Stabilized Feed-Back Amplifiers

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Classic Paper

This paper describes and explains the theory of the feed-back principle and demonstrates how stability of amplification, reduction of modulation products, and certain other advantages follow when stabilized feed-back is applied to an amplifier. The underlying principle of design by means of which "singing" is avoided also is set forth. The paper concludes with some examples of results obtained on amplifiers which have been built employing this new principle.

Due to advances in vacuum-tube development and amplifier technique, it now is possible to secure any desired amplification of the electrical waves used in the communication field. When many amplifiers are worked in tandem, however, it becomes difficult to keep the over-all circuit efficiency constant, variations in battery potentials and currents, small when considered individually, adding up to produce serious transmission changes for the over-all circuit. Furthermore, although it has remarkably linear properties, when the modern vacuum tube amplifier is used to handle a number of carrier telephone channels, extraneous frequencies are generated which cause interference between the channels. To keep this interference within proper bounds involves serious sacrifice of effective amplifier capacity or the use of a push-pull arrangement which, while giving some increase in capacity, adds to maintenance difficulty.

However, by building an amplifier whose gain is made deliberately, say 40 decibels higher than necessary (10000 fold excess on energy basis) and then feeding the output back to the input in such a way as to throw away the excess gain, it has been found possible to effect extraordinary improvement in constancy of amplification and freedom from nonlinearity. By employing this feed-back principle, amplifiers have been built and used whose gain varied less than 0.01 db with a change in plate voltage from 240 to 260 volts and whose modulation products were 75 db below the signal output at full load. For an amplifier of conventional design and comparable size this change in plate voltage would have produced about 0.7 db variation while the modulation products would have been only 35

db down; in other words, 40 db reduction in modulation products was effected. (On an energy basis the reduction was 10 000 fold.)

Stabilized feed-back possesses other advantages including reduced delay and delay distortion, reduced noise disturbance from the power supply circuits and various other features best appreciated by practical designers of amplifiers.

It is far from a simple proposition to employ feedback in this way because of the very special control required of phase shifts in the amplifier and feedback circuits, not only throughout the useful frequency band but for a wide range of frequencies above and below this band. Unless these relations are maintained, singing will occur, usually at frequencies outside the useful range. Once having achieved a design, however, in which proper phase relations are secured, experience has demonstrated that the performance obtained is perfectly reliable.

The carrier-in-cable system dealt with in a recent Institute paper (Carrier in Cables by A. B. Clark and B. W. Kendall. A.I.E.E. Trans., Dec. 1933, p. 1050) involves many amplifiers in tandem with many telephone channels passing through each amplifier and constitutes, therefore, an ideal field for application of this feed-back principle. A field trial of this system was made at Morristown, New Jersey, in which 70 of these amplifiers were operated in tandem. The results of this trial were highly satisfactory and demonstrated conclusively the correctness of the theory and the practicability of its commercial application.

CIRCUIT ARRANGEMENT

In the amplifier of Fig. 1, a portion of the output is returned to the input to produce feed-back action. The upper branch, called the μ circuit, is represented as containing active elements such as an amplifier while the lower branch, called the β-circuit, is shown as a passive network. The way a voltage is modified after once traversing each circuit is denoted μ and β, respectively, and the product, $\mu\beta$, represents how a voltage is modified after making a single journey around amplifier and feed-back circuits. Both μ and β are complex quantities, functions of frequency, and in the
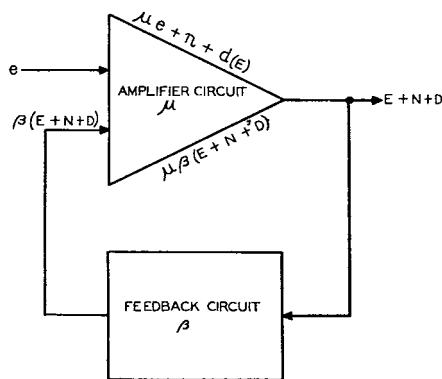
Fig. 1. Amplifier system with feed-back. e. Signal input voltage. μ. propagation of amplifier circuit. μe. signal output voltage without feed-back. n. noise output voltage without feed-back. d(E). distortion output voltage without feed-back. β. propagation of feed-back circuit. E. signal output voltage with feed-back. N. noise output voltage with feed-back. D. distortion output voltage with feed-back. The output voltage with feed-back is E + N + D and is the sum of $\mu e + n + d(E)$, the value without feed-back plus $\mu\beta[E + N + D]$ due to feed-back. $E + N + D = \mu e + n + d(E) + \mu\beta[E + N + D]$ [E + N + D]$(1 - \mu\beta) = \mu e + n + d(E)$ E+N+D $= \frac{\mu e}{1-\mu\beta} + \frac{n}{1-\mu\beta} + \frac{d(E)}{1-\mu\beta}$ If $|u\beta| \gg 1$, $E \doteq \frac{e}{\beta}$. Under this condition the amplification is independent of μ but does depend upon β. Consequently the over-all characteristic will be controlled by the feed-back circuit which may include equalizers or other corrective networks.
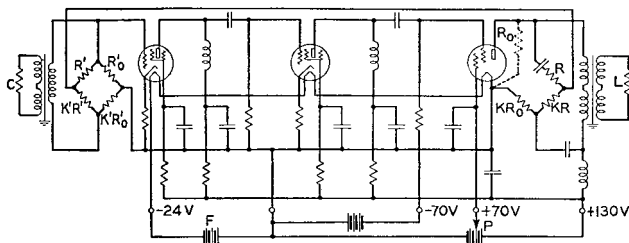

Fig. 2. Circuit of a negative feed-back amplifier.

generalized concept either or both may be greater or less in absolute value than unity: (μ is not used in the sense that it is used sometimes, namely, to denote the amplification constant of a particular tube, but as the complex ratio of the output to the input voltage of the amplifier circuit).

Fig. 2 shows an arrangement convenient for some purposes where, by using balanced bridges in the input and output circuits, interaction between the circuits that connect to the input and output is avoided. Thereby feed-back action and amplifier impedances are made independent of the properties of circuits connected to the amplifier.

GENERAL EQUATION

In Fig. 1, β is zero without feed-back and a signal voltage, e_0, applied to the input of the μ-circuit produces an output voltage. This is made up of what is wanted, the amplified signal, E_0, and components that are not wanted, namely, noise and distortion designated N_0 and D_0 and assumed to be generated within the amplifier. It is further assumed that the noise is independent of the signal and the distortion of modulation a function *only of the signal output*.

Using the notation of Fig. 1, the output without feed-back may be written as

$$E_0 + N_0 + D_0 = \mu e_0 + n + d(E_0) \qquad (1)$$

where zero subscripts refer to conditions without feed-back.

With feed-back, β is not zero and the input to the μ-circuit becomes $e_0 + \beta(E + N + D)$. The output is $E + N + D$ and is equal to $\mu[e_0 + \beta(E + N + D)] + n + dE$ or

$$E + N + D = \frac{\mu e_0}{1 - \mu\beta} + \frac{n}{1 - \mu\beta} + \frac{d(E)}{1 - \mu\beta}. \qquad (2)$$

In the output signal, noise and modulation are divided by $(1 - \mu\beta)$, and assuming $|1 - \mu\beta| > 1$, all are reduced.

CHANGE IN GAIN DUE TO FEED-BACK

From (2), the amplification with feed-back equals the amplification without feed-back divided by $(1 - \mu\beta)$. The effect of adding feed-back, therefore, usually is to change the gain of the amplifier and this change will be expressed as

$$G_{CF} = 20\log_{10}\left|\frac{1}{1 - \mu\beta}\right| \qquad (3)$$

where G_{CF} is *db change in gain due to feed-back*. As a quantitative measure of the effect of feed-back $1/1 - \mu\beta$ will be used and the feed-back referred to as *positive feed-back* or *negative feed-back* according as the absolute value of $1/1 - \mu\beta$ is greater or less than unity. Positive feed-back increases the gain of the amplifier; negative feed-back reduces it. The term feed-back is not limited merely to those cases where the absolute value of $1/1 - \mu\beta$ is other than unity.

From $\mu\beta = |\mu\beta|\Phi$ and (3), it may be shown that

$$10 - \frac{G_{CF}}{10} = 1 - 2|\mu\beta|\cos\Phi + |\mu\beta|^2 \qquad (4)$$

which is the equation for a family of concentric circles of radius $10 - G_{CF}/10$ about the point 1, 0. Fig. 3 is a polar diagram of the vector field of $\mu\beta = |\mu\beta|\Phi$. Using rectangular instead of polar coördinates, Fig. 4 corresponds to Fig. 3 and may be regarded as a diagram of the field of $\mu\beta$ where the parameter is db change in gain due to feed-back. From these diagrams all of the essential properties of feed-back action can be obtained such as change in amplification, effect on linearity, change in stability due to variations in various parts of the system, reduction of noise, etc. Certain significant boundaries have been designated similarly on both figures.

For example, boundary A is the locus of zero change in gain due to feed-back. Along this parametric contour line where the absolute magnitude of amplification is not changed by feed-back action, values of $|\mu\beta|$ range from zero to two and the phase shift, Φ around the amplifier and feed-back circuits equal $\cos^{-1}|\mu\beta/2$ and, therefore, lies between -90 deg and $+90$ deg. For all conditions inside or above this boundary, the gain with feed-back is increased; outside or below, the gain is decreased.
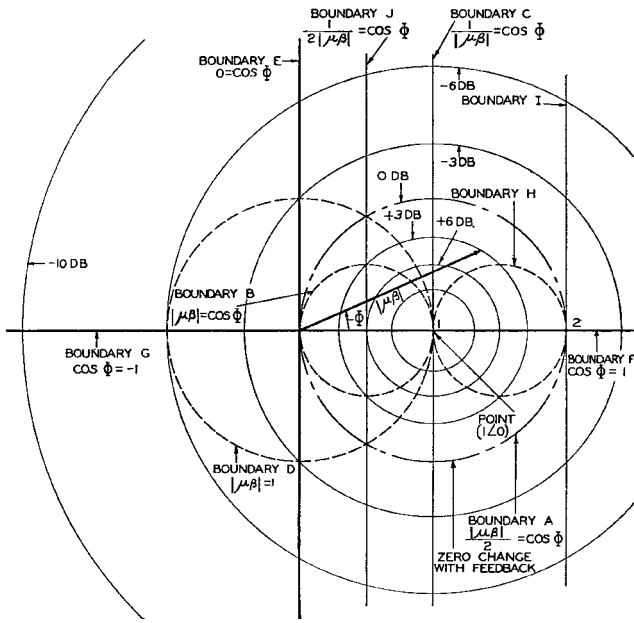
Fig. 3. The vector field of $\mu\beta$. The complex quantity $\mu\beta$ represents the ratio by which the amplifier and feed-back (or more generally μ and β) modify a voltage in a single trip around the closed path. First, there is a set of boundary curves indicated by letters which give either limiting or significant values of $|\mu\beta|$ and ϕ, Second, there is a family of curves in which db change in gain due to feed-back is the parameter. Boundaries. A. Conditions in which gain and modulation are unaffected by feed-back. B. Constant amplification ratio against small variations in $|\beta|$. Constant change in gain, $1/|1-\mu\beta|$, against variations in $|\mu|$ and $|\beta|$. Stable phase shift through the amplifier against variation in $\Phi\beta$. The boundary on which the stability of amplification is unaffected by feed-back.

STABILITY

From (2) $\mu e_o/1 - \mu\beta$ is the amplified signal with feed-back and $\mu/1 - \mu\beta$, therefore, is an index of the amplification. It is of course a complex ratio. It will be designated A_F and referred to as the amplification with feedback.

To consider the effect of feed-back upon stability of amplification, the stability will be viewed as the ratio of a change, δA_F, to A_F where δA_F is due to a change either in μ or β and the effects may be derived by assuming the variations are small

$$A_F = \frac{\mu}{1 - \mu\beta} \tag{5}$$

$$\left[\frac{\delta A_F}{A_F}\right]_\mu \doteq \frac{\left[\frac{\delta\mu}{\mu}\right]}{1 - \mu\beta} \tag{6}$$

$$\left[\frac{\delta A_F}{A_F}\right]_\beta \doteq \frac{\mu\beta}{1 - \mu\beta}\left[\frac{\delta\beta}{\beta}\right]. \tag{7}$$

If $\mu\beta \gg 1$, it is seen that μ or the μ-circuit is stabilized by an amount corresponding to the reduction in amplification and the effect of introducing a gain or loss in the μ-circuit is to produce no material change in the over-all amplification of the system; the stability of amplification as affected by β or the β-circuit is neither appreciably improved nor degraded since increasing the loss in the β-circuit raises the
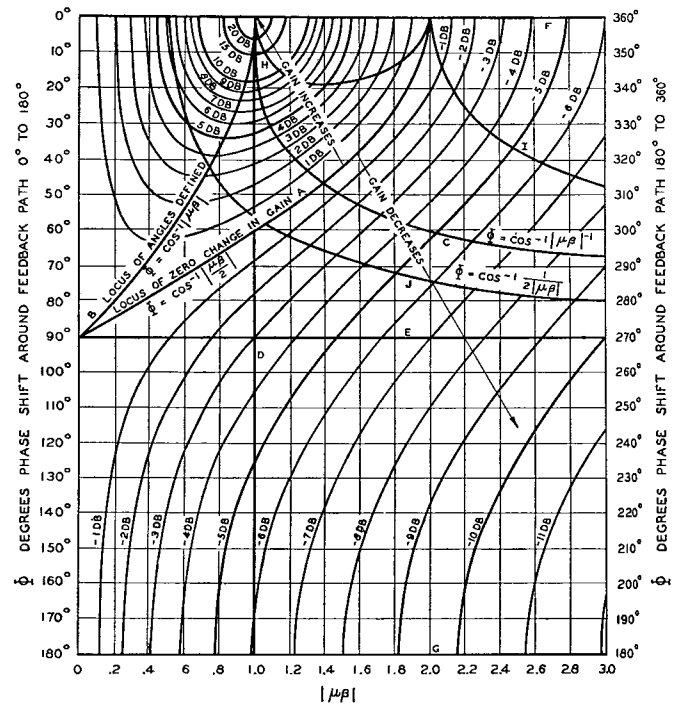

Fig. 4. Phase shift around the feed-back path plotted as a function of $|\mu\beta|$, absolute value of $\mu\beta$. C. Constant amplification ratio against small variations in $|\mu|$. Constant phase shift through amplifier against variations in $\Phi\mu$. The absolute magnitude of the voltage fed back $|\mu\beta|/|1 - \mu\beta|$ is constant against variations in $|\mu|$ and $|\beta|$. D. $|\mu\beta| = |$. E. $\Phi = 90°$. Improvement in gain stability corresponds to twice db reduction in gain. F. Constant amplification ratio against variations in Φ. G. Constant phase shift through the amplifier against variations in $|\mu|$ and $|\beta|$. H. Same properties as β. I. Same properties as E. J. Conditions in which $|\mu|/|1 - \mu\beta| = -1/|\beta|$ the overall-all gain is the exact negative inverse of the transmission through the β-circuit.

gain of the amplifier by an amount almost corresponding to the loss introduced and *vice versa*. If both μ and β are varied and the variations sufficiently small, the effect is the same as if each were changed separately and the two results then combined.

In certain practical applications of amplifiers it is the change in gain or ammeter or voltmeter reading at the output that is a measure of the stability rather than the complex ratio previously treated. The conditions surrounding gain stability may be examined by considering the absolute value of A_F. This is shown as follows:

Let (db) represent the gain in decibels corresponding to A_F. Then

$$(db) = 20\log_{10}|A_F|$$

$$\delta(db) \doteq 8.686\left[\frac{\delta|A_F|}{|A_F|}\right]. \tag{8}$$

To get the absolute value of the amplification:

$$\mu\beta = |\mu\beta|\Phi \tag{9}$$

$$|A_F| = \frac{|\mu|}{\sqrt{1 - 2|\mu\beta|\cos\Phi + |\mu\beta|^2}} \tag{10}$$

The stability of amplification which is proportional to the gain stability is given by

$$\left[\frac{\delta|A_F|}{|A_F|}\right]_{|\mu|} \doteq \frac{1-|\mu\beta|\cos\Phi}{|1-\mu\beta|^2}\left[\frac{\delta|\mu|}{|\mu|}\right] \quad (11)$$

$$\left[\frac{\delta|A_F|}{|A_F|}\right]_{|\beta|} \doteq \left|\frac{\mu\beta}{1-\mu\beta}\right|\left[\frac{\cos\Phi-|\mu\beta|}{|1-\mu\beta|}\right]\left[\frac{\delta|\beta|}{|\beta|}\right] \quad (12)$$

$$\left[\frac{\delta|A_F|}{|A_F|}\right]_{\Phi} \doteq -\left|\frac{\mu\beta}{1-\mu\beta}\right|\left[\frac{\sin\Phi}{|1-\mu\beta|}\right][\delta\Phi]. \quad (13)$$

A curious fact to be noted from (11) is that it is possible to choose a value of $\mu\beta$ (namely, $|\mu\beta| = \sec\Phi$) so that the numerator of the right hand term vanishes. This means that the gain stability is perfect, assuming differential variations in $|\mu|$. Referring to Figs. 3 and 4, contour C is the locus of $|\mu\beta| = \sec\Phi$ and it includes all amplifiers whose gain is unaffected by small variations in $|\mu|$. In this way it is possible even to stabilize an amplifier whose feed-back is positive, i.e., feed-back may be utilized to raise the gain of an amplifier and, at the same time, the gain stability with feed-back *need* not be degraded but on the contrary may be improved. If a similar procedure is followed with an amplifier whose feed-back is negative, the gain stability theoretically will be perfect and independent of the reductions in gain due to feed-back. Over too wide a frequency band practical difficulties will limit the improvements possible by these methods.

With negative feed-back, gain stability always is improved by an amount at least as great as corresponds to the reduction in gain and generally more; with positive feed-back, gain stability never is degraded by more than would correspond to the increase in gain and under appropriate conditions, assuming the variations are not too great is as good or much better than without feed-back. With positive feed-back, the variations in μ or β must not be permitted to become sufficiently great as to cause the amplifier to sing or give rise to instability as defined in the section devoted to the conditions for avoiding singing.

MODULATION

To determine the effect of feed-back action upon modulation produced in the amplifier circuit, it is convenient to assume that the output of undistorted signal is made the same with and without feed-back and that a comparison then is made of the difference in modulation with and without feed-back. Therefore, with feed-back, the input is changed to $e = e_0(1 - \mu\beta)$ and, referring to (2), the output voltage is μe_0 and the generated modulation, $d(E)$, assumes its value without feed-back, $d(E_0)$, and $d(E)/1 - \mu\beta$ becomes $d(E_0)/1 - \mu\beta$ which is $D_0/1 - \mu\beta$. This relationship is approximate because the voltage at the input without feed-back is free from distortion and with feed-back it is not and, hence, the assumption that the modulation is a function *only of the signal output* used in deriving (2) is not necessarily justified.

From the relationship $D = D_0/1 - \mu\beta$, it is to be concluded that modulation with feed-back will be reduced decibel for decibel as the effect of feed-back action causes an arbitrary db reduction in the gain of the amplifier; i.e., when the feed-back is negative. With positive feed-back the opposite is true, the modulation being increased by an amount corresponding to the increase in amplification.

If modulation in the β-circuit is a factor, it can be shown that usually in its effect on the output the modulation level at the output due to nonlinearity of the β-circuit is approximately $\mu\beta/1 - \mu\beta$ multiplied by the modulation generated in the β-circuit acting alone and without feedback.

ADDITIONAL EFFECTS

Noise: A criterion of the worth of a reduction in noise is the reduction in signal-to-noise ratio at the output of an amplifier. Assuming that the amount of noise introduced is the same in two systems, for example, with and without feed-back, respectively, and that the signal outputs are the same, a comparison of the signal-to-noise ratios will be affected by the amplification between the place at which the noise enters and the output. Denoting this amplification by a and a_0, respectively, it can be shown that the relation between the 2 noise ratios is $a_0/a(1 - \mu\beta)$. This is called the *noise index*.

If noise is introduced in the power supply circuits of the last tube, $a_0/a = 1$ and the noise index is $(1 - \mu\beta)$. As a result of this relation less expensive power supply filters are possible in the last stage.

Phase Shift, Envelope Delay, Delay Distortion: In the expression $A_F = [\mu/1 - \mu\beta]\theta$, θ is the over-all phase shift with feed-back, and it can be shown that the *phase shift through the amplifier with feed-back may be made to approach the phase shift through the β-circuit plus 180 deg.* The effect of phase shift in the β-circuit is not reduced correspondingly. It will be recalled that in reducing the change in phase shift with frequency, envelope delay, which is the slope of the phase shift with respect to the angular velocity, $\omega = 2\pi f$, also is reduced. The *delay* distortion likewise is reduced because a measure of delay distortion at a particular frequency is the difference between the envelope delay at that frequency and the least envelope delay in the band.

β-Circuit Equalization: Referring to (2), the output voltage E approaches $-e_0/\beta$ as $1 - \mu\beta = -\mu\beta$ and equals it in absolute value if $\cos\Phi = 1/2|\mu\beta|$ where $\mu\beta = |\mu\beta|\,\Phi$. Under these circumstances increasing the loss in the β-circuit 1 db raises the gain of the amplifier 1 db, and *vice-versa*, thus giving any gain-frequency characteristic for which a like loss-frequency characteristic can be inserted in the β-circuit. This procedure has been termed β-circuit equalization. It possesses other advantages and properties which are beyond the scope of this paper.

AVOID SINGING

Having considered the theory up to this point, experimental evidence was readily acquired to demonstrate that $\mu\beta$
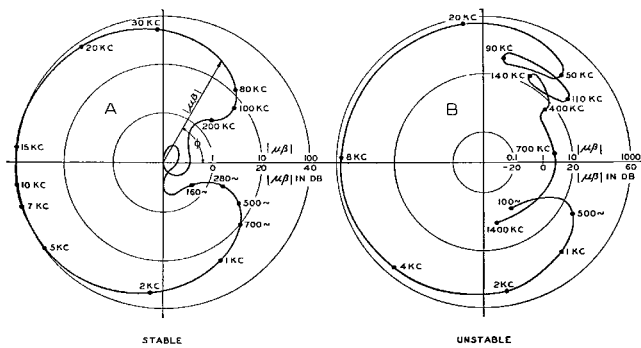
Fig. 5. Measured $\mu\beta$ characteristics of 2 amplifiers.

Fig. 6. Gain-frequency characteristics with and without feed-back of amplifier of Fig. 2.

Fig. 7. Modulation characteristics with and without feed-back for the amplifier of Fig. 2.

might assume large values, 10 to 10000, provided Φ was not at the same time zero. However, one noticeable feature about the field of $\mu\beta$ (Figs. 3 and 4) is that it implies that even though the phase shift is zero and the absolute value of $\mu\beta$ exceeds unity, self-oscillations or singing will not result. This may or may not be true. When first thinking about this matter it was suspected that owing to practical nonlinearity, singing would result whenever the gain around the closed loop equaled or exceeded the loss and simultaneously the phase shift was zero; i.e., $\mu\beta = |\mu\beta| + jo \geq 1$. Results of experiments, however, seemed to indicate something more was involved and these matters were described to H. Nyquist who developed a more general criterion for freedom from instability applicable to an amplifier having linear positive constants. (For a complete description of the criterion for stability and instability and exactly what is meant by enclosing the point (1, 0), reference should be made to Regeneration Theory, by H. Nyquist. *Bell System Technical Journal*, v. XI, July 1932, p. 126–47.)

To use this criterion, plot $\mu\beta$ (the modulus and argument vary with frequency) and its complex conjugate in polar coördinates for all values of frequency from 0 to $+\infty$. If the resulting loop or loops do not enclose the point (1, 0) the system will be stable, otherwise not. The envelope of the transient response of a stable amplifier always dies away exponentially with time; that of an unstable amplifier in all physically realizable cases increases with time. Characteristics A and B in Fig. 5 are results of measurements on 2 different amplifiers; the amplifier having $\mu\beta$ characteristic denoted A was stable, the other unstable.

The number of stages of amplification that can be used in a single amplifier is not significant except in so far as it affects the question of avoiding singing. Amplifiers with considerable negative feed-back have been tested where the number of stages ranged from one to five, inclusive. In every case the feed-back path was from the output of the last tube to the input of the first tube.

EXPERIMENTAL RESULTS

Figs. 6 and 7 show how the gain-frequency and modulation characteristics of the three-stage impedance coupled amplifier of Fig. 2 are improved by negative feed-back. In Fig. 7 the improvement in harmonics is not equal exactly to the decibel reduction in gain. Fig. 8 shows measurements
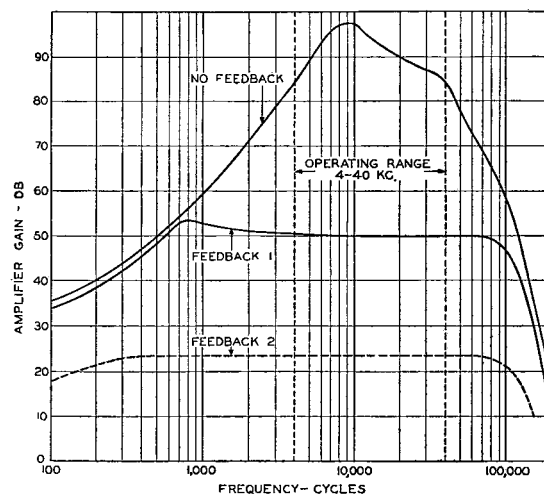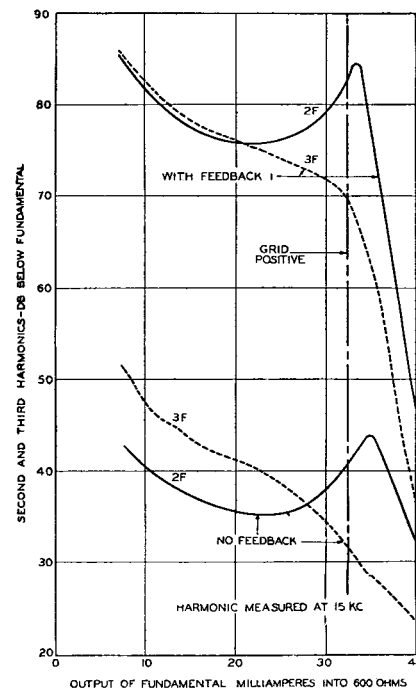
on a different amplifier in which harmonics are reduced as negative feed-back is increased, decibel for decibel over a 65-db range.

That the gain with frequency practically is independent of small variations in $|\mu|$ is shown by Fig. 9. This is a characteristic of the Morristown amplifier, described in the paper by Clark and Kendall referred to previously, which meets the severe requirements imposed upon a repeater amplifier for use in cable carrier systems. Designed to amplify frequencies from 4 kc to 40 kc the maximum change in gain due to variations in plate voltage does not exceed 7/10 000 db per volt and at 20 kc the change is only 1/20 000 db per volt. This illustrates that for small changes in $|\mu|$, the ratio of the stability without feed-back
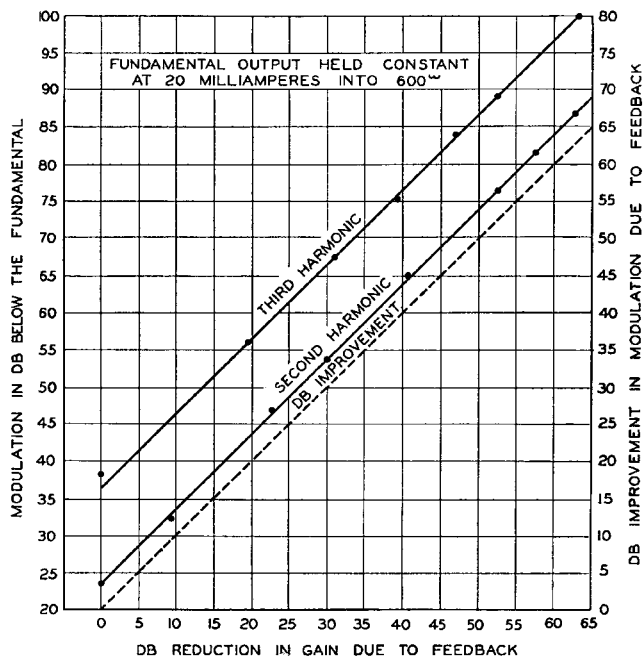
Fig. 8. Improvement of harmonics with feed-back. One example of another amplifier in which, with 60-db feed-back, harmonic currents in the output are only 1 thousandth and their energy 1 millionth of the values without feed-back.
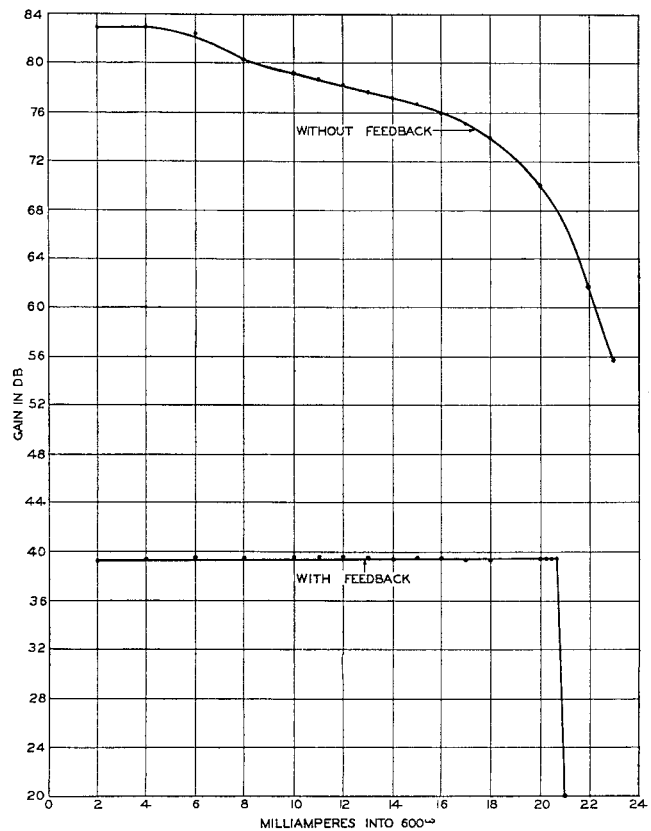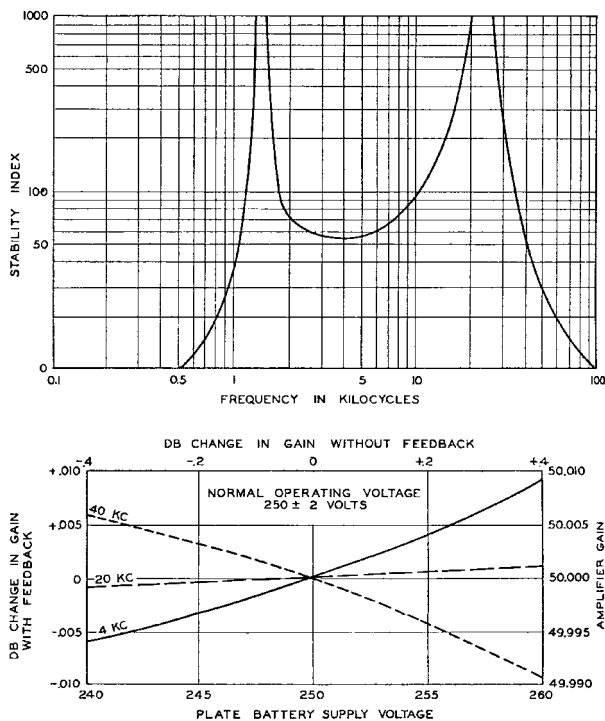

Fig. 9. Representative gain stability of a single amplifier as determined by measuring 69 feed-back amplifiers in tandem at Morristown, New Jersey. The upper figure shows the absolute value of the stability index. It can be seen that between 20 and 25 kc the improvement in stability is more than 1,000 to 1 yet the reduction in gain was less than 35 db. The lower figure shows change in gain of the feed-back amplifier with changes in the plate battery voltage and the corresponding changes in gain without feed-back. At some frequencies the change in gain is of the same sign as without feed-back and at others it is of opposite sign and it can be seen that near 23 kc the stability must be perfect.
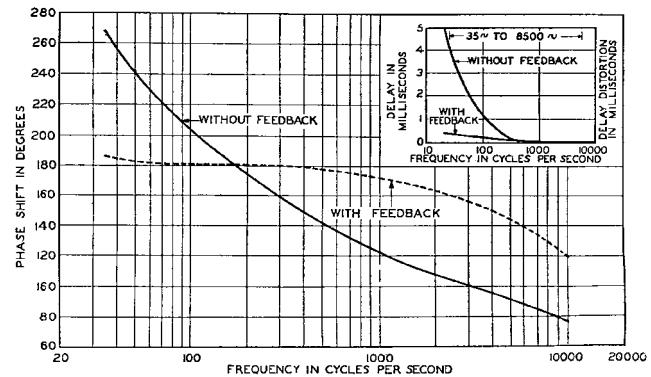

Fig. 10. Gain-load characteristic with and without feed-back for a low level amplifier designed to amplify frequencies from 3.5 to 50 kc.

Fig. 11. Phase shift, delay, and delay distortion with and without feed-back for a single tube voice frequency amplifier.

to the stability with feed-back, called the *stability index*, approaches $|1 - \mu\beta|^2/1 - |\mu\beta|\cos\Phi$ and gain stability is improved at least as much as the gain is reduced and usually more, and is theoretically perfect if $\cos\Phi = 1/|\mu\beta|$.

In Fig. 10 is indicated the effectiveness with which the gain of a feed-back amplifier can be made independent of variations in input amplitude practically up to the overload point of the amplifier. These measurements were made on a three-stage amplifier designed to work from 3.3kc to 50 kc.

As shown in Fig. 11, the negative feed-back may be used to improve phase shift and reduce delay and delay distortion. These measurements were made on an experimental one-tube amplifier, 35–8,500 cycles, feeding back
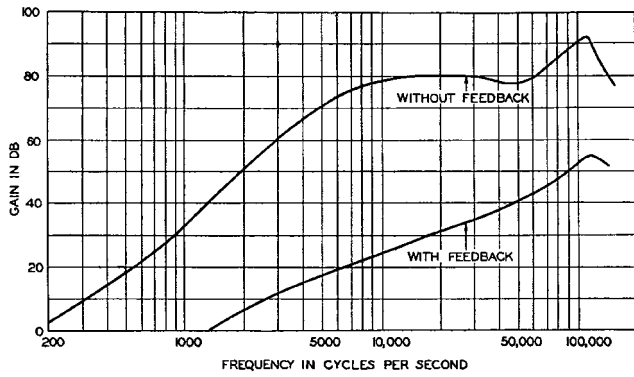
Fig. 12. Gain-frequency characteristic of an amplifier with an equalizer in the β-circuit. This was designed to have a gain frequency characteristic with feed-back of the same shape as the loss frequency characteristic of a nonloaded telephone cable.

around the low side windings of the input and output transformers.

In Fig. 12 is given the gain-frequency characteristic of an amplifier with and without feed-back when in the β-circuit there is an equalizer designed to make the gain-frequency characteristic of the amplifier with feed-back of the same shape as the loss-frequency characteristic of a nonloaded telephone cable.

CONCLUSION

The feed-back amplifier dealt with in this paper was developed primarily with requirements in mind for a cable carrier telephone system, involving many amplifiers in tandem with many telephone channels passing through each amplifier. Most of the examples of feed-back amplifier performance naturally have been drawn from amplifiers designed for this field of operation. In this field, vacuum tube amplifiers normally possessing good characteristics with respect to stability and freedom from distortion are made to possess superlatively good characteristics by application of the feed-back principle.

However, certain types of amplifiers, in which economy has been secured by sacrificing performance characteristics, particularly as regards distortion, can be made to possess improved characteristics by the application of feed-back. Discussion of these amplifiers is beyond the scope of this paper.