pi_{10}^1,\\
&(d)\Pi_n^m = \frac{1}{m}\frac{1}{(n+1)^2} < \Pi_{10}^1\\
&\forall (n,m): \{n+m=11, n \geq 3\}.
\end{aligned} \tag{D.2}$$

According to Ray/Vohra (1999), p. 302, Theorem 3.6., the unique stable coalition structure involves a single merger with nine firms and one singleton firm remaining. Any smaller coalition will collapse leaving everybody with the smaller profits P_{10}^1. Full cartelization is not sustainable, since one firm could leave without having to fear that the remaining coalition P_2^9 collapses, since $P_2^9 > P_{10}^1$. Note that this coalition structure is not a Nash equilibrium since $P_3^1 > P_2^9$! Similarly, any combination of several smaller mergers will also collapse.

Appendix E: A Complete Merger Partition with Variable Cost Synergies: Example 2

In a Cournot oligopoly with linear demand, homogeneous goods and constant cost, let $\{L = 1, a = 1, b = 1, f = 0, n_m = 0.5\,(z\text{-}m+1)/z, z = 10\}$.

In Figure E.1. below, m_1 is on the horizontal axis, profits are on the vertical axis, and the left panel shows the case of two mergers with possible outsiders, while the right panel shows the case with only a single merger with possible outsiders. In each panel, the lowest profit curve is that of the outsider singleton.

Panel (a) shows 10 firms and two (potential) mergers. For merger $m_2 = 5$, profits of merger m_1 are strictly increasing in m_1, profits of merger m_2 are strictly decreasing in m_1 and profit of any firm staying outsider is strictly decreasing in m_1 for all $m_1 \leq 6$.

Note that for $M^* = \{5, 5\}$, Propositions 1, 2 and 3 hold! $M^* = \{5, 5\}$ is a Nash-equilibrium merger partition with individually coalition-proof mergers. However, it is Pareto-dominated by full cartelization, i.e. by $M^{**} = \{10\}$! Panel (b) shows 10 firms and one (potential) merger. Profits of merger m_1 are strictly increasing in m_1 while profit of any firm staying outsider is strictly decreasing in m_1 for all $m_1 \leq 10$. $M^{**} = \{10\}$ is a stable coalition structure!

Figure E.1: Profits with Variable Cost Synergies: Example 2