Empirical Rules of Thumb for Choice under Uncertainty

by

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Abstract

A substantial body of empirical evidence shows that individuals overweight extreme events and act in conflict with the expected utility theory. These findings were the primary motivation behind the development of the rank-dependent utility theory. The purpose of this paper is to demonstrate that some plausible empirical rules of thumb for choice under uncertainty can be rationalized by the rank-dependent utility theory.

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1. Introduction

The arithmetic mean was for a long period considered as a good rule of thumb for choice under uncertainty. However, as illustrated by the famous St. Petersburg paradox no one would pay any very large fee for a lottery that offers a prize of 2^{n-1} with a probability of 2^{-n}, even though its expected payoff in money terms is infinite. Daniel Bernoulli (1738) responded to the challenge from this paradox by introducing expected utility, instead of the arithmetic mean, as the proper criterion for choice under uncertainty. In particular, he suggested to employ the geometric mean as the certainty equivalent, that is, the amount of money which, when received with certainty, is considered equally attractive as the given lottery.

In order to develop a theoretical foundation for expected utility as a criterion for choice under uncertainty, it appears convenient to introduce probability distributions as a formalization of uncertainty. Then the individual's decision problem is to choose between distribution functions. Consequently, a preference relation defined on the family of distribution functions may represent an individual’s choice over uncertain prospects. An individual who adopts expected utility has to support certain behavioral axioms that imply a strict structure on the preference relation. In general, there are two approaches for a preference relation to have an expected utility representation, depending on whether one treats the distribution function as objective or subjective. Von Neumann and Morgenstern (1944) introduced the former approach by assuming that the agents know the distribution functions. An alternative approach proposed by Ramsey (1926) and Savage (1954) considers the distribution function as unknown and subjective to the economic agents.

Even though the expected utility representation has been credited for its normative appeal and convenient mathematical form, there is fairly strong empirical evidence in disfavor of this theory, whether it is considered objective or subjective. The criticism against the expected utility theory originates from Allais (1953) and his famous paradox. Similar experiments have been constructed by MacCrimmon (1968), Kahneman and Tversky (1979), and others, and they all found results inconsistent with the expected utility hypothesis. As a result, numerous alternative theories to the expected utility have been proposed, see e.g. Karni and Schmeidler (1991) for a survey of alternative theories for choice under uncertainty.

Since economic agents solely in exceptional cases are faced with known distribution functions their decisions have to rely on less informative data. At most, observations, which may be considered as independent draws from distribution functions, will be available. In that case the agent may start out with the objective approach and adopt a decision criterion together with conventional statistical inference theory to arrive at a decision. Alternatively, the agent may view the statistical decision problem as an integral part of his preferences. In that case the agent's beliefs over probability distributions form the basis for the choice behavior. However, it can be argued that the complexity of these rules will make it difficult for individuals to follow them, especially when complex computations are involved. It appears more likely that individuals normally base their decisions under uncertainty on simple criterions like maximin and maximax. For example, when individuals choose between lotteries, the lottery that offers the highest possible prize appears to be particular attractive, even though this lottery offers very few medium-size prizes. This is in line with a maximax-type of behavior. By contrast, there is extensive empirical evidence concerning the willingness to purchase insurance, which suggests a maximin-type of behavior. On the other hand, the widely observed coincidence between insurance and gambling discussed by Friedman and Savage (1948) may be due to a mixture of maximin- and maximax-type of behavior. Moreover, experimental evidence shows that individuals overweight extreme events. These findings were the primary motivation behind the development of the rank-dependent (anticipated) utility theory proposed by Quiggin (1982). This theory, which has been given a convincing intuitive justification by Diecidue and Wakker (2001), proves to be appropriate for explaining the purchase of insurance side by side with engaging in gambling. However, as for the alternative theories for choice under uncertainty the complexity of the corresponding decision criterion may make the theory less plausible unless there turns out to exist a simple empirical counterpart.

In this paper we assume that economic agents base their decisions on data which can be considered as independent outcomes from unknown distribution functions. Moreover, we assume that the agents utilize the available information in a way that corresponds to dividing the total sample of observations into an appropriate set of subsamples. Provided that the agents are primarily concerned with the smallest and/or the largest observations in each subsample three simple decision rules emerge. These are the averages of the smallest, the largest and the sum of the smallest and largest observations of each subsample. In Sections 2 and 3 we shall show that there is a close relationship between these rules of thumb and the theories of Quiggin (1982) and Yaari (1987).

2. Maximin- and maximax-type of behavior

Standard decision-criteria, like expected and rank-dependent utility, can be employed in situations where probabilities, either objective or subjective, can be assigned to the different states/outcomes. However, when the feasible information is insufficient for estimating the probability distributions economic agents have to rely on alternative criteria to expected and rank-dependent utility. Starting out from complete and partial uncertainty, Kelsey (1993) and Barrett and Pattanaik (1994) demonstrate the plausibility of maximin- and maximax-types of decision criteria in these cases. By contrast, this section discusses the plausibility of employing criteria based on extreme values when samples of independent observations from unknown distribution functions are feasible.

Without loss of generality we restrict the decision problem to the choice between two distribution functions F_1 and F_2. Assume that N independent outcomes have been generated from each of the distributions. Consider an agent that is primarily concerned with escaping losses or small outcomes. This agent may find it plausible to choose the distribution/prospect, which in the long run appears to offer the highest minimum value. We will demonstrate that this particular decision rule is consistent with the theory of Yaari (1987).

Assume that the agent divides each of the two data sets into r random sub-samples, each consisting of n observations. Thus, $N = nr$. Let $X_{i1}^k, X_{i2}^k, \ldots, X_{in}^k, i = 1, 2, \ldots, r$ be r random sub-samples of independent observations from $F_k, k = 1, 2$. As suggested above the individual prefers F_1 for F_2 if

$$\frac{1}{r}\sum_{i=1}^{r} \min_{j \le n} X_{ij}^1 \ge \frac{1}{r}\sum_{i=1}^{r} \min_{j \le n} X_{ij}^2 .$$

For notational simplicity we suppress the superscription k in the discussion. For large r we know that

$$\frac{1}{r}\sum_{i=1}^{r} \min_{j \le n} X_{ij} \tag{1}$$

approaches $E \min_{j \le n} X_{ij}$.

Now, since

$$\Pr\left(\min_{j \le n} X_{ij} \le x\right) = 1 - \left(1 - F(x)\right)^n \tag{2}$$

we get that

$$E \min_{j \le n} X_{ij} = \int x \, d P_{1,n}\left(F(x)\right), \tag{3}$$

where

$$P_{1,n}(t) = 1 - \left(1 - t\right)^n , \tag{4}$$

it follows that the rule of thumb (1) for large r is consistent with Segal's (1987b) proposal of concave weighting-functions, and moreover can be rationalized by Yaari's theory for choice under uncertainty. Note that the number of observations (n) in each sub-sample determines the degree of risk aversion. Moreover, the degree of risk aversion proves to increase with increasing n. Note that the number of sub-samples depends on the overall sample size (N) and the individual's degree of inequality aversion.

By contrast, consider an individual who is primarily concerned with the highest payoffs from a set of feasible distributions. Based on N observations from each of the distributions the decision rule may then be given by

$$\frac{1}{r}\sum_{i=1}^{r} \max_{j \le n} X_{ij} , \tag{5}$$

which for large r approaches

$$E \max_{j \le n} X_{ij} = \int \left(1 - F^n(x)\right) dx = \int x \, d\, P_{2,n}\left(F(x)\right), \qquad (6)$$

where

$$P_{2,n}(t) = t^n. \qquad (7)$$

Thus, for large r the rule of thumb (5) proves to have a theoretical basis of the type discussed by Yaari (1987). The convexity of $P_{2,n}$ shows that individuals who adopt (5) as basis for making decisions under uncertainty are risk lovers.

3. Decision-making based on the mid-range

Experimental evidence as well as observed behavior suggests that people who in many cases are risk averse appear to be willing to purchase lottery tickets as well. Allais (1953) and Edwards (1955, 1962) suggest that the explanation of this behavior may be due to a substitution of decision weights for probabilities. Moreover, Allais (1953) demonstrated that decision-making based on a weighted sum of utilities was consistent with experimental evidence. Later Quiggin (1982), Yaari (1987), Segal (1987a) and Quiggin and Wakker (1994) developed an axiomatic basis for various non-expected utility criteria. Quiggin (1981, 1982) refers to experimental evidence, which suggests that economic agents are solely local risk averse and overweighs extreme events. Based on a survey of risk attitudes amongst Australian farmers Quiggin (1981) found indication of overweighing extreme outcomes with low probabilities. Although inconsistent with global risk-aversion, overweighing extreme events with low probabilities appear to be quite widespread, as illustrated by the widely observed simultaneous purchase of insurance and lottery tickets. In order to describe this type of behavior Quiggin (1982) introduced decision weights in a theory of choice under uncertainty. To deal with the propensity to overweighing extreme events a symmetric concave-convex weighting-function was incorporated into the criterion for choice under uncertainty. As will be demonstrated below adopting a particular version of this criterion turns out to be consistent with applying a rule of thumb based on the mid-range, i.e. the average of the lowest and largest values of sub-samples of a sample of observations. The average mid-range is defined by

$$\frac{1}{2r} \sum_{i=1}^{r} \left(\min_{j \le n} X_{ij} + \max_{j \le n} X_{ij} \right). \qquad (8)$$

For large r the mid-range approaches

$$\frac{1}{2}\left(E\min_{j\le n}X_{ij}+E\max_{j\le n}X_{ij}\right)=\int x\,d\,P_{3,n}\left(F(x)\right), \qquad (9)$$

where

$$P_{3,n}(t)=\frac{1}{2}\left(1-(1-t)^{n}+t^{n}\right). \qquad (10)$$

Note that $P_{3,n}$ assigns value 1/2 to probability 1/2, is symmetric about 1/2 and has a concave-convex functional form. Thus, the weighting function $P_{3,n}$ corresponds to the weighting functions that Quiggin (1982) introduced in order to deal with overweighing of extreme events. Note that the overweighing of extreme events increases when n increases. Thus, for large n the derivative $P'_{3,n}(t)$ is near 0 over most of its range, which means that the smallest and largest outcome will receive very high weights.

Although Segal (1986) has provided some plausible arguments against the condition $P\left(\frac{1}{2}\right)=\frac{1}{2}$, Quiggin and Wakker (1994) show that this condition is favorable in many respects. However, as suggested by Quiggin (1987) less restrictive concave-convex specifications of P can be obtained by assuming that P is concave on $[0,a]$ and convex on $[a,1]$ where $a\in\langle 0,1\rangle$. Empirical rules of thumb that are consistent with this type of weighting functions are obtained by dividing the set of outcomes into two different sets of random sub-samples. As above we firstly divide the overall sample into r random sub-samples. Next, we divide the overall sample into s random sub-samples, each consisting of m outcomes. Let $\tilde{X}_{i1},\tilde{X}_{i2},\ldots,\tilde{X}_{im},\ i=1,2,\ldots,s$ be $N=ms$ outcomes from F. Thus, we may introduce the following alternative rule of thumb to the average mid-range

$$\frac{1}{2}\left(\frac{1}{r}\sum_{i=1}^{r}\min_{j\le n}X_{ij}+\frac{1}{s}\sum_{i=1}^{s}\max_{j\le m}\tilde{X}_{ij}\right). \qquad (11)$$

For large r and s this statistic approaches

$$\frac{1}{2}\left(E\min_{j\le n}X_{ij}+E\max_{j\le m}\tilde{X}_{ij}\right)=\int x\,d\,P_{4,n,m}\left(F(x)\right) \qquad (12)$$

where

$$P_{4,n,m}(t)=\frac{1}{2}\left(1-(1-t)^{n}+t^{m}\right).$$

When $n>m$ the concave curvature in the lower part of P is more strict than the convex curvature in the upper part of P. Thus, in this case the agent will give larger weight to the worst outcomes than to the best outcomes.

Note that the decision criteria defined by (9) and (12) may be considered as special cases of Quiggin's rank-dependent utility model since utility is linear. However, by introducing a concave utility function U followed by replacing each outcome X_{ij} in (8) and (11) by $U(X_{ij})$ and $\tilde{X}_{ij}$ in (11) by $U(\tilde{X}_{ij})$, the more general rank-dependent utility form emerges in (9) and (12). Even though this model proves to possess several attractive properties, see Chew et al. (1987) and Quiggin (1992), the simplicity exhibited by the rule of thumb defined by (8) and (11) is partly lost. Since most individuals probably base their decisions on simple rules of thumb and moreover by paying particular attention to and treasuring up extreme events the criteria defined by (8) and (11) appear plausible. By considering the r and s sub-samples of observations as the feasible information, the individual is supposed to exclusively preserve the knowledge of the lowest and the largest value of each sub-sample.

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