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By Sree Likhita Gavini

Entitled
CONTROL OF NON-MINIMUM PHASE POWER CONVERTERS

For the degree of Master of Science in Electrical and Computer Engineering

Is approved by the final examining committee:

Afshin Izadian
Chair

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CONTROL OF NON-MINIMUM PHASE POWER CONVERTERS

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ABSTRACT

Gavini, Sree Likhita. M.S.E.C.E., Purdue University, May 2012. Control of Non-minimum Phase Power Converters. Major Professor: Afshin Izadian.

The inner structural characteristics of non-minimum phase DC-DC converters pose a severe limitation in direct regulation of voltage when addressed from a control perspective. This constraint is reflected by the presence of right half plane zeros or the unstable zero dynamics of the output voltage of these converters. The existing controllers make use of one-to-one correspondence between the voltage and current equilibriums of the non-minimum phase converters and exploit the property that when the average output of these converters is the inductor current, the system dynamics are stable and hence they indirectly regulate the voltage. As a result, the system performance is susceptible to circuit parameter and load variation and require additional controllers, which in turn increase the system complexity.

In this thesis, a novel approach to this problem is proposed for second order non-minimum phase converters such as Boost and Buck-Boost Converter. Different solutions have been suggested to the problem based on whether the converter is modeled as a linear system or as a nonlinear system. For the converter modeled as a linear system, the non-minimum phase part of the system is decoupled and its transfer function is converted to minimum phase using a parallel compensator. Then the control action is achieved by using a simple proportional gain controller.

This method accelerates the transient response of the converter, reduces the initial undershoot in the response, and considerably reduces the oscillations in the transient response. Simulation results demonstrate the effectiveness of the proposed approach.

When the converter is modeled as a bilinear system, it preserves the stabilizing nonlinearities of the system. Hence, a more effective control approach is adopted by using Passivity properties. In this approach, the non-minimum phase converter system is viewed from an energy-based perspective and the property of passivity is used to achieve stable zero dynamics of the output voltage. A system is passive if its rate of energy storage is less than the supply rate i.e. the system dissipates more energy than stores. As a result, the energy storage function of the system is less than the supply rate function. Non-minimum phase systems are not passive, and passivation of non-minimum phase power converters is an attractive solution to the posed problem. Stability of non-minimum phase systems can also be investigated by defining the passivity indices.

This research approaches the problem by characterizing the degree of passivity i.e. the amount of damping in the system, from passivity indices. Thus, the problem is viewed from a system level rather than from a circuit level description. This method uses feed-forward passivation to compensate for the shortage of passivity in the non-minimum phase converter and makes use of a parallel interconnection to the open-loop system to attain exponentially stable zero dynamics of the output voltage. Detailed analytical analysis regarding the control structure and passivation process is performed on a buck-boost converter. Simulation and experimental results carried out on the test bed validate the effectiveness of the proposed method.

1. INTRODUCTION

1.1 Introduction

This thesis focuses on the control of second order non-minimum phase DC-DC power converters and the analysis is carried out on a buck-boost converter as an example. Buck-Boost converters are nonlinear switching systems with non-minimum phase output voltage and can either step-up or step-down the output voltage. They are the result of cascading the buck and boost converter circuits [1]. The inner structural characteristics of these converters pose a severe limitation on the transient response of the converter [2]. This constraint is reflected by the presence of right half plane zeroes in the converter control-to-output transfer function. When addressed from a control perspective, the right half plane zeros in the transfer function or the non-minimum phase zeros of the buck-boost converter complicate the control design scheme [3, 4, 5]. This research proposes a novel approach to resolve this problem.

1.2 Previous Work

Using the classical control procedures, this problem is overcome by using either a low gain feedback with reduced performance [6], or by using cascaded voltage and indirect current control approach [7]. However, the response of the system is characterized by significant undershoots and overshoots. The presence of non-minimum phase zeroes in the system result in an initial undershoot in the output voltage response of the system [8]. Overshoots are also observed in the response of the converter [9]. A gain control approach results in limited bandwidth of the system.

A number of effective non-linear controllers [10] such as sliding mode control, passivity based control, feedback linearization are also used. They indirectly control the output voltage by regulating the inductor current. Consequently, these controllers make use of one-to-one correspondence between the voltage and current equilibriums and exploit the property that when the average output of the buck-boost converter is the inductor current, the system dynamics are stable. This results in large sensitivity of the controller to circuit parameter and load variations. As a result, adaptive controllers [10] are incorporated to achieve a satisfactory performance, and result in complex control systems.

1.3 Objectives

In this work, the drawbacks of the above approaches are addressed and different methods are proposed based on either linear or nonlinear models. The objective of this research is to achieve a high profile transient output voltage response i.e. considerable reduction in the overshoots and undershoots, by achieving direct regulation of non-minimum phase voltage of the converter. The system sensitivity to load variations is reduced.

1.4 About This Thesis

In this thesis, for the linear buck-boost converter model, the mathematical model of the buck-boost converter is derived using the state space averaging technique. Then, the non-minimum phase converter control-to-output transfer function is decoupled from the non-minimum phase converter line-to-output transfer function. A parallel compensator is connected in parallel to the converter control-to-output transfer function to obtain a new minimum phase replacement plant, and result in an almost strictly positive real system. In the last stage, output voltage of the compensated buck-boost converter system can be effectively controlled using a proportional gain controller. The main advantages of this technique are the reductions of initial undershoot and overshoot, expand the control bandwidth, and enhance the effectiveness of the control. The simulation results demonstrate the performance of this method compared with a proportional integral controller and with the parallel compensator.

For the nonlinear converter model, the problem is approached by characterizing the degree of passivity in the system from passivity indices. A complementary system level approach is introduced in this research and a simple linear controller is used to enhance the output voltage profile and attain robustness against load variations. This method makes use of a parallel interconnection to the open-loop system to achieve exponentially stable zero dynamics of the output voltage. An excess passive system is used to compensate for the shortage of passivity in the buck-boost converter system to reduce the non-minimum phase behavior. Simulations as well as experimental results validate the effectiveness of this control approach.

2. DYNAMICS OF DC-DC POWER CONVERTERS

2.1 Modeling Procedures

Power converters are nonlinear switching circuits that are able to buck, boost or buck and boost the output voltage as needed. Boost and buck-boost converters can be made in different forms as Cuk, Sepic, and Zeta and they are fourth order systems. The focus of this thesis is on second-order non-minimum phase converters such as Boost or Buck-Boost. Modeling of converters is challenging due to their continuously varying switching behavior. Averaging techniques [11] are usually employed to mathematically represent an approximate behavior of the converters. The averaging techniques can be broadly classified into frequency dependent [12-14] and frequency independent methods. The frequency dependent averaging takes into account the frequency of the switching waveform and gives an estimate of ripple in the averaged state variables, thus provides a more accurate representation of the converter than frequency independent averaging, however the models are complex in nature.

The most widely used model of these converters is the frequency independent State Space Averaging method. In state space averaging, the various circuit configurations of the converter are averaged to obtain a unified representation of the converter. The output voltage in this model is controlled by a variable duty cycle at the gate of switching transistor. The averaged model obtained is a non-linear large signal model and has to be perturbed and linearized around an operating point to derive the small-signal ac equations. The state space-averaging model does not provide any information regarding the amount of ripple in the averaged state variable. It is more suitable for low ripple converters, which have small variations around the operating points.

The other types of frequency dependent modeling techniques applied to power converters are Generalized State Space Averaging [13, 14], and Multi-Frequency Averaging [12]. These modeling approaches are usually used with converters with high ripple content like resonant converters or large ripple PWM converters. Fourier series are used to approximate the behavior of circuit state variables. Depending on the degree of accuracy required, the required number of harmonics is determined. In case of small ripple approximation, the dc content of the signal dominates the Fourier series, and the frequency selective averaging technique essentially becomes equal to state space averaging.

Another modeling procedure is based on PWM switch modeling [18] which is a circuit based technique. Unlike state space averaging PWM switch modeling averages the time variant pulses and is modeled independent of the load. This technique is especially useful when analyzing circuits containing constant power loads (CPL). When state space averaging is used to model CPL, one of the state-variables is inverted in the state-space equations and results in nonlinearities. In this thesis, state-space averaging models are used as they provide a good approximation for Boost and Buck-Boost Converters. PWM switch model for the buck-boost converter is also derived to illustrate the PWM modeling procedure. However, state space averaging models are adopted in this thesis, owing to the simplicity of the models.

2.1.1 State Space Averaging

In this section, the small-signal transfer functions for a continuous conduction mode (CCM) buck-boost converter are derived based on state space averaging method. Figure 2.1 shows the circuit diagram of a buck-boost converter.

Buck-Boost converters step up or step down the output voltage depending on the ratio of applied duty cycle. It is an inverting circuit, whose output polarity is opposite of the input voltage.

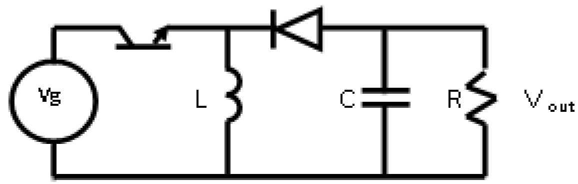

Figure 2.1 Buck-Boost converter

The circuit equations of the converter, when the switch is ON, are given as:

$$\begin{cases} -V_{in} + L\dfrac{di_L}{dt} = 0 \\ C\dfrac{dv_C}{dt} + \dfrac{v_C}{R} = 0 \end{cases} \tag{2.1}$$

The circuit equations when the switch is OFF are given as:

$$\begin{cases} -L\dfrac{di_L}{dt} + v_C = 0 \\ i_L + C\dfrac{dv_C}{dt} + \dfrac{v_C}{R} = 0 \end{cases} \tag{2.2}$$

where C , L and R are the values of the capacitance, inductance and resistance respectively of the buck-boost converter as shown in Figure 2.1.

Equations 2.1 and 2.2 are averaged so that the duty cycle u and $1-u$ are used as weights respectively. Then the state space averaged model of buck-boost converter is given as:

$$\begin{cases} L\dfrac{di_L}{dt} = uV_{in} + (1-u)v_C \\ C\dfrac{dv_C}{dt} = -\dfrac{v_C}{R} - (1-u)i_L \end{cases} \tag{2.3}$$

The above equation gives a large signal non-linear averaged model of the buck-boost converter. Equation 2.3 is perturbed and linearized, i.e. the state variable is expressed as a DC value with a superimposed small ac variation such that only linear terms are considered in the resulting equation, around the quiescent values of the circuit.

The following small-signal ac equations are obtained:

$$\begin{cases} L\dfrac{d\hat{i}(t)}{dt} = D\hat{v}_{in}(t) + D'\hat{v}(t) + (V_{in} - V)\hat{d}(t) \\ C\dfrac{d\hat{v}(t)}{dt} = -D'\hat{i}(t) - \dfrac{\hat{v}(t)}{R} + I\hat{d}(t) \end{cases} \tag{2.4}$$

where D is the steady state value of the duty cycle driving the converter to obtain a steady state voltage V and current I and $\hat{d}(t)$, $\hat{v}(t)$ and $\hat{i}(t)$ are small ac variations of duty cycle, voltage and current respectively around the steady state operating point.

The V, I and D' are as follows:

$$\begin{cases} V = -\dfrac{D}{D'}V \\ I = -\dfrac{V}{D'R} \\ D' = 1 - D \end{cases} \tag{2.5}$$

Then deriving the transfer function from the small-signal ac equations we get:

$$\hat{v}(s) = \frac{-DD'}{D'^2 + s\dfrac{L}{R} + s^2 LC}\hat{v}_g(s) - \frac{V_g - V - sLI}{D'^2 + s\dfrac{L}{R} + s^2 LC}\hat{d}(s) \tag{2.6}$$

Decoupling the converter control-to-output transfer function from the converter line-to-output transfer function results in two transfer functions [17].

The converter line-to-output transfer function is given as:

$$G_{v_{in}}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_{in}(s)}\right|_{\hat{d}(s)=0} = \left(-\frac{D}{D'}\right)\frac{1}{1 + s\dfrac{L}{D'^2 R} + s^2\dfrac{LC}{D'^2}} \tag{2.7}$$

The converter control-to-output transfer function is given as:

$$G_{vd}(s)=\left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_{in}(s)=0}=\left(-\frac{V_g-V}{D'^2}\right)\frac{\left(1-s\frac{LI}{V_g-V}\right)}{\left(1+s\frac{L}{D'^2R}+s^2\frac{LC}{D'^2}\right)} \tag{2.8}$$

Hence, the converter power stage small-signal model can be depicted graphically as shown in Figure 2.2.

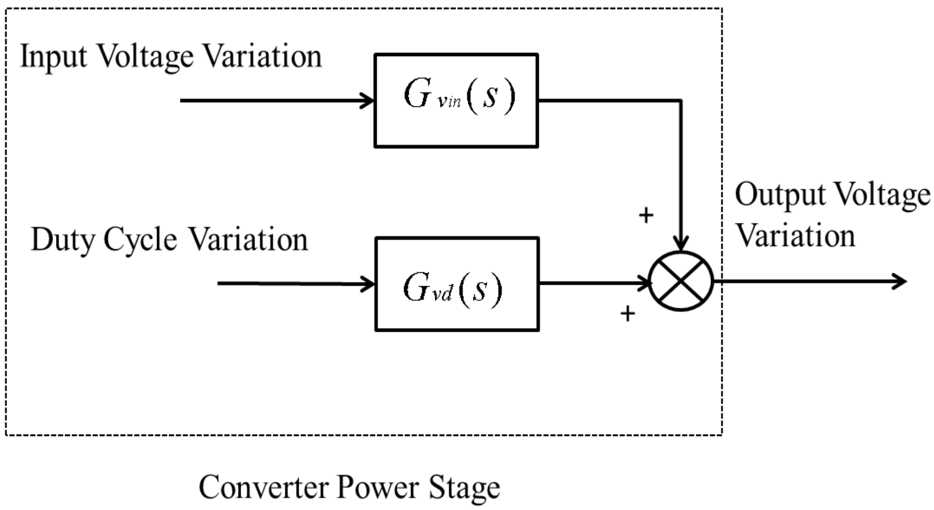

Figure 2.2 Small-signal equivalent model of power converters

2.1.2 PWM Switch Modeling

In this section the PWM switch model of a CCM buck-boost converter is developed. In this method, a three-terminal device called the PWM switch is used to replace the active and passive switches of the PWM converter. The equivalent model for the PWM switch [18] is as shown in Figure 2.2. PWM switch modeling gives the DC and small-signal model of the converter. In this modeling procedure, it is considered that the non-linearity in the power converter is due to the switching device and by replacing the nonlinear switch with its equivalent DC and small-signal model, a small-signal model for the entire converter can be obtained.

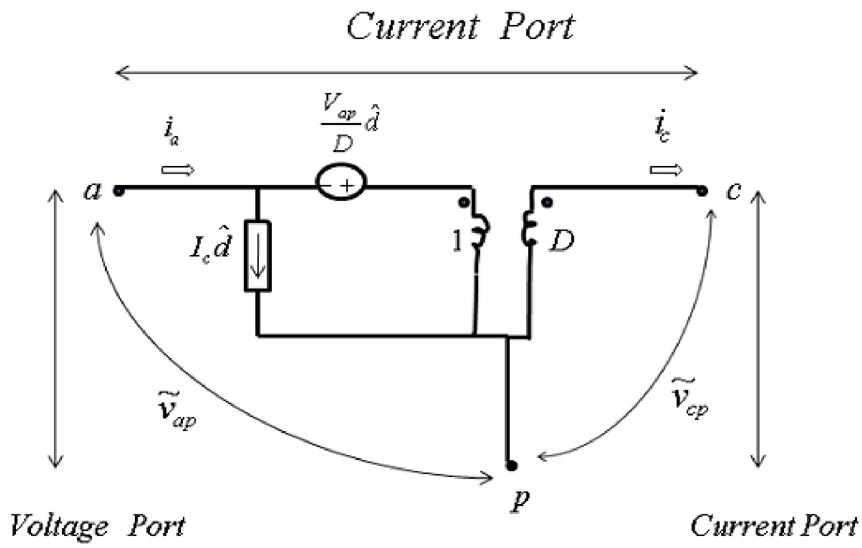

Figure 2.3 Equivalent DC and small signal model of PWM switch

By replacing this model in the buck-boost converter, the small signal transfer functions [19] can be obtained.

The line to output transfer function of the buck-boost converter is given as:

$$G_{v_{in}}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_{in}(s)}\right|_{\hat{d}(s)=0} = \frac{\frac{s}{\omega_{zn}}+1}{\frac{s^2}{\omega_0^2}+\frac{2\xi}{\omega_0}s+1} \tag{2.9}$$

Where,

$$\begin{cases} \xi = \dfrac{C\left[(r_L + DD'r_e)(R+r_C) + D'^2 R r_C\right] + L}{2\sqrt{LC(R+r_C)(r_L + D'^2 R)}} \\ \omega_0 = \sqrt{\dfrac{r_L + DD'r_e + D'^2 R}{LC(R+r_C)}} \\ \omega_{zn} = \dfrac{1}{Cr_C} \quad , r_e = r_C \| R \end{cases} \tag{2.10}$$

The control to output transfer function is given as:

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_{in}(s)=0} = -\frac{V_o}{DD'} \frac{\left(\frac{s}{\omega_{zn}}+1\right)\left(\frac{s}{\omega_{zp}}+1\right)}{\frac{s^2}{\omega_0^2}+\frac{2\xi}{\omega_0}s+1} \tag{2.11}$$

where

$$\begin{cases} \omega_{zp} = \dfrac{-D'^2R + r + DD'r_e}{DL} \\ M_{VDC} = \dfrac{V_o}{V_{in}} = \dfrac{D}{D'} \dfrac{1}{\left[1+\dfrac{Dr_e}{D'R}+\dfrac{r_L}{D'^2R}\right]} \end{cases} \tag{2.12}$$

The parameters r_C and r_L are the equivalent series resistances of the capacitor and inductor respectively. As the transfer functions of the buck-boost converter derived from PWM switch modeling are cumbersome compared to the state space averaging transfer function, in this research state space averaging models are used for buck-boost converters in Chapter 3.

2.1.3 Bilinear Model

In this section, the averaged converter modeled obtained from state space averaging is not perturbed and linearized. The original non-linear model is taken into consideration, as linearizing a system cancels the stabilizing nonlinearities in a system. The nonlinear model of the buck-boost converter is bilinear in nature.

A bilinear system is a close approximation of nonlinear behavior for a class of nonlinear systems [20, 21]. They demonstrate linear behavior of their states and their controls separately, but are nonlinear when analyzed together [22].

A general structure of bilinear systems is given by the following equation [22]:

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u + N(t)xu \\ y &= C(t)x \end{aligned} \tag{2.13}$$

where *A*, *B*, and *C* are time-varying system matrices, and *N* is the bilinear matrix that relates the states and the control command.

State space averaging model (Equation 2.3) of a buck-Boost converter, shown in Figure 1, is repeated here for convenience as,

$$\begin{aligned} \frac{di}{dt} &= \frac{V_{in}}{L}u + (1-u)\frac{V_{out}}{L} \\ \frac{dv}{dt} &= -\frac{V_{out}}{RC} - (1-u)\frac{i}{C} \end{aligned} \tag{2.14}$$

Considering the inductor current as the state x_1 and the output capacitor voltage as state x_2, the bilinear model of a buck-boost converter can be represented as,

$$\begin{aligned} \dot{x} &= \begin{pmatrix} \dfrac{x_2}{L} \\ -\dfrac{x_1}{C} - \dfrac{x_2}{RC} \end{pmatrix} + \begin{pmatrix} \dfrac{V_{in}}{L} - \dfrac{x_2}{L} \\ \dfrac{x_1}{C} \end{pmatrix} u \\ y &= x_2 \end{aligned} \tag{2.15}$$

where $x = [x_1 \quad x_2]$ is the vector of state variables, and x_2 is negative as the system is an inverting circuit.

Considering a general form of nonlinear system [23] as

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2.16}$$

The functions $f(x)$, $g(x)$, and $h(x)$ of the bilinear buck-boost converter can be represented as

$$f(x)=\begin{pmatrix} \dfrac{x_2}{L} \\ -\dfrac{x_1}{C}-\dfrac{x_2}{RC} \end{pmatrix} \quad , \quad g(x)=\begin{pmatrix} \dfrac{V_{in}}{L}-\dfrac{x_2}{L} \\ \dfrac{x_1}{C} \end{pmatrix} \tag{2.17}$$

$$h(x)=x_2$$

2.2 Nonlinear System Structure

The bilinear model of the buck-boost converter can be converted into Isidori's Normal form if there exist a relative degree r for the system i.e. a non-singular co-ordinate transformation.

A single-input single-output nonlinear system, of the form as given in Equation 2.16, is said to have a relative degree r at a point x° [23], where x° is the state of the system at time t° such that $L_g L_f^{r-1} h(x^\circ) \neq 0$.

i. $L_g L_f^k h(x)=0$ for all x in a neighborhood of x^o and all $k<r-1$

ii. $L_g L_f^{r-1} h(x^o) \neq 0$

where

$$L_f \lambda(x)=\sum_{i=1}^{n} \frac{\partial \lambda}{\partial x_i} f_i(x) \tag{2.18}$$

The function (Equation 2.18) is the derivative of a vector λ along the direction of another vector field f.

The function $\frac{\partial \lambda}{\partial x}$ is called the Jacobian and is given as

$$\frac{\partial \lambda}{\partial x} = \begin{pmatrix} \frac{\partial \lambda_1}{\partial x_1} & \frac{\partial \lambda_1}{\partial x_2} & \cdots & \frac{\partial \lambda_1}{\partial x_n} \\ \frac{\partial \lambda_2}{\partial x_1} & \frac{\partial \lambda_2}{\partial x_2} & \cdots & \frac{\partial \lambda_2}{\partial x_n} \\ . & . & \cdots & . \\ \frac{\partial \lambda_n}{\partial x_1} & \frac{\partial \lambda_n}{\partial x_2} & \cdots & \frac{\partial \lambda_n}{\partial x_n} \end{pmatrix} \tag{2.19}$$

and

$$L_g L_f \lambda(x) = \frac{\partial (L_f \lambda)}{\partial x} g(x) \tag{2.20}$$

is the derivative of the vector λ along the vector field f and then along a vector field g.

Finally, $L_f^k \lambda(x)$ is the derivative of λ differentiated k times along f and is given as:

$$L_f^k \lambda(x) = \frac{\partial (L_f^{k-1} \lambda)}{\partial x} f(x) \tag{2.21}$$

The relative degree of a non-linear system can be interpreted to be equivalent to the number of zeroes of the linearized system i.e. for a linear system the relative degree is considered to be the difference between the order of numerator and the denominator. Another interpretation of the relative degree r of the non-linear system is that it requires the output be differentiated r times to explicitly show the relationship between the input $u(t)$ and the output $y(t)$.

Then the following Lemma 2.1 [23] holds.

Lemma 2.1 [23]: For a non-linear system in the form of Equation 2.16 with relative degree r, the row vectors $dh(x^o), dL_f h(x^o), \cdots, dL_f^{r-1} h(x^o)$ are linearly independent, where d is the differential or gradient of a real-valued function.

It can be concluded that a system of order n is of relative degree r if the following proposition holds.

Proposition 2.1 [23]: Suppose the system has relative degree r at x^o. Then $r \le n$ and the co-ordinate transformation is given as

$$\begin{aligned} \phi_1(x) &= h(x) \\ \phi_2(x) &= L_f h(x) \\ &\cdots \\ \phi_r(x) &= L_f^{r-1} h(x) \end{aligned} \tag{2.22}$$

If r is strictly less than n, it is always possible to find $n-r$ functions $\phi_{r+1}(x),\cdots\phi_n(x)$ such that the mapping

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \cdots \\ \phi_n(x) \end{pmatrix} \tag{2.23}$$

has a jacobian matrix nonsingular at x^o and qualifies as a local coordinate transformation in a neighborhood of x^o. The value at x^o of these additional functions can be fixed arbitrarily. Moreover, it is always possible to choose $\phi_{r+1}(x),\cdots\phi_n(x)$ such that $L_g\phi_i(x) = 0$ for all $r+1 \le i \le n$ and all x around x^o.

Hence, the state-space description of the system (Equation 2.16) in the normal form [23] is given as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(z) + a(z)u \\ \dot{z}_{r+1} &= q_{r+1}(z) \\ &\cdots \\ \dot{z}_n &= q_n(z) \end{aligned} \tag{2.24}$$

For a buck-boost converter system (Equation 2.17), the relative degree is $r = 1$ as

$$L_g h(x) = \frac{\partial h(x)}{\partial x} g(x) = [0 \quad 1] \begin{bmatrix} \frac{V_{in}}{L} - \frac{x_2}{L} \\ \frac{x_1}{C} \end{bmatrix} = \frac{x_1}{C} \neq 0 \tag{2.25}$$

As the relative degree is one for a second order system, there exist a non-singular co-ordinate transformation and therefore the normal form is

$$\begin{cases} z_1 = \phi_1(x) = h(x) = x_2 \\ z_2 = \phi_2(x) = \frac{x_1^{\ 2}}{2C} + \frac{x_2^{\ 2}}{2L} - \frac{x_2}{L} V_{in} \\ y = h(x) = x_2 \end{cases} \tag{2.26}$$

The output voltage of an inverting buck-boost converter is $x_2 < 0$ and as the buck-boost converter is considered to be operating in CCM $x_1 > 0$. Therefore, $z_2 > 0$ for all values and the inverse transformation exists and can be obtained as

$$\begin{cases} x_1 = \sqrt{2C\left(z_2 + z_1 \frac{V_{in}}{L} - \frac{z_1^{\ 2}}{2L} \right)} \\ x_2 = z_1 \end{cases} \tag{2.27}$$

The buck-boost converter system in the new coordinates (z_1, z_2), i.e. the normal form, is represented by

$$\begin{cases} \dot{z}_1 = -\frac{1}{RC} z_1 - \frac{(1-u)}{C} \sqrt{2C\left(z_2 + \frac{V_{in}}{L} z_1 - \frac{z_1^{\ 2}}{2L} \right)} \\ \dot{z}_2 = \frac{V_{in} z_1 - z_1^2}{RLC} + \frac{V_{in}}{LC} \sqrt{2C\left(z_2 + \frac{V_{in}}{L} z_1 - \frac{z_1^{\ 2}}{2L} \right)} \end{cases} \tag{2.28}$$

$$y = z_1$$

The advantage of the normal form of the system is the structure, which is taken advantage of in different controls like exact linearization, due to which many important properties of the system can be obtained just by inspection. From Equation 2.28, it can be said that the relative degree of the buck-boost converter system is one, as the first equation explicitly contains the input term $u(t)$. The zero dynamics of a non-linear system can also be easily obtained from the normal form.

2.3 Zero Dynamics

The zero dynamics of a non-linear system are analogous to the zeros of the transfer function in a linear system. A system with relative degree r that is strictly less than the order of the system n is said to have zero dynamics of the order $n-r$. If $r=n$ the system does not have any unseen internal dynamics and it also implies that the transfer function of the linear system has no zeros. Zero dynamics of a system are the hidden internal dynamics when the initial conditions of the system and the input are constrained to make the output of the system zero [23].

The zero dynamics of a nonlinear system can be easily obtained from the Isidori's Normal form, by zeroing the output. If a nonlinear system (Equation 2.16) has a relative degree r then the state vector z can be grouped into the following two vectors as given below:

$$\xi = \begin{pmatrix} z_1 \\ \cdots \\ z_r \end{pmatrix}, \quad \eta = \begin{pmatrix} z_{r+1} \\ \cdots \\ z_n \end{pmatrix} \tag{2.29}$$

Then the normal form can be written as:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(\xi,\eta) + a(\xi,\eta)u \\ \dot{\eta} &= q(\xi,\eta) \end{aligned} \tag{2.30}$$

When the output of the system is identical to zero its state is constrained to evolve such that $\xi(t)$ is identically zero [23]. Then the zero dynamics are given by the following equation as:

$$\dot{\eta}(t) = q(0, \eta(t)) \tag{2.31}$$

then the unique input to the system is given as:

$$u(t) = -\frac{b(0, \eta(t))}{a(0, \eta(t))} \tag{2.32}$$

For the buck-boost converter system the zero dynamics are obtained by zeroing the output in Equation 2.26, then the following equation is obtained,

$$\dot{z}_2 = \frac{V_{in}}{LC}\sqrt{2Cz_2} \tag{2.33}$$

Phase plane [25] is a tool used to analyze the stability of zero dynamics. This method is used to graphically analyze how the system trajectories change with the state variable from which the stability of the system can be commented. The phase plane plot of the zero dynamics is shown in Figure 2.4.

As the phase plane trajectories diverge in Figure 2.4, the zero dynamics of the capacitor voltage in a buck-boost converter are not stable; this behavior is due to the non-minimum phase nature of these converters with respect to capacitor voltage. Hence, the zero dynamics of the non-minimum phase voltage of the buck-boost converter are unstable.

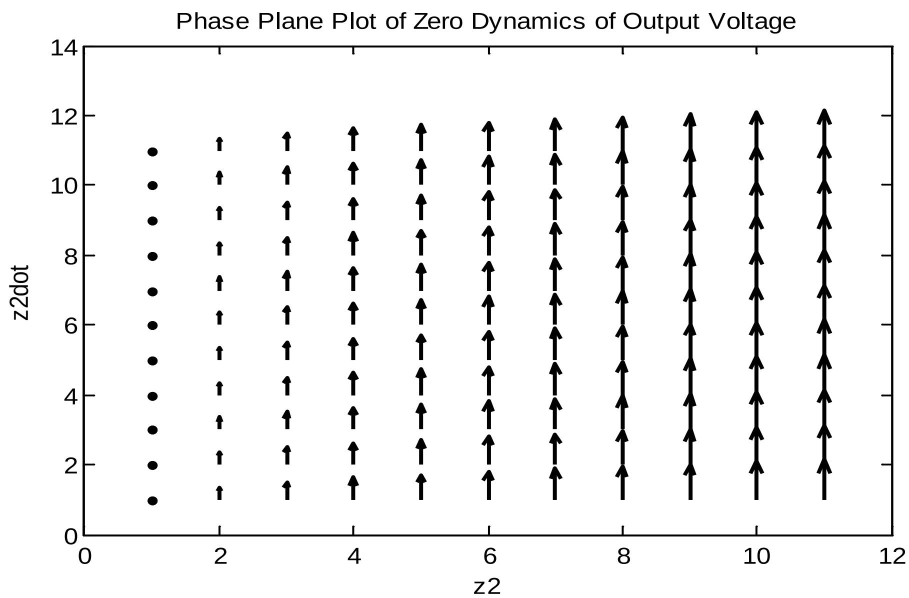

Figure 2.4 Phase plane plot of zero dynamics of capacitor voltage

2.4 Passivity

A system is said to be passive if its rate of energy storage is less than the rate of supply energy i.e. the system dissipates more energy than it stores. As a result, the energy storage function of the system is less than the supply rate energy function. Passivity can be formalized by the following definitions. Consider a nonlinear system, H, defined as

$$H = \begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases} \tag{2.34}$$

Definition 2.1 Supply Rate [25]: The supply rate $w(t) = w(u(t), y(t))$ is a real valued function defined on $U \times Y$, such that for any $u(t) \in U$ and $x_o \in X$ and $y(t) = h(\phi(t, t_0, x_0, u))$, $w(t)$ satisfies

$$\int_{t_0}^{t_1} |w(t)| dt < \infty \tag{2.35}$$

for all $t_1 \geq t_0 \geq 0$.

Definition 2.2 Dissipative Systems [25]: System H with supply rate $w(t)$ is said to be dissipative if there exists a nonnegative real function $S(x): X \rightarrow \Re^{+}$, called the storage function, such that, for all $t_1 \geq t \geq 0$, $x_0 \in X$ and $u \in U$,

$$S(x_1) - S(x_0) \leq \int_{t_0}^{t_1} w(t)dt \tag{2.36}$$

or

$$\frac{dS(x(t))}{dt} \leq w(t) \tag{2.37}$$

Where $x_1 = \phi(t_1, t_0, x_0, u)$ and $\Re^{+}$, is a set of nonnegative real numbers. A Lyapunov candidate of the system can be used as Storage Function $S(x)$.

Definition 2.3 Passive Systems [25]: A system is said to be passive if it is dissipative with respect to the following supply rate

$$w(u(t), y(t)) = u^T(t)y(t) \tag{2.38}$$

and the storage function $S(x)$ satisfies $S(0) = 0$.

Passivity is an input-output property, and it depends on how the output of the system is defined. Consider the buck-boost converter system in its original representation (Equation 2.16) with a positive definite storage function $S(x) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2$. In order to investigate this input-output property, consider, for now, the output as the current and define a bilinear supply rate $w(t) = u^T y = u \cdot V_{in} \cdot x_1$, where $u \cdot V_{in}$ is the input to the circuit with current x_1 as output. Then the following can be shown:

$$\dot{S}(x) = uV_{in}x_1 - \frac{x_2^2}{R} \leq w(t) \tag{2.39}$$

Hence, the buck-boost converter is passive when the output defined is the inductor current.

The following theorem encapsulates the passivity properties for nonlinear systems as

Theorem 2.1 [25]: A system H with a constant rank $\{L_g h(x)\}$ in a neighborhood of $x = 0$, with a C^2 storage function $S(x)$ which is positive definite, is passive, then:

1. $L_g h(0)$ is nonsingular and H has relative degree $\{1, \cdots, 1\}$.
2. The zero dynamics of H exists locally at $x = 0$, and H is weakly minimum phase.

As the phase plane plot (Fig. 2.4) reveals, the output voltage zero dynamics are unstable, i.e. non minimum phase. Therefore from theorem 2.1 it can be inferred that the buck-boost converter is not passive respect to the output voltage.

Passivity is an input-output property, and does not provide information about the state of the system. But, the Kalman-Yacubovich-Popov property relates passivity of the system with the state of the system. It relates the storage function of the system with the Lyapunov candidate of the system and is given as follows:

Definition 2.4 Kalman-Yacubovich-Popov Property [25]: Consider a control affine system without throughput (2.16) where $x \in X \in \Re^n$, $u \in U \subset \Re^m$ and $y \in Y \subset \Re^m$. It is said to have the Kalman-Yacubovich-Popov (KYP) property if there exists a C^1 nonnegative function $S(x): X \to \Re^+$, with $S(0) = 0$ such that

$$L_f S(x) = \frac{\partial S(x)}{\partial x} f(x) \leq 0 \tag{2.40}$$

$$L_g S(x) = \frac{\partial S(x)}{\partial x} g(x) = h^T(x) \tag{2.41}$$

for each $x \in X$.

A system H of the form (Equation 2.16) which has the KYP property is passive, with a storage function $S(x)$ and conversely, a passive system having a C^1 storage function has the KYP property [25]. This proposition is used in Chapter 4 to derive the gain constraints for a parallel interconnection in a feed-forward passivation procedure. .

3. PARALLEL COMPENSATION APPROACH

3.1 Motivation

In this chapter, the parallel compensation approach is introduced to overcome the drawbacks of the existing linear control methods [6, 7] discussed in Chapter 1. It will be shown at the end of this chapter that this method accelerates the transient response of the converter, considerably removes undershoot in the response, reduces the oscillations in the transient response, and directly regulates the voltage instead of using multi-loop current mode control [7]. The effectiveness of this method can be verified from the simulation results carried out on the buck-boost converter.

State space averaging method is used to represent the buck-boost converter mathematically. The small-signal control-to-output transfer function and the line-to-output transfer function of the buck-boost converter (Equations 2.6-2.8), derived in Chapter 2, are used in the process of designing a compensator. The non-minimum phase control-to-output transfer function is decoupled from the minimum phase line-to-output transfer function and a parallel compensator is connected in parallel to the non-minimum phase transfer function. The new replacement plant is compensated in such a way that the system exhibits minimum phase characteristics.

An effective way to design a compensator for the non-minimum phase systems is to use strictly positive real form of the system [27, 28]. Parallel feed-forward compensators can be used to convert any plant to an almost strictly positive real system [29]. Then the control can be achieved by using output feedback techniques.

Parallel compensation techniques have been successfully implemented [30, 31] and have proven to be efficient and stable control approaches for non-minimum phase systems than pole-zero cancellation technique [32]. Pole-zero cancellation generates hidden modes that may cause instabilities, thus, cannot be universally used in non-minimum phase systems.

There are different methods in deriving the transfer function of an augmented plant and the parallel compensator. The principle of transforming the configuration of a non-minimum phase plant to minimum phase is introduced in [33]. This technique uses a feed through compensation to obtain a minimum-phase augmented plant and uses a high gain feedback control to stabilize the system [33]. The transfer function of the compensator in [33] is derived by using the transmission-zero-assignment technique [34]. Gessing [35, 36] have also proposed ways of changing non-minimum phase plants into minimum phase. They have classified the control problems of regulation, tracking or disturbance rejection and specially designed replacement plants for each problem at hand. General approach to compensator design depends on the application purposes [34-36]. The control problem of the converter, considered in this thesis, falls under the category of voltage regulation [35] and the method outlined in [35] is adopted here.

3.2 Undershoot and Non-minimum Phase Zeros

It has been studied and observed [9, 38] that in a non-minimum phase system with odd number of positive zeros, the system response gives rise to an initial undershoot. In such a case, the coefficients of the numerator polynomial of a system are not all positive [38], thus non-Hurwitz. This can be verified in the case of buck-boost converter system from its control-to-output transfer function (Equation 2.8) given by:

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_{in}(s)=0} = \left(-\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{\left(1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}\right)} \tag{3.1}$$

It can be observed that all of the coefficients of the numerator polynomial in Equation 3.1 are not positive. The undershoot in a system is undesirable as the output initially tends in the opposite direction of the final value, as a result delay is introduced into the system.

The physical significance behind the undershoot due to the right half-plane zeroes in converters can be explained as follows. The average diode current of the converter in Figure 2.1 is related to the average inductor current [17] as follows:

$$\left\langle i_D \right\rangle_{T_s} = d' \left\langle i_L \right\rangle_{T_s} \tag{3.2}$$

where T_s is the switching interval.

The average diode current $\left\langle i_D \right\rangle$ is equal to the load current. When a step increase in duty cycle is applied, $\left\langle i_D \right\rangle$ decreases and the capacitor begin to discharge and the voltage across the capacitor drops. However, the increased duty cycle causes a slow increase in the inductor current as a result $\left\langle i_D \right\rangle$ increases again and the voltage across the capacitor starts increasing. This delay in the rise of the inductor current is not a desirable phenomenon as it causes undershoot in the system and is a destabilizing effect. In this research, this problem is addressed and the parallel compensation approach is provided as a solution to the problem. The following behavior is observed in the simulations of the buck-boost converter without any compensation as shown in Figure 3.1.

3.3 Positive Real Transfer Function

Positive real systems have many important properties with regard to stability analysis and in the generation of Lyapunov functions. A passive linear system (Equation 2.38) is strictly positive real, and vice versa. The following definition can be given with regard to a positive real transfer function.

Definition 3.1 Positive Real Transfer Function [26]: A transfer function $G(s)$ is positive real if

i. $G(s)$ is analytic in $\mathrm{Re}(s) > 0$

ii. $G(j\omega) + G^*(j\omega) \geq 0$ for any frequency ω that $j\omega$ is not a pole of $G(s)$. If there are poles $p_1, p_2, \cdots, p_q$ of $G(s)$ on the imaginary axis, they are non-repeated and the residue matrix at the poles $\lim_{s \to p_i}(s - p_i)G(s) \quad (i = 1, \cdots, q)$ is Hermitian and positive semi-definite.

Transfer function $G(s)$ is said to be strictly positive real (SPR) if

i. $G(s)$ is analytic in $\mathrm{Re}(s) \geq 0$

ii. $G(j\omega) + G^*(j\omega) > 0 \, \forall \omega \in (-\infty, +\infty)$

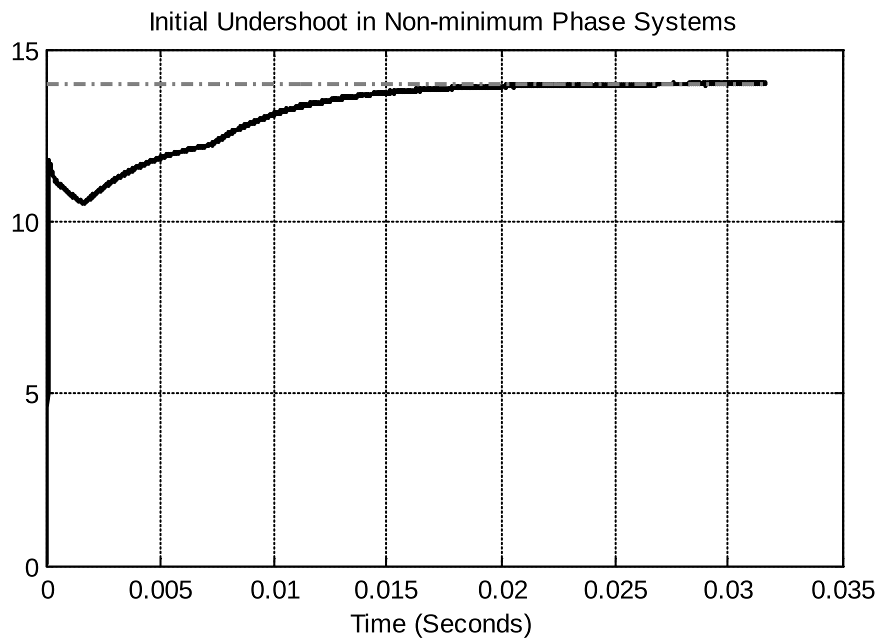

Figure 3.1 Initial undershoot in non-minimum phase buck-boost converter

The above definition implies that the transfer function $G(s)$ is strictly stable, has relative degree 0 or 1, and is strictly minimum-phase. In addition, the Nyquist plot of

$G(j\omega)$ lies entirely in the right half complex plane or the phase shift of the system is always within the range of $\left(-90^{\circ}, 90^{\circ}\right)$.

In order to achieve the objectives as specified in Chapter 1, it is desired that the non-minimum phase control-to-output transfer function of buck-boost converter be converted to SPR/PR, such that the system be converted to a minimum phase system. The transfer function approach is considered here, as a linear small-signal model of buck-boost converter is adopted in parallel compensation approach. The process of designing a compensator is detailed in the following sections.

3.4 Control Structure

The control scheme given to achieve the stated objectives is shown in Figure 3.2. As can be observed in Figure 3.2, the compensation is applied only to the decoupled non-minimum phase control-to-output transfer function. Then voltage regulation can be achieved by the use of a simple proportional controller. The control structure for the buck-boost converter plant is as shown in Figure 3.3. The next section gives the design procedure for the parallel compensator.

3.5 Design of Parallel Compensator for Voltage Regulation Application

In this section the parallel compensator for the control-to-output transfer function is derived. The technique outlined in [35] is adopted for deriving the replacement plant for a buck-boost converter. There are other techniques to accomplish this task such as the transmission zero assignment technique [34]. This technique uses Eigen value assignment technique to place the transmission zeros in the desired positions. Moreover, the complexity of the procedure increases as buck-boost converter is also non-linear in nature. So the following method [35] of deriving the replacement plant for the buck-boost converter control-to-output transfer function has been adopted.

A first order shunt system is taken as the transfer function for the replacement plant. Therefore, the replacement plant is selected in such a way that it is SPR, then the non-minimum phase control-to-output transfer function is compensated to become a minimum phase system. The only criterion to be satisfied for the application of using this kind of replacement plant is that the original open-loop converter control-to-output transfer function should be stable.

The control-to-output transfer function of buck-boost converter (Equation 2.8) is stable as the denominator polynomial is Hurwitz. Hence, all the poles of the buck-boost converter are in the left-half plane and the open-loop transfer function is stable.

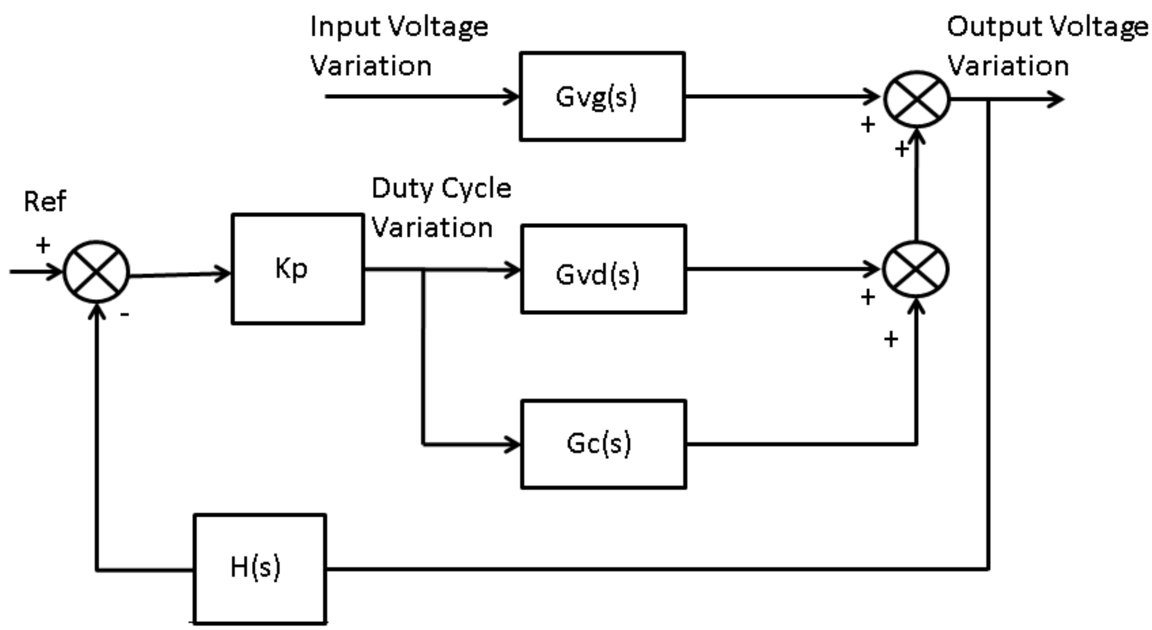

Figure 3.2 Proposed control scheme for a small-signal buck-boost converter model

Figure 3.4 shows the interconnection structure of the parallel compensator $G_c(s)$ [35] to the non-minimum phase plant $G(s)$ such that the replacement plant, shown by the dashed box is SPR

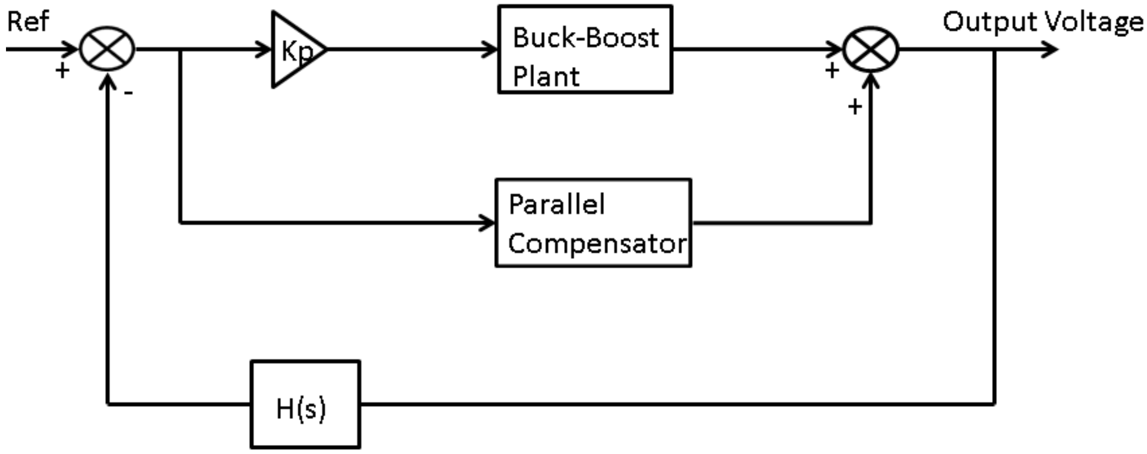

Figure 3.3 Control structure for buck-boost converter plant

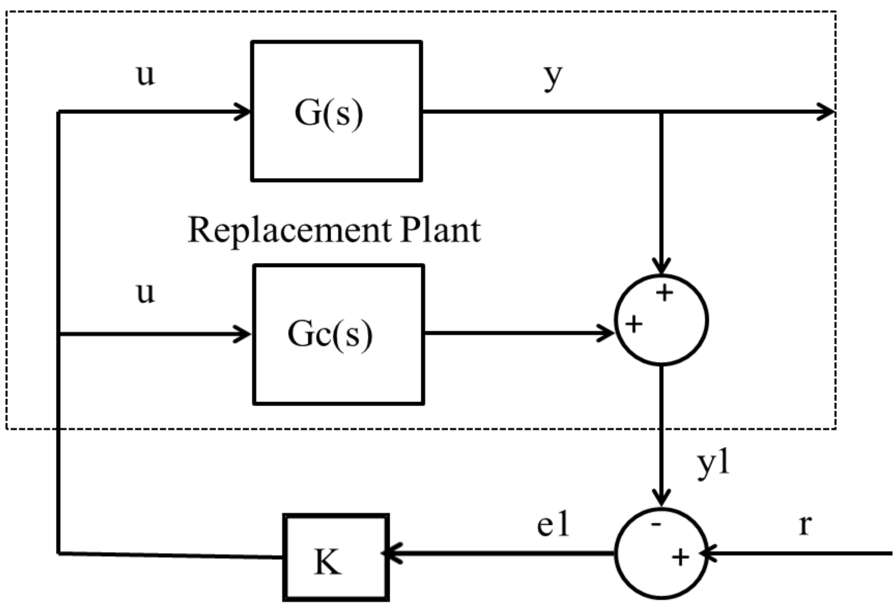

Figure 3.4 Replacement plant structure as shown in dashed box [35]

The non-minimum phase transfer function for which a replacement transfer function is to be derived is given as below:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{L(s)}{M(s)} \tag{3.3}$$

Then the transfer function of the parallel compensator which is to be connected in parallel to the above non-minimum phase plant is given as:

$$G_c(s) = \frac{Y_c(s)}{U(s)} = G_1(s) - G(s) \tag{3.4}$$

The transfer function of the replacement plant, which is minimum phase, is given by

$$\begin{aligned} G_r(s) = \frac{Y_1(s)}{U(s)} &= G(s) + G_c(s) \\ &= G(s) + G_1(s) - G(s) = G_1(s) \end{aligned} \tag{3.5}$$

The transfer function of the stable replacement plant $G_1(s)$ is chosen as,

$$G_1(s) = \frac{k_o}{Ts+1} \tag{3.6}$$

where the constant k_o is given [35] as:

$$k_o = G(0) \tag{3.7}$$

From Equation 3.6 it can be said that $G_1(s)$ is SPR as the relative degree is one, minimum-phase and stable. The condition to be satisfied by the original plant and the replacement plant is that steady state value of the original plant should be equal to the steady state value of the replacement plant as:

$$G_1(0) = G(0) \tag{3.8}$$

Now for the buck-boost control-to-output non-minimum phase transfer function given as

$$G_{vd}(s) = k\left(-\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{\left(1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}\right)} \tag{3.9}$$

The replacement plant transfer function is as given below and is derived based on the procedure outlined above:

$$G_{vd1}(s) = \frac{l}{1 + sT} \quad , \quad l = -\frac{(Vg - V)}{D'^2}k \tag{3.10}$$

Thus, the parallel compensator using Equation 3.4 for the buck-boost converter control-to-output transfer function is given as:

$$G_c(s) = \frac{l}{1 + sT} - \left(-\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{1 + \frac{sL}{D'^2 R} + s^2\frac{LC}{D'^2}} \tag{3.11}$$

where the gain k is a proportional constant which is known and the time constant T is a positive number. It has been observed in simulations that the smaller the time constant T, the faster the response of the system.

Therefore, the equivalent compensation structure is as shown in Figure 3.6 and the design procedure for the compensator can be illustrated as shown in Figure 3.5.

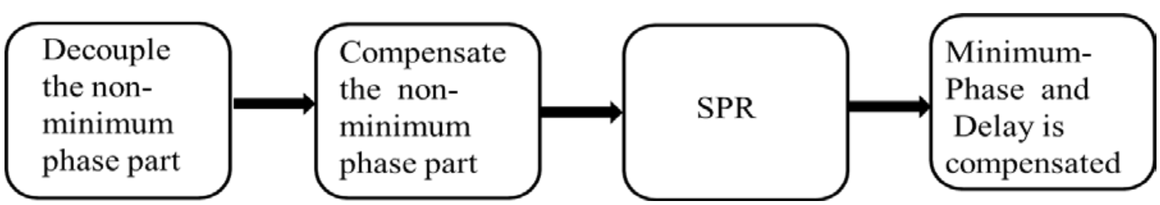

Figure 3.5 Parallel compensation design process

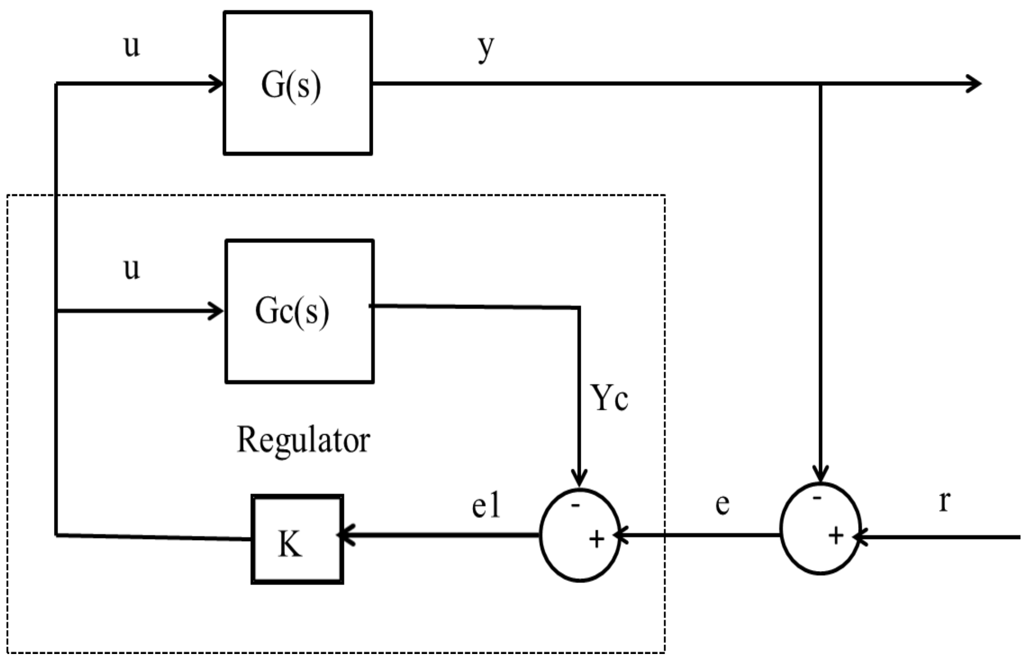

Figure 3.6 Equivalent compensation structure [35]

3.6 Stability

To understand the effect of the parallel compensator on the closed loop stability of a buck-boost converter, the bode diagram of the converter control-to-output transfer functions with and without compensation is provided in Figure 3.7. It can be observed that the phase of the compensated system is in the range of $\left(-90^\circ, 90^\circ\right)$ satisfying the requirement of SPR system.

3.7 Simulation Results

The simulations are carried out on the buck-boost converter with the following parameters: $R = 10\Omega, L = 100\mu H, C = 1mF, V_g = 30V$. The derived math model of the buck-boost converter using state-space averaging (Equations 2.7-2.8) is represented mathematically in Matlab.

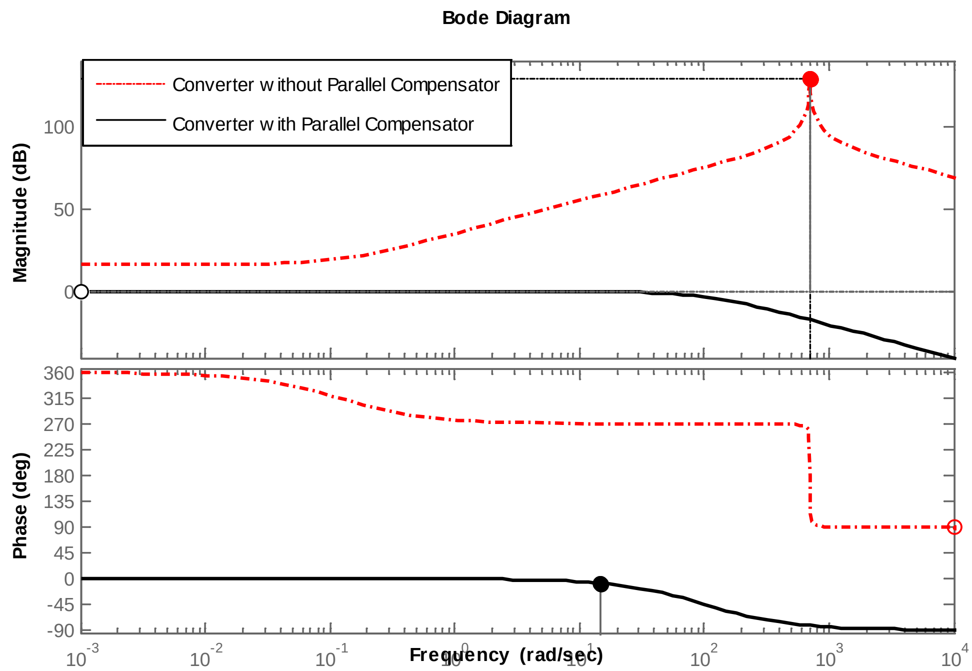

Figure 3.7 Bode plot of the compensated and uncompensated System

Figures 3.8 - 3.10 show the converter math models in Matlab. The compensator designed for a buck-boost converter is incorporated into these math models as shown in Figure 3.8. The effect of the compensation can be observed in Figure 3.11. This figure also illustrates the comparison of the effect of parallel compensation on the voltage regulation profile of the converter. It was assumed that the input voltage was 30 V with 1 V variation at frequency 1000 rad/sec.

Figure 3.11 demonstrates the output voltage profile of the converter controlled by a parallel compensator approach. The non-minimum phase zero generated significant transients when the reference changed from buck to boost. However, when the compensator added, it neutralized the effect of the transients and as the Figure 3.11 illustrates, the oscillations resulted from the input source variation were removed.

Figure 3.12 shows the performance comparison between the buck-boost converter controlled by the parallel compensation approach and by the proportional integral controller on a buck-boost SimPowersystem simulation model with the following parameters: $R = 100\ \Omega$, $L = 1\ mH$, $C = 1\ mF$ and $V_g = 12$ V.

It is clearly observed that the undershoots and overshoots in the system have been compensated and the delay in the transient response is reduced considerably. The controller has resulted in a longer settling time, which is limited using parallel compensators. The best design will have a tradeoff between overshoot and settling time [37, 38].

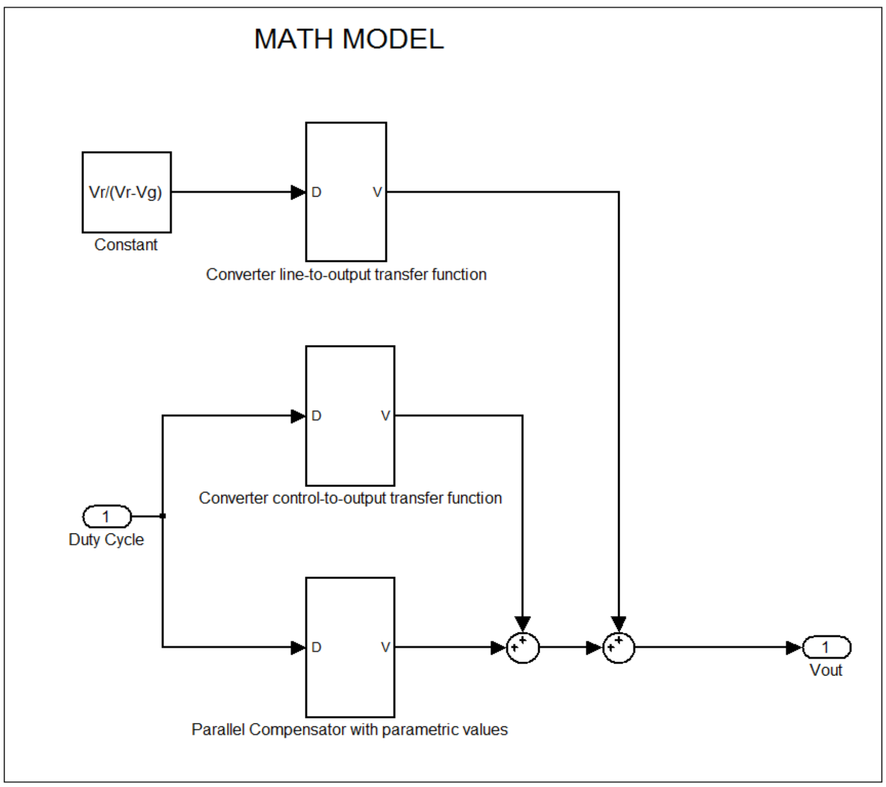

Figure 3.8 Math model of buck-boost converter in Simulink

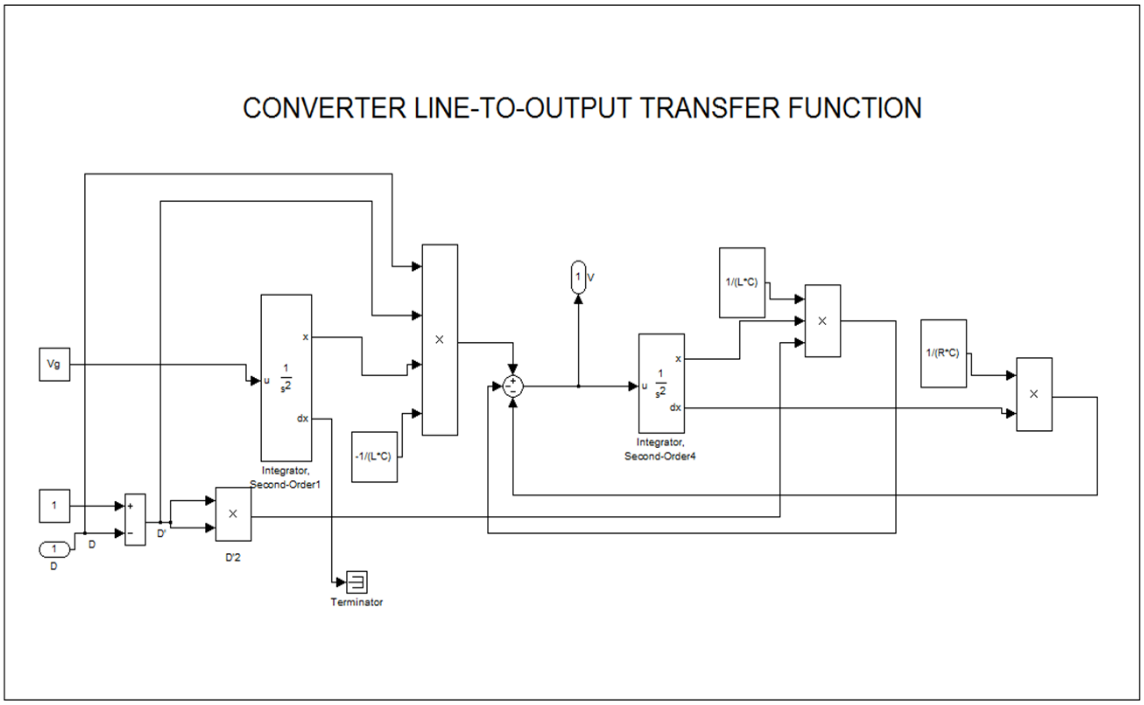

Figure 3.9 Math model of the converter line-to-output transfer function

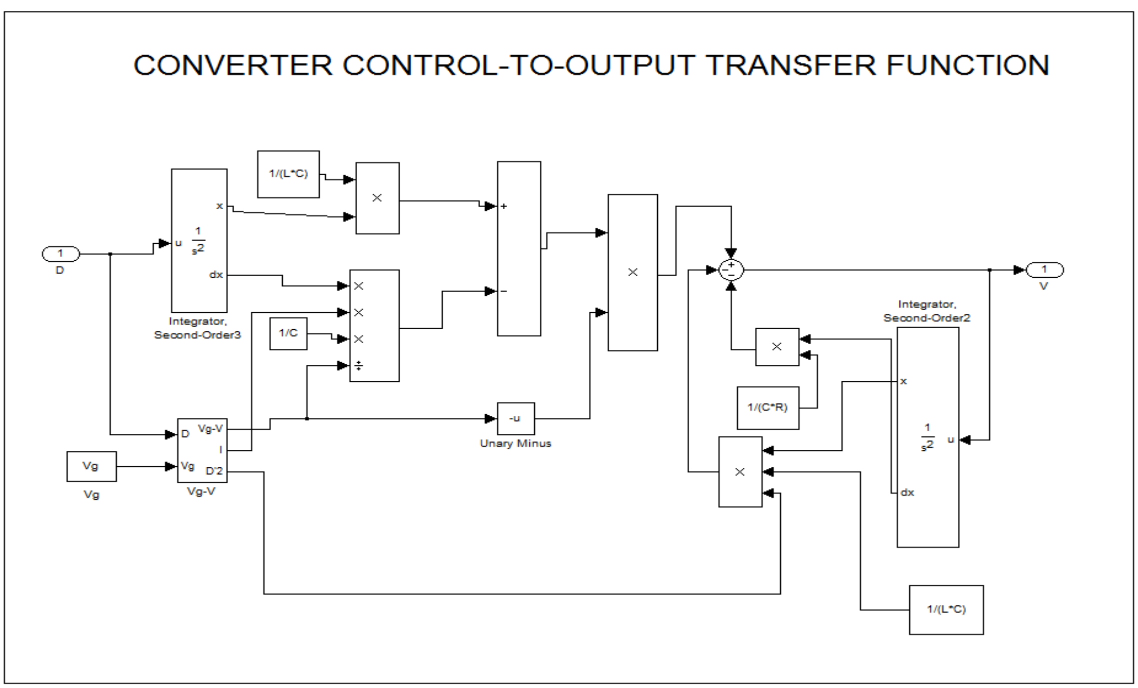

Figure 3.10 Math model of the converter control-to-output transfer function

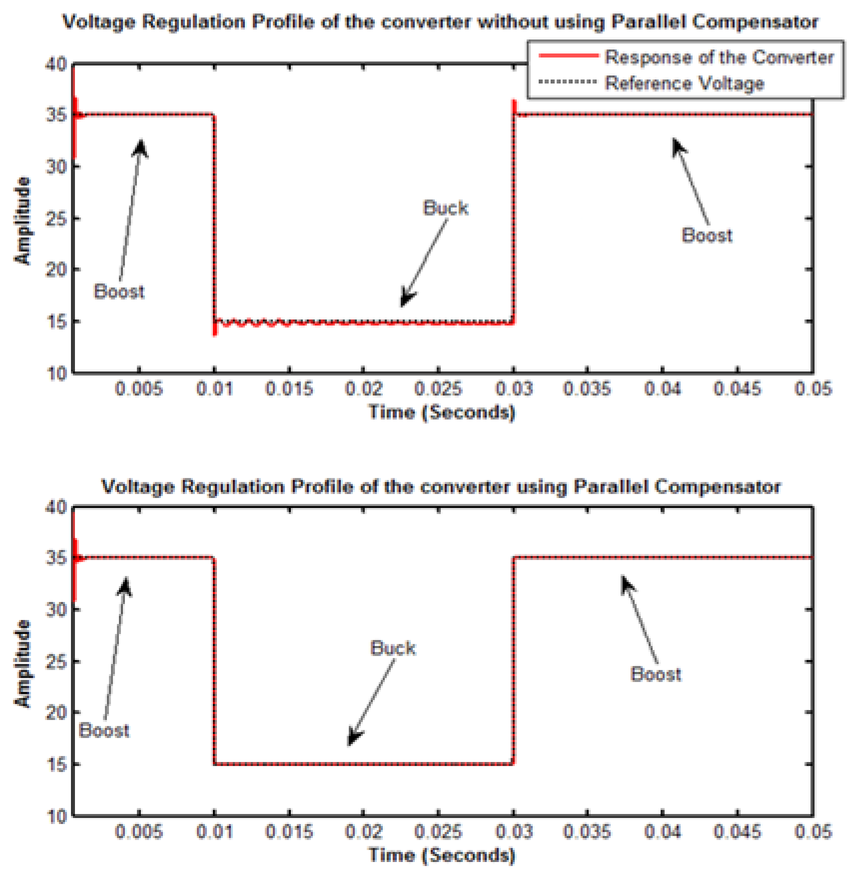

Figure 3.11 Comparisons of voltage profiles of compensated and uncompensated system

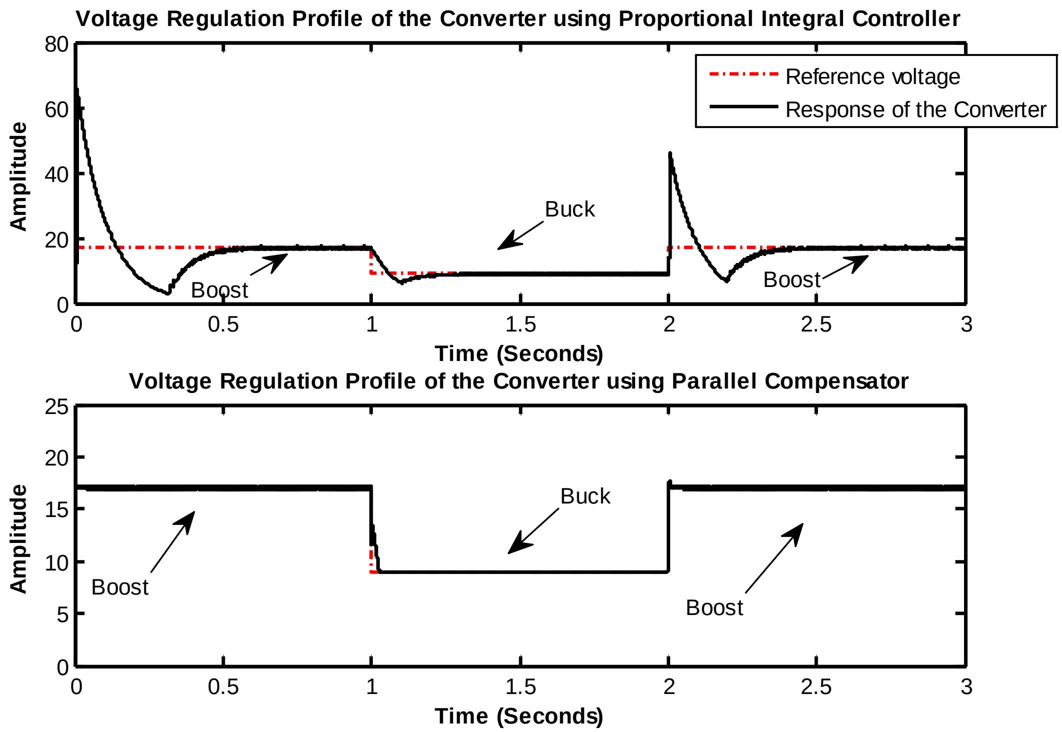

Figure 3.12 Comparison of voltage profile with a compensator and a PI controller

4. FEEDFORWARD EXCESS PASSIVITY BASED CONTROL (FFEP)

4.1 Motivation

In Chapter 3, the parallel compensation approach was introduced and the effectiveness of the approach was validated through the simulation results. The compensator in Equation 3.11 derived for the buck-boost converter resulted in a good performance. However, the design procedure for the compensator was based on a linearized small-signal model and cumbersome. A linear system model does not accurately depict the behavior of the original system, leaving nonlinearities un-controlled or over simplified. As a solution to this problem, feed-forward excess passivity based control is proposed in this chapter. This control approach is also carried out on buck-boost converter as an example.

Feed-forward excess passivity based (FFEP) control is an energy shaping approach [39], which exploits the damping of the system. A large-signal non-linear model for the buck-boost converter, instead of a small-signal linearized model, is considered as the system model. For a non-linear buck-boost converter, as seen in Chapter 2, the output capacitor voltage exhibits non-minimum phase behavior. As a result, the zero dynamics of the output capacitor voltage are unstable and the simple problem of regulating the output voltage becomes challenging.

Existing controllers [10] indirectly regulate the voltage through the inductor current, as the zero dynamics of the inductor current are stable. However, in the indirect approach, the system performance is subject to circuit parameter and load variations and additional adaptive controllers are required to achieve a satisfactory performance.

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This, in turn, tends to increase the complexity of the system. In this research, FFEP control is proposed as a solution and aims to achieve direct regulation of non-minimum output voltage. It has also been verified through simulation and experimental results that the system performance is not significantly affected by the load variations and does not call for additional advanced controllers.

FFEP control uses the principle of passivity as a design tool. It has been investigated in Chapter 2 that the buck-boost converter system is not passive when the output is the capacitor voltage. It is due to the non-minimum phase behavior of the capacitor voltage. In FFEP control, direct regulation of voltage is achieved by making the open-loop buck-boost converter passive when the output is the capacitor voltage. In order to attain passivity in the converters, the damping of the converter system i.e. the degree of passivity need to be modified.

In FFEP control, it is shown that a parallel interconnection to the open-loop system can achieve exponential stability of the zero dynamics of the output voltage. An excess passive system is used to compensate for the shortage of passivity in the buck-boost converter to reduce the non-minimum phase behavior. To achieve passive system, the degree of passivity in the system is characterized from passivity indices rather than from the system's energy function.

The noteworthy feature of FFEP control lies in its simplicity and its effectiveness. Though, different solutions [40, 41] to the problem of direct regulation of non-minimum phase voltage have been previously proposed, they rely on circuit level energy descriptions whereas FFEP control which is based on a system level description. Chapter 5 compares the two methods of approach and shows the merit of FFEP over [40, 41]. The following sections of Chapter 4 provide detailed information regarding the FFEP control

It has to be pointed out that the motivation for FFEP control for non-minimum phase power converters is inspired from process control [44] and networked control systems

[43]. The concept of compensating for the shortage of passivity through system interconnection has been thoroughly explored in applications relating to pH process control, heat distillation columns, robotic manipulators [42], and in large interconnected systems. The novelty of this research lies in the identification and application of this concept to non-minimum phase power converters, in order to achieve simple effective solution for the problem of direct regulation of non-minimum phase voltage.

4.2 Passivity Indices

In FFEP control, the degree of passivity in a system [24, 26, 45], as an index to measure the passivity, is quantified by passivity indices. The passivity indices indicate either excess or shortage of passivity. The following two definitions [26] can be given with regard to excess and shortage of passivity.

Definition 4.1: **Output Feedback Passive (OFP)** [26] : System H is said to be OFP, if it is dissipative with respect to supply rate $w(u,y) = u^T y - \rho y^T y$, for some $\rho \in \Re$. Where ρ is the largest gain that can be placed in positive feedback with a system, such that the interconnected system is passive, as shown in Figure 4.1.

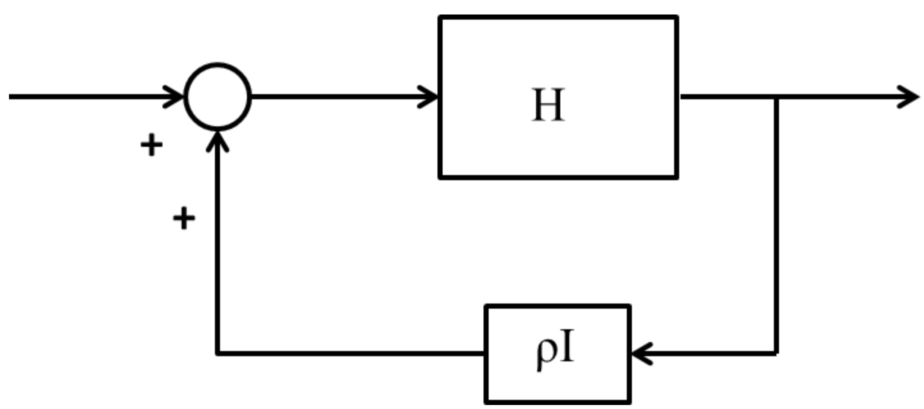

Figure 4.1 Output feedback passivity [26]

Definition 4.2: **Input Feedforward Passive (IFP)** [26] : System H is said to be IFP, if it is dissipative with respect to supply rate $w(u,y)=u^T y-\upsilon u^T u$, for some $\upsilon \in \Re$. Where υ is the largest gain that can be put in negative parallel interconnection with a system, such that the interconnected system is passive, as shown in Figure 4.2.

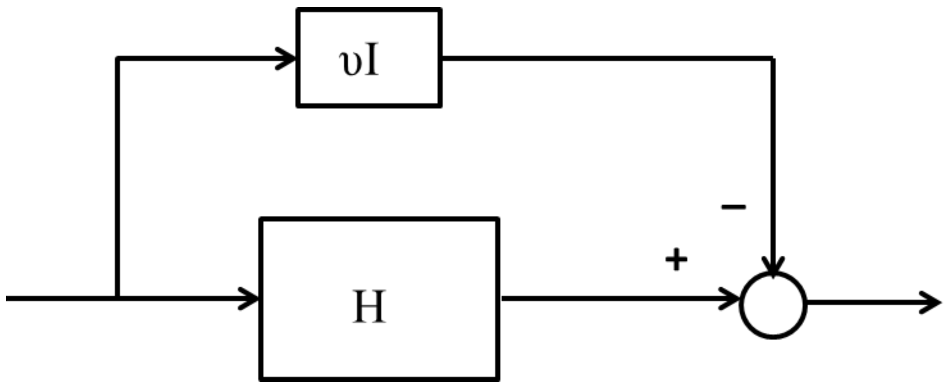

Figure 4.2 Input feedforward passivity [26]

The indices ρ and υ indicate the level of passivity in the system. A passive system has both the indices ρ and υ positive or zero. A system which is not passive has one of the indices positive and the other negative. Positive indices indicate that the system has an excess of passivity and negative indices indicate shortage of passivity.

A system which is not passive can be made passive by either feedback passivation or by feedforward passivation [26]. An unstable system has shortage of OFP and is characterized by OFP$(-\rho)$. This system can be made passive by negative feedback, only if the system is minimum phase and has relative degree one or zero.

A non-minimum phase system has a shortage of IFP and is characterized by IFP$(-\upsilon)$. This system can be made passive by positive feedforward only if the system is stable. A system which is unstable and non-minimum phase cannot be made passive with any combination of feedback and feedforward gains [26].

The technique of feedback passivation [46] is quite popular and makes use of feedback to render a system passive. However, the relative degree and zero-dynamics of a system are invariant under feedback, and any system irrespective of its relative degree whose zero dynamics with respect to the output are not minimum phase, cannot be made passive via feedback. Buck-boost converters fall under the category of a system whose relative degrees is one and has unstable output voltage zero dynamics. Therefore, direct voltage regulation of non-minimum buck-boost converter can be made possible only by feedforward passivation [26].

It should be noted that more general definitions for the supply rate functions can be used other than given in Definition 1 and Definition 2, to simultaneously obtain the IFP and OFP indices. Some such supply rates [26] can be given as:

$$w(u,y) = y^T u - \upsilon^T(u)u - \rho^T(y) \tag{4.1}$$

or

$$w(u(t), y(t)) = y^T(t)Qy(t) + 2u^T(t)Sy(t) + u^T(t)Ru(t) \tag{4.2}$$

where $Q, R, S \in \Re^{m\times m}$ are constant weighting matrices, with matrices Q and R being symmetrical.

Note 4.1: The following general statement should be recalled with regard to classical control. Feedback structure in a system affects the poles of the system i.e. the system stability, and Feedforward structure in a system effects the zeros of the system i.e. zero dynamics.. Therefore, feedback passivation applied on a system does not influence the zero dynamics and relative degree of the system. Similarly feedforward passivation cannot influence the free dynamics of the system i.e. the stability of the system.

4.3 Passivity in Relation to Stability and Positive Realness

The concept of stability, passivity and positive realness are interrelated and this section explores their relationship. A passive system is stable when it is unforced i.e. when the input $u = 0$. This relation between passivity and stability can be formally given prior to which certain definitions like zero-state observability (ZSO) and zero-state detectability (ZSD) are to be stated.

Definition 4.3 ZSO and ZSD [26]: A system $H:\begin{cases}\dot{x}=f(x,u)\\ y=h(x,u)\end{cases}$ is ZSO if for any $x\in X$,

$$y(t)=h(\phi(t,t_0,x,0))=0,\quad \forall t\geq t_0\geq 0\qquad \textit{implies}\ x=0 \tag{4.3}$$

and the system is locally ZSO if there exists a neighborhood X_n of 0, such that for all $x\in X_n$, Equation 4.3 holds. The system is ZSD if for any $x\in X$,

$$y(t)=h(\phi(t,t_0,x,0))=0,\quad \forall t\geq t_0\geq 0\quad \textit{implies}\quad \lim_{t\to\infty}\phi(t,t_0,x,0)=0 \tag{4.4}$$

and the system is locally ZSD if there exists a neighborhood X_n of 0, such that for all $x\in X_n$ Equation 4.4 holds.

The following theorem relates passivity and stability.

Theorem 4.1 Passivity and Stability [24]: The passive system H with a C^1 storage function S and $h(x,u)$ be C^1 in u for all x. Then the following properties hold:

i. If S is positive definite, then the equilibrium $x=0$ of H with $u=0$ is stable
ii. If H is ZDS, then the equilibrium $x=0$ of H with $u=0$ is stable
iii. When there is no throughput, $y=h(x)$, then the feedback $u=-y$ achieves asymptotic stability of $x=0$ if and only if H is ZSD.

where C^1 of a function indicates that the derivate of the function exists and is continuous.

A system is said to be positive real if the following relation holds [26]:

$$\int_{t_0}^{t_1} y^T(t)u(t)dt\geq 0,\quad \forall t_1\geq t_0\geq 0 \tag{4.5}$$

From Equation 2.36 it can be inferred that a positive real system is passive and vice versa.

4.4 Exponential Stability and Passivity Indices of Buck-Boost Converter

The stability of a nonlinear system can be given by various definitions, depending on the degree of stability in the system. A nonlinear system can have different equilibriums unlike a linear system [26]. So the stability in a non-linear system is with respect to the individual equilibrium point. It is very likely that some equilibrium points in a non-linear system are stable and some of them from the same system are unstable. The following definition is the stability of the system in the sense of Lyapunov.

Definition 4.4 Stability in the sense of Lyapunov [25]: The equilibrium state $X = 0$ is said to be stable if for any $R > 0$, there exists $r > 0$, such that if $\|x(0) < r\|$, then $\|x(t)\| < R$ for all $t \geq 0$. Otherwise, the equilibrium point is unstable.

Definition 4.4 implies that for a stable system the state trajectories, originating from a set of initial states, are confined to a bounded region of a certain radius. Definition 4.4 can be said to be an unconstrained definition of stability and does not give indicate how fast the system trajectories converge to the bounded region.

The following definition can be given with regard to asymptotic stability.

Definition 4.5 Asymptotic Stability [25]: An equilibrium point 0 is asymptotically stable if it is stable, and if in addition there exists some $r > 0$ such that $\|X(0)\| < r$ implies that $X(t) \rightarrow 0$ as $t \rightarrow \infty$.

Asymptotic stability gives an estimate about the convergence of the system state trajectories, and is a strong definition of stability than the definition of stability in the sense of Lyapunov. Exponential stability of the system has faster rate of convergence of system states than asymptotic stability and is given by Definition 4.6.

Definition 4.6 Exponential Stability [25]: A system is exponentially stable [25] if there exist two strictly positive numbers α and λ such that,

$$\forall\ t > 0, \quad \|x(t)\| \le \alpha \|x(0)\| e^{-\lambda t} \tag{4.6}$$

In some ball B_r around the origin.

Hence the following order of precedence can be given with regard to how fast the system trajectories converge.

$$\textit{Stability in the sense of Lyapunov} \Rightarrow \textit{Aymptotic Stability} \Rightarrow \textit{Exponential Stability}$$

The above distinction between the degrees of stability in a system is vital, especially when the system comprises of several interconnections between different sub-systems. The interactions between the sub-systems influence the stability of the overall system, so it is critical to mark the degree of stability of each sub-system.

In this research on FFEP control, a more constrained definition of stability is imposed on buck-boost converter, which is justified in Section 4.10. The following arguments can be given with regard to exponential stability of the buck-boost converter.

To demonstrate the exponential stability of the output voltage of the buck-boost converter, characteristic equation of the buck-boost converter is used. When the switch Q in Figure 2.1 is on or off mode, the output voltage x_2 is in charging or discharging mode of operation respectively. In either case, a general form of this state variable can be expressed as

$$x_2 = Ae^{-\lambda t} \tag{4.7}$$

where A is a negative coefficient, as the buck-boost converter is an inverting circuit, and λ is the time constant of the circuit defined as:

$$\lambda = \frac{1}{RC} \tag{4.8}$$

In an RLC circuit, analyzed separately in two modes of buck-boost operation, x_2 can be expressed as

$$x_2 = A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t} \tag{4.9}$$

where λ_1 and λ_2 are the circuit time constants and

$$\begin{aligned} \lambda_1 &= \frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \\ \lambda_2 &= \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \end{aligned} \tag{4.10}$$

In a physically realizable circuit, the time constants λ_1, λ_2 are strictly positive values. Hence, the state x_2 of the buck-boost system, which is the output voltage, converges to the origin exponentially. Thus, buck-boost converter is exponentially stable.

Note 4.2: The argument (Equations 4.7-4.10) with regard to exponential stability of the buck-boost converter when the output is the capacitor voltage may be misleading, with the statement that the zero dynamics of the output capacitor voltage (Equation 2.33) are unstable. In order to clarify, another definition of zero dynamics need to be highlighted from Brynes and Isidori's pioneering work [49] on zero dynamics of a nonlinear system. For a nonlinear system of the form (Equation 2.16), which is decomposed into observable and unobservable components, the zero dynamics of a system can be considered as the internal dynamics of the unobservable component. Hence, stability of the system i.e. observable part of the system does not imply stability of the unobservable component i.e. the zero dynamics, which is the cause of all complications.

IFP and OFP passivity indices of the buck-boost converter system can be obtained by considering the buck-boost converter system in its original bilinear representation (Equation 2.15) with a positive definite storage function,

$$S(x) = \frac{1}{2} L x_1^2 + \frac{1}{2} C x_2^2 \tag{4.11}$$

and by considering a more general definition for the supply rate (Equation 4.1)

$$w(t) = u^T y - \rho y^T y - \upsilon u^T u = uV_{in} x_2 - \rho x_2^2 - \upsilon (uV_{in})^2 \tag{4.12}$$

and the following relation is given

$$\dot{S}(x) = uV_{in} x_1 - \frac{x_2^2}{R} \le |\upsilon| (uV_{in})^2 - \left(\rho x_2^2 - uV_{in} x_2\right) \tag{4.13}$$

It can be observed that $\rho = \frac{1}{R}$ is positive and $|\upsilon| > x_1$ is negative, depicting a shortage of IFP $(-\upsilon)$ and an excess of OFP (ρ) in the output voltage of buck-boost converter. From the Definitions 4.1 and 4.2, since the buck-boost converter with output defined as capacitor voltage is IFP $(-\upsilon)$ and exponentially stable from Equations 4.7 - 4.10, the buck-boost converter with the output capacitor voltage can be made passive by feedforward passivation. Dynamic feedforward compensation is used to passivate the buck-boost converter instead of static gain compensation.

4.5 FFEP Control Structure

The following background is provided with regard to the design of the dynamic feedforward compensation. Polynomial systems are large classes of nonlinear dynamic systems which can be categorized into subclasses of bilinear and quadratic systems. These subclass systems can approximate an analytic system with linear inputs (ALS) [47]. An interesting property exists for ALS systems and is based on Theorem 4.2 [48] and is repeated here for convenience of the readers.

Theorem 4.2 [48]: The Jacobi linearized zero dynamics of an ALS at an operating point x_o can be changed at least in the neighborhood of x_o by a linear compensator. If the differential degree d_c of the linear compensator with the transfer function $F_c(s)$ is sufficiently large, a possibly unstable zero dynamics of a given ALS can be changed by the free designable parameters of the compensator in such a way that the substitute system $\sum_s$ has a stable zero dynamics.

Bilinear systems can approximate ALS, therefore, unstable zero dynamics of a buck-boost converter can be stabilized by linear compensators connected in parallel to the converter. Therefore, the control structure for the ALS systems is as given in Figure 4.3 [48].

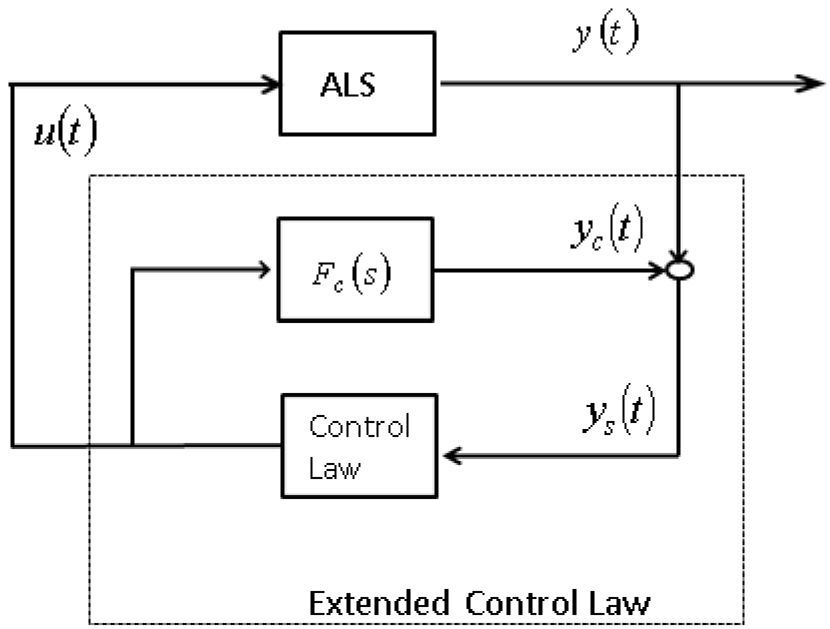

Figure 4.3 ALS system compensation structure [48]

This justifies that a linear compensator can be used to achieve dynamic feedforward passivation of the open-loop voltage control of buck-boost converters. Figure 4.4 shows the structure of the proposed FFEP control.

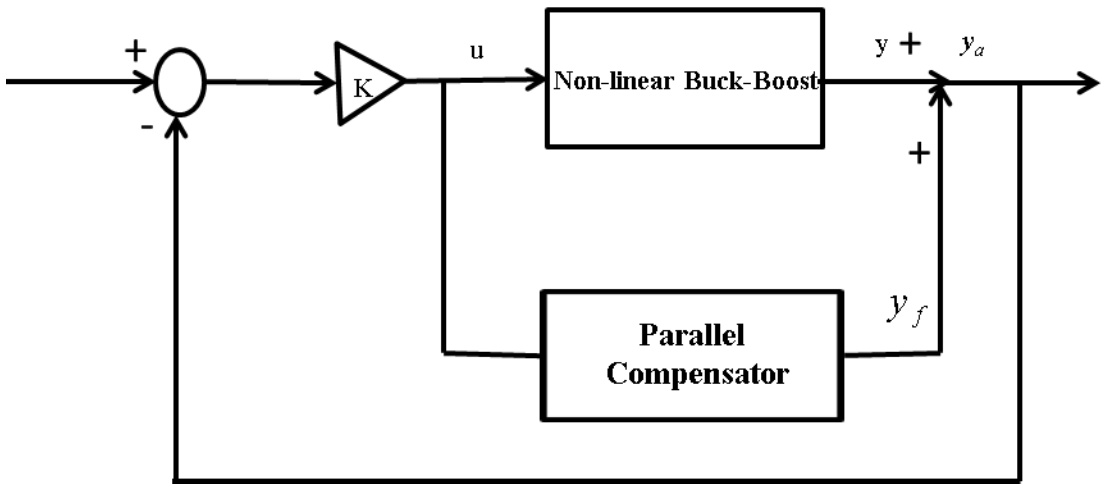

Figure 4.4 Proposed FFEP control structure

4.6 On the Choice of Linear Compensator

The objective of FFEP control is to achieve direct regulation of non-minimum phase voltage of the buck-boost converters. As stated in previous chapters, buck-boost converters have shortage of passivity i.e. $IFP(-\upsilon)$ and can be made passive by dynamic feedforward passivation. Prior to carrying out dynamic feedforward passivation, it was verified that the buck-boost converter open-loop system was stable. A tighter bound on the stability of buck-boost converter was provided by imposing exponential stability. Then, it has been shown that a linear compensator could be used to achieve dynamic feedforward compensation. This section further explains the reasons involved in the choice of linear compensator for the dynamic feedforward passivation.

It is a known fact that a non-minimum phase system has inherent delay in the system i.e. phase lag. Usually proportional derivative (PD) controllers are used to improve the phase of the system and decrease the settling time of the system. However, it is a widely known fact that PD controllers degrade the stability of the system [50], as a result are seldom used in control purposes. In [51] a solution to this problem has been intuitively provided, which is taken into account in the design of dynamic feedforward compensator for the buck-boost converter system.

[51] states that an effect of a derivative controller of the form,

$$H(s) = K\left(1 + \frac{s}{s_0}\right) \tag{4.14}$$

can still be achieved without affecting the stability of the system, by using the inverse of derivative controller

$$H^{-1}(s) = \frac{D}{\left(1 + \frac{s}{s_0}\right)} \tag{4.15}$$

where $D = \frac{1}{K}$ is a feedforward configuration of the system.

Hence, the linear compensator in the design of dynamic feedforward compensation is considered to be of the form given by

$$G(s) = \frac{k_f}{s - f_1} \tag{4.16}$$

where f_1 is a negative constant, whose state-space description is given as:

$$\begin{aligned} \dot{z}_f &= f_1 z_f + k_f u \\ y_f &= z_f \end{aligned} \tag{4.17}$$

It is note worth pointing out that the linear compensator is strictly positive real, as the relative degree is equal to one and the poles and zeros of $G(s)$ are in the left-half plane. As $G(s)$is strictly positive real, it can be recalled from section 4.3 that $G(s)$is passive. PID controllers are also passive [26], hence $G^{-1}(s)$ i.e. the derivative controller is passive.

The following sections further detail as to how to constrain the gains of the linear compensator such that the augmented buck-boost converter system becomes and remains passive.

4.7 Lipschitz Continuity

A brief discussion on Lipschitz Continuity is carried out to the constrain the gains of the linear parallel compensator. A nonlinear system is described by a set of differential equations. Consider a system given by the following equation:

$$\dot{x} = f(t, x), \quad x(t_0) = x_0 \tag{4.18}$$

The solution to Equation 4.18 with the given initial condition exists and is unique if and only if the function satisfies the following inequality, called the Lipschitz condition [26]

$$\| f(t, x) - f(t, y) \| \le L \| x - y \| \tag{4.19}$$

where L is called Lipschitz constant. Equation (4.19) can also be written as,

$$\left| \frac{f(x) - f(y)}{x - y} \right| \le L \tag{4.20}$$

From the Equation (4.20), it can be said that the derivative of the function f is bounded by the Lipschitz constant. However, Lipschitz continuity does not imply that the function f is differentiable.

Lipschitz constant depends on the function. It basically defines how the function f varies with changes in the input. If f undergoes a large variation for a small change in input then the function f has a large Lipschitz constant. Therefore, Lipschitz continuity quantifies the continuity of a function f and Lipschitz constant provides an upper bound on the amount of variation. The Lipschitz constant of function also depends on the interval that the function is defined at. If the interval on which the function is defined is changed, then the Lipschitz constant for the same function is different.

If a function f is defined as $f : R^m \times R \to R$ then the following definition can be given:

Definition 4.7: A function f is locally Lipschitz continuous with respect to its first argument if it is continuous and if for each $(x,t) \in R^m \times R$ there exists a $L > 0$ such that

$$\| f(y,s) - f(z,s) \| \le L \| z - y \| \tag{4.21}$$

The above definition is used to obtain the Lipschitz constants of the buck-boost converter system (Equation 4.22)

4.8 Exponential Minimum-phaseness

A system whose zero dynamics are stable is called a minimum phase system. It is shown in Chapter 2 that the zero dynamics of the buck-boost converter are unstable when the output of the buck-boost converter system is the voltage across the capacitor. In FFEP control, the linear compensator (Equation 4.17) in parallel to the buck-boost converter makes the augmented system passive i.e. dynamic feedforward passivation. From Theorem 2.1 in Chapter 2, a passive system has relative degree not greater than one and the zero dynamics are minimum phase. Hence, the gains of the linear compensator are constrained to make the augmented zero dynamics stable.

Now the normal form of the buck-boost converter is given as:

$$\begin{cases} \dot{z}_1(t) = a(z_1, z_2) + b(z_1, z_2)u \\ \dot{z}_2(t) = q(z_1, z_2) \\ y(t) = z_1(t) \end{cases} \tag{4.22}$$

Where,

$$\begin{cases} a(z_1, z_2) = -\dfrac{1}{RC} z_1 - \dfrac{1}{C}\sqrt{2C\left(z_2 + \dfrac{V_{in}}{L} z_1 - \dfrac{z_1^2}{2L}\right)} \\ b(z_1, z_2) = \dfrac{1}{C}\sqrt{2C\left(z_2 + \dfrac{V_{in} z_1}{L} - \dfrac{z_1^2}{2L}\right)} \\ q(z_1, z_2) = \dfrac{V_{in} z_1 - z_1^2}{RLC} + \dfrac{V_{in}}{LC}\sqrt{2C\left(z_2 + \dfrac{V_{in} z_1}{L} - \dfrac{z_1^2}{2L}\right)} \end{cases} \tag{4.23}$$

Then, the augmented system i.e. the parallel compensator (Equation 4.17) in shunt with buck-boost converter is represented as

$$\begin{bmatrix} \dot{z} \\ \dot{z}f \end{bmatrix} = \begin{bmatrix} f_z(z) \\ f_1 z_f \end{bmatrix} + \begin{bmatrix} b_o b(z) \\ b_f \end{bmatrix} u \qquad (4.24)$$
$$y_a = z_1 + z_f$$

where,

$$\begin{aligned} z &= \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T \\ f_z(z) &= \begin{bmatrix} a(z) & q(z) \end{bmatrix}^T \\ b_o &= \begin{bmatrix} 1 & 0 \end{bmatrix}^T \end{aligned} \qquad (4.25)$$

By using the following non-singular co-ordinate transformation,

$$\begin{bmatrix} z_{a\xi} & \eta_a^T \end{bmatrix}^T = \phi_z(z, z_f) \qquad (4.26)$$

where

$$\begin{aligned} z_{a\xi} &= z_{a1} \\ \eta_a &= \begin{bmatrix} \eta_{a1} & \eta_{a2} \end{bmatrix}^T \\ z_{a1} &= y_1 + y_f = z_1 + z_f \\ \eta_{a1} &= z_1 - \frac{1}{k_f} b(z,t) z_f \\ \eta_{a2} &= z_2 \end{aligned} \qquad (4.27)$$

The normal form of the augmented system is given by

$$\begin{cases} \dot{z}_{a\xi} = \dot{z}_{a1} = a(z) + f_1 z_f + \left(b(z,t) + k_f\right) u \\ \dot{\eta}_a = \begin{bmatrix} \dot{\eta}_{a1} \\ \dot{\eta}_{a2} \end{bmatrix} = \begin{bmatrix} a(z) - \frac{1}{k_f} b(z,t) f_1 z_f - \frac{1}{k_f} \dot{b}(z,t) z_f \\ q(z) \end{bmatrix} \\ y = z_{a1} \end{cases} \qquad (4.28)$$

From Equation 4.28 it can be seen that the relative degree of the augmented system is one. The zero dynamics of the augmented system are obtained by zeroing out the output $y = z_{a1} = z_1 + z_f = 0$.

Then the zero dynamics $\dot{\eta}_a$ of the augmented system are given as,

$$\dot{\eta}_a = \psi(0,\eta_a) = \begin{bmatrix} \dot{\eta}_{a1} \\ \dot{\eta}_{a2} \end{bmatrix} = \begin{bmatrix} a(z) - \dfrac{z_f}{k_f}\left[\dot{b}(z,t) + f_1 b(z,t)\right] \\ q(z) \end{bmatrix} \tag{4.29}$$

The zero dynamics $\dot{\eta}_a$ can be expressed as,

$$\begin{aligned} \dot{\eta}_{a1} &= a(z_1,\eta_{a2}) + \frac{f_1}{k_f} b(z_1,\eta_{a2})(z_{a1} - z_1) + \\ &\quad \frac{1}{k_f}\dot{b}(z_1,\eta_{a2})(z_{a1} - z_1) \\ &= a(\eta_{a1},\eta_{a2}) + \{a(z_1,\eta_{a2}) - a(\eta_{a1},\eta_{a2})\} + \\ &\quad \frac{f_1}{k_f} b(z_1,\eta_{a2})(z_{a1} - z_1) + \frac{1}{k_f}\dot{b}(z_1,\eta_{a2})(z_{a1} - z_1) \\ \dot{\eta}_{a2} &= q(\eta_{a1},\eta_{a2}) + \{q(z_1,\eta_{a2}) - q(\eta_{a1},\eta_{a2})\} \end{aligned} \tag{4.30}$$

which can be further written as $z_{a1} = 0$

$$\begin{aligned} \dot{\eta}_a = {} & f_z(\eta_a) + \{a(\eta_{a1}^*,\eta_{a2}) - a(\eta_{a1},\eta_{a2})\}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \\ & \frac{f_1}{k_f} b(\eta_{a1}^*,\eta_{a2})\eta_{a1}^*\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{k_f}\dot{b}(\eta_{a1}^*,\eta_{a2})\eta_{a1}^*\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \\ & \{q(\eta_{a1}^*,\eta_{a2}) - q(\eta_{a1},\eta_{a2})\}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned} \tag{4.31}$$

where

$$f_z(\eta_a) = [a(\eta_{a1},\eta_{a2}) \quad q(\eta_{a1},\eta_{a2})]^T \tag{4.32}$$

and $\eta_{a1}^* = z_1$ as another operating point.

A system (Equation 2.16) is exponentially minimum phase i.e., has exponentially stable zero dynamics [52] if the Lyapunov energy function of the zero dynamics $V_o(z)$ of the system satisfy the following relations [25]:

$$\begin{cases} \alpha_1 \|z\|^2 \le V_o(z,t) \le \alpha_2 \|z\|^2 \\ \dot{V}_o \le -\alpha_3 \|z\|^2 \\ \left\| \dfrac{\partial V_o}{\partial z} \right\| \le \alpha_4 \|z\| \end{cases} \tag{4.33}$$

The zero dynamics of the augmented system (Equation 4.31) are exponentially stable if there exists a positive definite function $V_o(\eta_a)$ such that the time derivative of $V_o(\eta_a)$ satisfies (Equation 4.33).

Note 4.3: Here, exponential stability of the zero dynamics is considered instead of asymptotic stability due to an important stability result given in [49]. For asymptotic stability of zero dynamics of a system to achieve closed-loop stability full state feedback of the system is needed. However if the zero dynamics of the system are exponentially stable, then the closed-loop stability of the system can be achieved using output feedback. Therefore, closed-loop stability of the buck-boost converter requires that the zero dynamics be exponentially stable rather than asymptotically stable.

Time derivative of $V_o(\eta_a)$ results in

$$\dot{V}_o(\eta_a) = \nabla V(\eta_a)^T \cdot \dot{\eta}_a \tag{4.34}$$

The derivative of Lyapunov candidate of the zero dynamics of the augmented system is,

$$\begin{aligned} \dot{V}_o(\eta_a) = \nabla V_o(\eta_a)^T \cdot \dot{\eta}_a = \frac{\partial V_o}{\partial \eta_a} \cdot \dot{\eta}_a = \frac{\partial V_o}{\partial \eta_a} f_z(\eta_a) + \\ \frac{\partial V_o}{\partial \eta_a} \cdot \left\{ a(\eta_{a1}^*, \eta_{a2}) - a(\eta_{a1}, \eta_{a2}) \right\} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \\ \frac{\partial V_o}{\partial \eta_a} \cdot \frac{f_1}{k_f} b(\eta_{a1}^*, \eta_{a2}) \eta_{a1}^* \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \\ \frac{\partial V_o}{\partial \eta_a} \cdot \frac{1}{k_f} \dot{b}(\eta_{a1}^*, \eta_{a2}) \eta_{a1}^* \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \\ \frac{\partial V_o}{\partial \eta_a} \cdot \left\{ q(\eta_{a1}^*, \eta_{a2}) - q(\eta_{a1}, \eta_{a2}) \right\} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned} \tag{4.35}$$

The non-linear differential equations (Equation 4.22) representing the buck-boost converter system ensures the existence and uniqueness of a solution if and only if the system functions satisfy the Lipschitz condition [26]. The Lipschitz condition for the system functions with respect to the first argument are evaluated as,

$$\begin{aligned}
\left\| a\left(\eta_{a1}^{*},\eta_{a2}\right)-a\left(\eta_{a1},\eta_{a2}\right)\right\| &\leq \left(\frac{1}{RC}+\frac{1}{\sqrt{2C}}\frac{V_{in}}{L}\right)\left\|\eta_{a1}^{*}-\eta_{a1}\right\| \\
\left\| b\left(\eta_{a1}^{*},\eta_{a2}\right)-b\left(\eta_{a1},\eta_{a2}\right)\right\| &\leq \left(\frac{1}{\sqrt{2C}}\frac{V_{in}}{L}\right)\left\|\eta_{a1}^{*}-\eta_{a1}\right\| \\
\left\| q\left(\eta_{a1}^{*},\eta_{a2}\right)-q\left(\eta_{a1}^{*},\eta_{a2}\right)\right\| &\leq \left(\frac{V_{in}}{RLC}+\frac{1}{\sqrt{2C}}\frac{{V_{in}}^{2}}{L^{2}}\right)\left\|\eta_{a1}^{*}-\eta_{a1}\right\|
\end{aligned} \tag{4.36}$$

Since the system function $b\left(\eta_{a1},\eta_{a2}\right)$ is Lipschitz continuous, the function $b\left(\eta_{a1},\eta_{a2}\right)$ is bounded, and an upper limit to the bound of the function is evaluated as follows. $b(z,t)$ is expressed as,

$$b(z,t)=\frac{1}{RC}\sqrt{\frac{L}{C}}z_{1}\left(z_{1}-1\right) \tag{4.37}$$

So the upper bound for the function can be found as,

$$b\left(\eta_{a1},\eta_{a2}\right)<\frac{1}{RC}\sqrt{\frac{L}{C}} \tag{4.38}$$

Moreover as the function $b\left(\eta_{a1},\eta_{a2}\right)$ is differentiable and Lipschitz continuous the derivative of $b\left(\eta_{a1},\eta_{a2}\right)$ is bounded by the Lipschitz constant and is given by,

$$\dot{b}\left(\eta_{a1},\eta_{a2}\right)\leq\frac{1}{\sqrt{2C}}\frac{V_{in}}{L} \tag{4.39}$$

A feedforward passivation technique makes the buck-boost converter system passive. It can be inferred that the Lyapunov candidate of the open-loop system contains an energy level less than the maximum energy stored in the augmented system.

Therefore, by evaluating the Lipschitz continuity equations and solving Equation 4.35 and using the relations in Equation 4.33 the following relation for the gain constraints of the parallel compensator are obtained such that the open-loop augmented system is passive:

$$k_f > \frac{V_{in}}{L}\left(\frac{V_{in}R}{L}\sqrt{\frac{C}{2}} + R\sqrt{2C} + 1\right) + 1 \tag{4.40}$$

4.9 Closed-Loop Stability

In section 4.8, the detailed procedure to make the augmented buck-boost converter system passive was outlined. As the control of a passive system can be relatively easy, the control action is achieved by using a simple proportional gain controller as shown in Figure 4.4. From Theorem 4.3 and from Note 4.3 the closed-loop stability of buck-boost converter can be guaranteed.

Theorem 4.3 [24]: If two systems H1 and H2 are passive, then their parallel and feedback interconnections are also passive.

We can infer that the closed-loop system is passive, where H2 is the positive gain, which guarantees the closed-loop stability of the system.

4.10 Bias Effect

Application of a parallel interconnection to the buck-boost converter, in dynamic feedforward passivation, results in a steady state error or bias effect. In order to overcome the offset in the steady state response, a pre-filter [53] of the following form, is used in cascade with the plant.

$$G_f(s) = \frac{a_f s + b_f}{s + \delta} \tag{4.41}$$

where a_f, b_f, δ are positive constants which satisfy

$$\frac{b_f}{a_f} - \delta > 0 \tag{4.42}$$

Then the structure of the buck-boost converter plant with the pre-filter and a linear compensator is as given in Figure 4.5.

Now the dynamic feedforward passivation is carried out on the cascade interconnection of pre-filter and the buck-boost converter. For this to be possible, the modified buck-boost converter system should be stable i.e. cascade connection of the pre-filter to buck-boost converter should be stable. This is required because feedforward passivation cannot influence the stability of the system, recall from Note 4.1. Then the modified FFEP control structure is given in Figure 4.6.

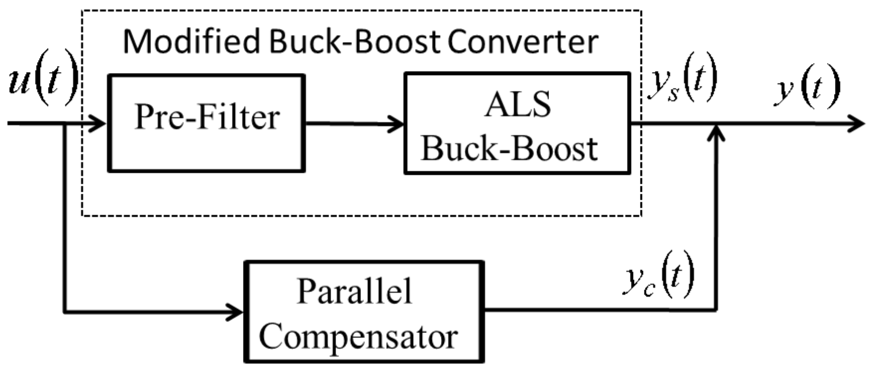

Figure 4.5 Structure of the plant with pre-filter and linear parallel compensator

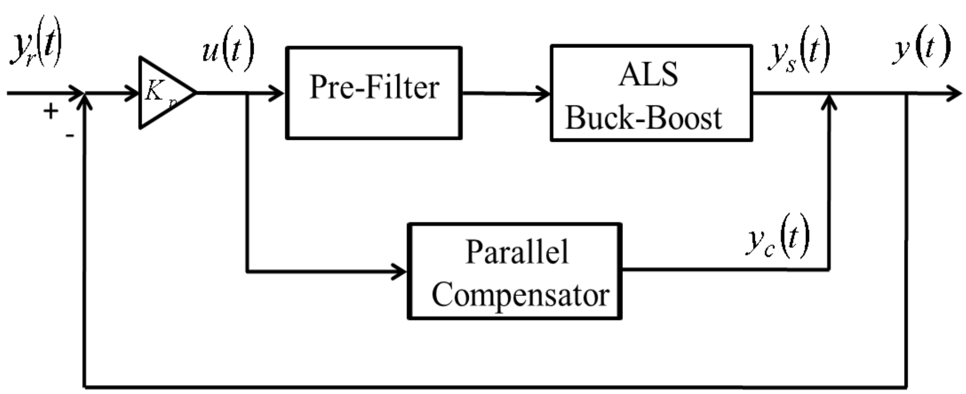

Figure 4.6 Modified FFEP control structure

It is to be recalled that the reason for the imposition of a stronger definition of stability for buck-boost converter in Section 4.4 has been postponed until Section 4.10. The stability of the cascade interconnection of the non-linear buck-boost converter system with the linear stable pre-filter can be achieved only when the buck-boost converter system is exponentially stable. This condition is imposed on buck-boost converter system to avoid a phenomenon called the peaking effect [24].

Peaking phenomenon usually occurs in non-minimum phase systems [24], in which the response of the system has high transients and might lead to instability when high gains are applied. Therefore, in order to decrease the peaking phenomenon to a certain degree, exponential stability of the buck-boost converter is considered. In an exponentially stable system the rate of convergence of system trajectories (Equation 4.6) is rapid, hence peaking and system instability can be avoided to a large extent when a tighter exponential stability is imposed upon the buck boost converter system.

Note 4.4: It is shown in Chapter 4 of [24] that minimum-phase systems are not peaking systems. Hence, when the augmented buck-boost converter system is made passive by dynamic feedforward compensation, it can be inferred that the transient response is not characterized by any peaks. This result can be verified from the buck-boost converter simulation in Figure 4.7.

The following analysis is given with regard to modified buck-boost converter system. The resulting system with buck-boost and pre-filter is given as:

$$\begin{cases} \dot{z}_1 = a(z) + b(z,t)(b_f - \delta a_f)w + b(z,t)a_f u \\ \dot{z}_2 = q(z) \\ \dot{w} = -\delta w + u \\ y = z_1 \end{cases} \tag{4.43}$$

where $z = [z_1 \quad z_2]^T$. Then using the following transformation,

$$w_e = z_1 - b(z,t)a_f w \tag{4.44}$$

the cascaded system is obtained as:

$$\begin{cases} \dot{z}_1 = a_e(z, w_e) + b_e(z,t)u \\ \dot{z}_2 = q(z) \\ \dot{w}_e = q_e(z, w_e) \\ y = z_1 \end{cases} \tag{4.45}$$

where

$$\begin{cases} a_e(z, w_e) = a(z) + \left(\dfrac{b_f}{a_f} - \delta \right)(z_1 - w_e) \\ b_e(z,t) = a_f b(z,t) \\ q_e(z, w_e) = a(z) + \left\{ \dfrac{b_f}{a_f} - \dfrac{\dot{b}(z,t)}{b(z,t)} \right\}(z_1 - w_e) \end{cases} \tag{4.46}$$

Then, the augmented system i.e. the parallel compensator (Equation 4.17) in shunt with buck-boost converter and the pre-filter in cascade is represented as,

$$\begin{bmatrix} \dot{z}_e \\ \dot{z}f \end{bmatrix} = \begin{bmatrix} f_z(z, w_e) \\ f_1 z_f \end{bmatrix} + \begin{bmatrix} b_o b(z) \\ b_f \end{bmatrix} u$$
$$y_a = z_1 + z_f \tag{4.47}$$

where,

$$z_e = [z_1 \quad z_2 \quad w_\varepsilon]^T$$
$$f_z(z, w_e) = [a_e(z, w_e) \quad q(z) \quad q_e(z, w_e)]^T \tag{4.48}$$
$$b_o = [1 \quad 0 \quad 0]^T$$

By using the following non-singular co-ordinate transformation,

$$z' = [z_{a\xi} \quad \eta_a^T]^T = \phi_z(z_e, z_f) \tag{4.49}$$

where

$$\begin{aligned} z_{a\xi} &= z_{a1} \\ \eta_a &= \left[\eta_{a1} \quad \eta_{a2}^T\right] \\ \eta_{a1} &= z_{a2} \\ \eta_{a2} &= \left[z_{a3} \quad z_{a4}\right]^T \end{aligned} \tag{4.50}$$

given as:

$$\begin{aligned} z_{a\xi} &= z_{a1} = z_1 + z_f \\ \eta_{a1} &= z_{a2} = z_1 - \frac{1}{k_f} b_\varepsilon(z,t) z_f \\ \eta_{a2} &= \begin{bmatrix} z_{a3} \\ z_{a4} \end{bmatrix} = \begin{bmatrix} z_2 \\ w_e \end{bmatrix} \end{aligned} \tag{4.51}$$

The normal form of the modified buck-boost converter augmented system is given by:

$$\left\{ \begin{aligned} \dot{z}_{a1} &= \dot{z}_1 + \dot{z}_f = a_e(z,w_e) + f_1 z_f + \left(b_e(z,t) + k_f\right)u \\ \dot{z}_{a2} &= \dot{z}_1 - \frac{1}{k_f} b_e(z,t)\dot{z}_f - \frac{1}{k_f}\dot{b}_e(z,t) z_f \\ &= a_e(z,w_e) - \frac{1}{k_f} b_e(z,t) f_1 z_f - \frac{1}{k_f}\dot{b}_e(z,t) z_f \\ \dot{z}_{a3} &= \dot{z}_2 = q(z) \\ \dot{z}_{a4} &= \dot{w}_e = q_e(z,w_e) \end{aligned} \right. \tag{4.52}$$

Now by following the same procedure, outlined in Section 4.8 to obtain the zero dynamics and constrain the gains of the linear compensator, a new gain constraint relation will be obtained. The next section summarizes the design process of FFEP control.

4.11 FFEP Control Procedure

The following flow chart shows the step-by-step design process of FFEP control approach as shown in Figure 4.7.

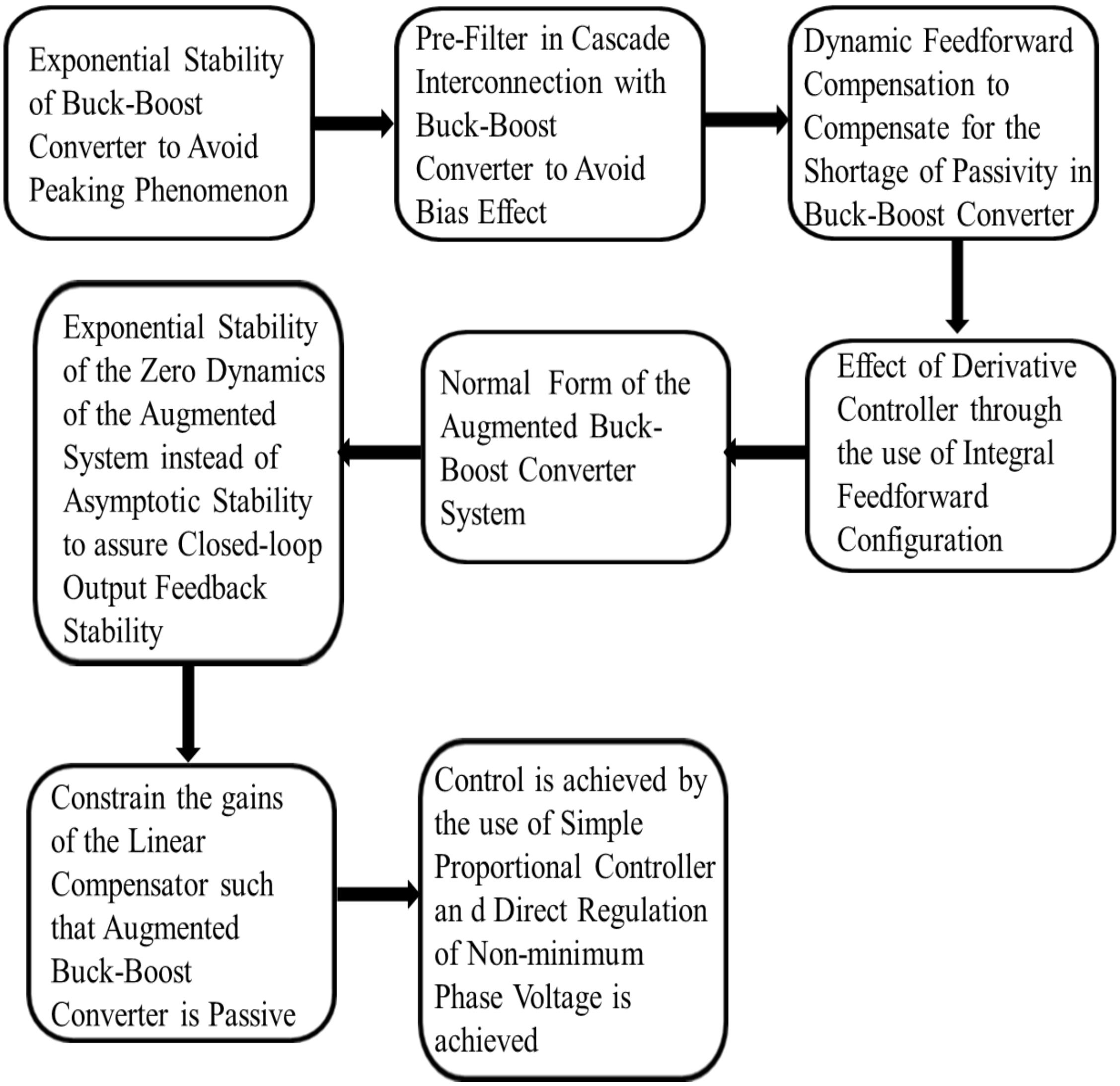

Figure 4.7 FFEP control procedure

5. DISCUSSIONS

5.1 Simulations and Experimental Results

This section discusses the simulations and experimental results obtained using FFEP control approach on buck-boost converter. The FFEP control for direct regulation of voltage is carried out on a buck-boost converter, simulated using SimPowerSystems Toolbox in Matlab environment. The following parameters are adopted for buck-boost converter, resistance $R = 100\Omega$, capacitance $C = 50\mu F$, inductance $L = 10\mu H$, switching frequency $f_s = 20KHz$ and input voltage $V_{in} = 5V$.

Figure 5.1 shows the voltage regulation profile of the buck-boost converter. It can be observed that the delay due to over-shoot and under-shoot has been compensated. Figure 5.2 shows the controller effort and Figure 5.3 shows the contribution of the linear compensator and it can be observed that the bias effect due to the compensator is negligible. Simulations demonstrate the effectiveness of FFEP control and direct regulation of non-minimum phase voltage is achieved.

Experiments were also carried out on the buck-boost converter test bed and tested by implementing the FFEP control using dSPACE rapid-prototyping device. Figure 5.4 shows the voltage regulation profile of the buck-boost converter. The buck-boost converter test bed is operated at 1 KHz switching frequency and the design parameters of the test bed are given in the Appendix.

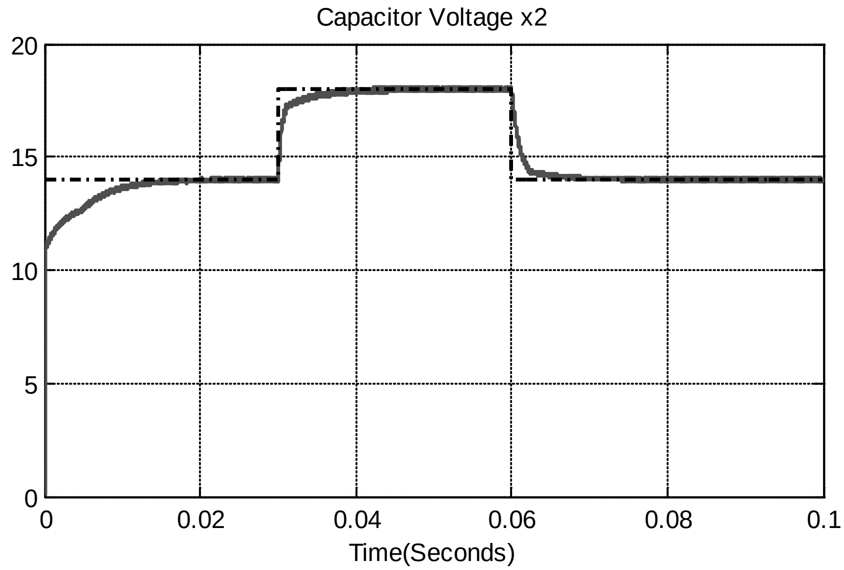

Figure 5.1 Voltage regulation profile of converter after FFEP compensation

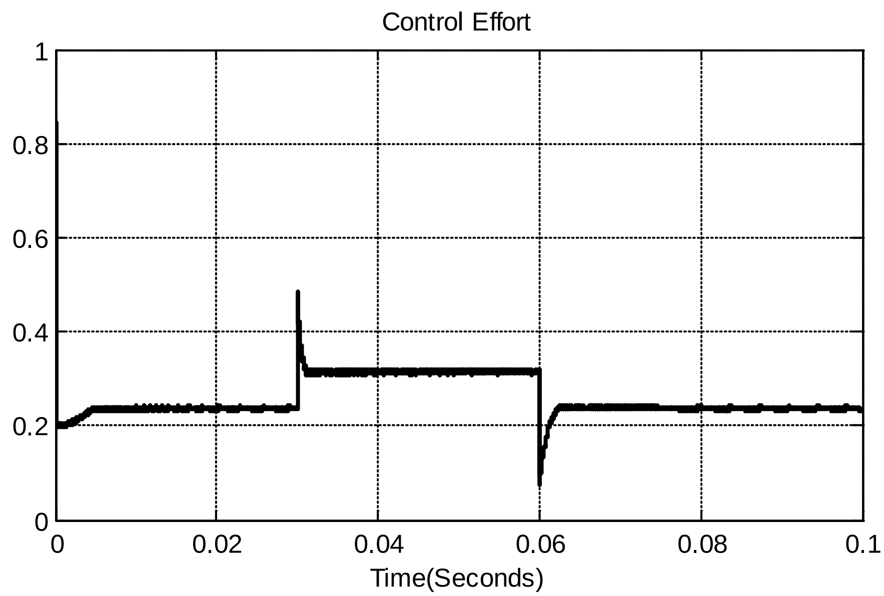

Figure 5.2 Control effort

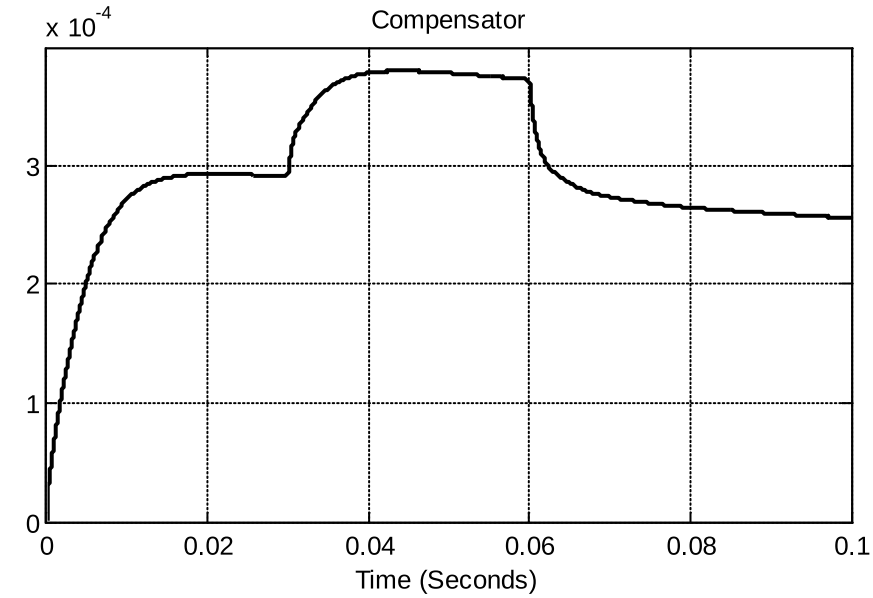

Figure 5.3 Compensator contribution

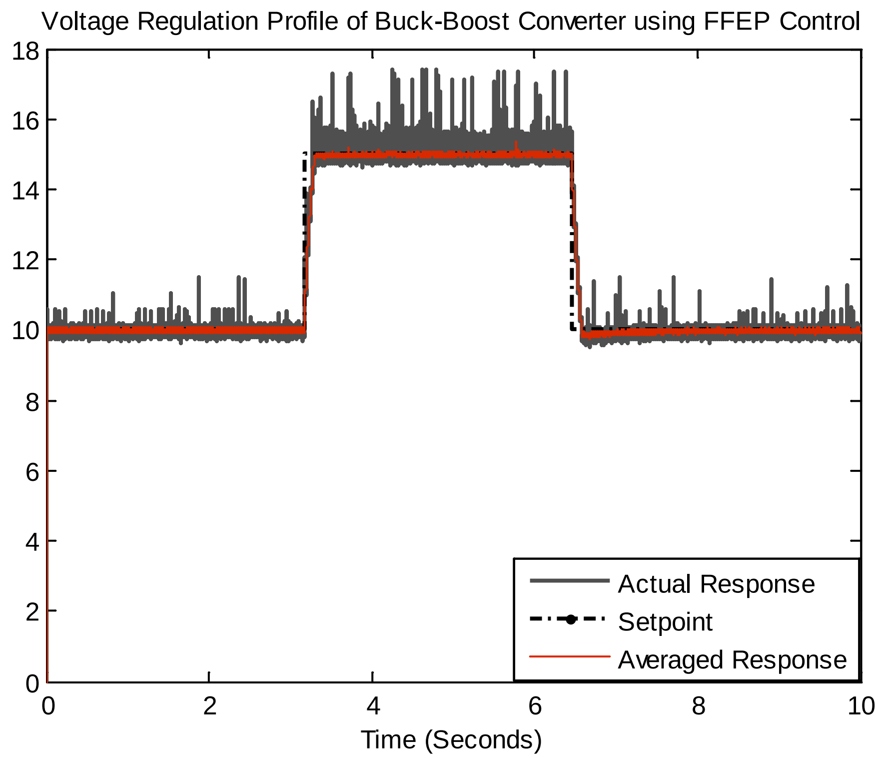

Figure 5.4 Experimental verification of FFEP control

5.2 Performance of FFEP Control in the Presence of Load Variations

In the objectives of the thesis it was stated that using FFEP control the sensitivity of the converter to load variation was reduced significantly. This can be shown by the following argument. It has been shown in Section 4.4 that the buck-boost converter is IFP $(-\upsilon)$ by using a general definition for the supply rate. By considering the actual definition of supply rate given by

$$w(u,y) = u^T y - \upsilon u^T u = uV_{in}x_2 - \upsilon(uV_{in})^2 \tag{5.1}$$

and

$$S(x) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2 \tag{5.2}$$

The following inequalities are obtained,

$$\begin{aligned} &\dot{S}(x) < w(u,y) \\ &uV_{in}x_1 - \frac{x_2^2}{R} < (uV_{in})^2|\upsilon| - uV_{in}|x_2| \end{aligned} \tag{5.3}$$

Then it can be seen by inspection that it is always possible to find a $|\upsilon| \in \Re$, as buck-boost converter is IFP $(-\upsilon)$, such that

$$uV_{in}x_1 < (uV_{in})^2|\upsilon| \tag{5.4}$$

and

$$\frac{x_2^2}{R} > uV_{in}|x_2| \quad if \quad R < \frac{1}{1-u} \tag{5.5}$$

The above analysis implies that at high loads (small R), Equation 5.3 holds i.e. the buck-boost converter has shortage of passivity at high loads or there is insufficient damping in the system. However, from Chapter 4 in FFEP control dynamic feedforward passivation is used to make the buck-boost converter system passive. This implies that irrespective of the load applied to the buck-boost converter system the system is made passive through dynamic feedforward passivation. Therefore if the load is perturbed, the system response returns to its nominal behavior as the FFEP control maintains the buck-boost converter passive through dynamic feedforward passivation.

This behavior of the converter in the presence of load variations has been verified using Simulations. Figure 5.5 and 5.6 shows how the converter returns to its nominal behavior quickly when the load is decreased by 30% and 50% respectively. Figure 5.7 also shows the converter's response when the load is increased by 50%.

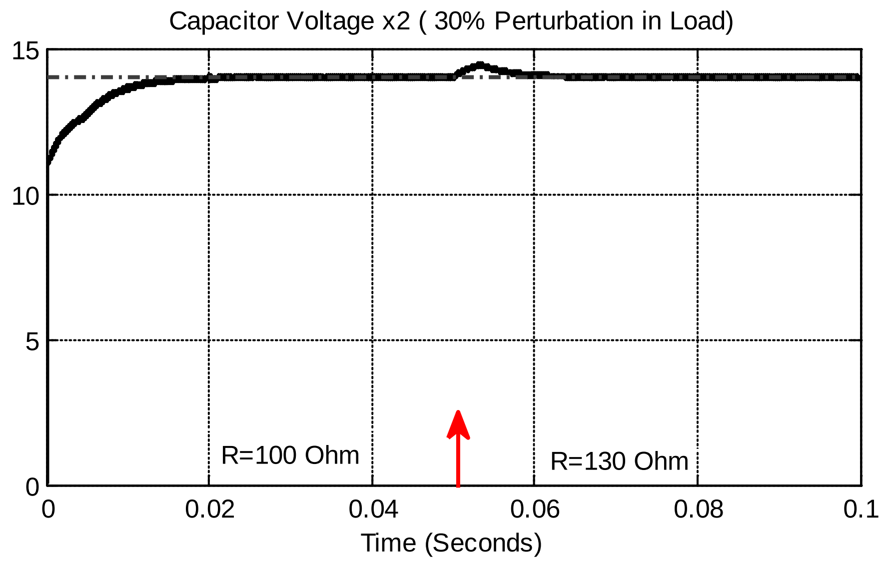

Figure 5.5 Response of the converter in the presence of 30% decrease in load

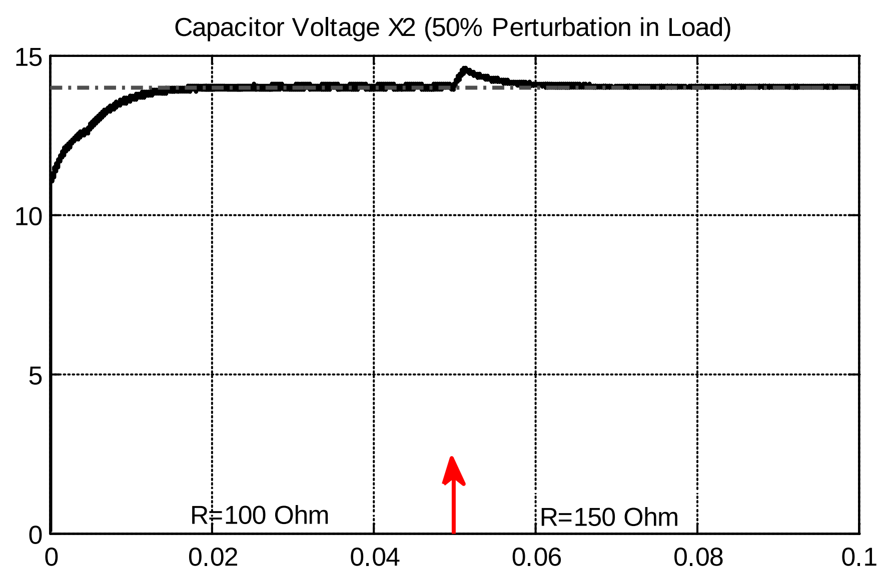

Figure 5.6 Response of the converter in the presence of 50% decrease in load

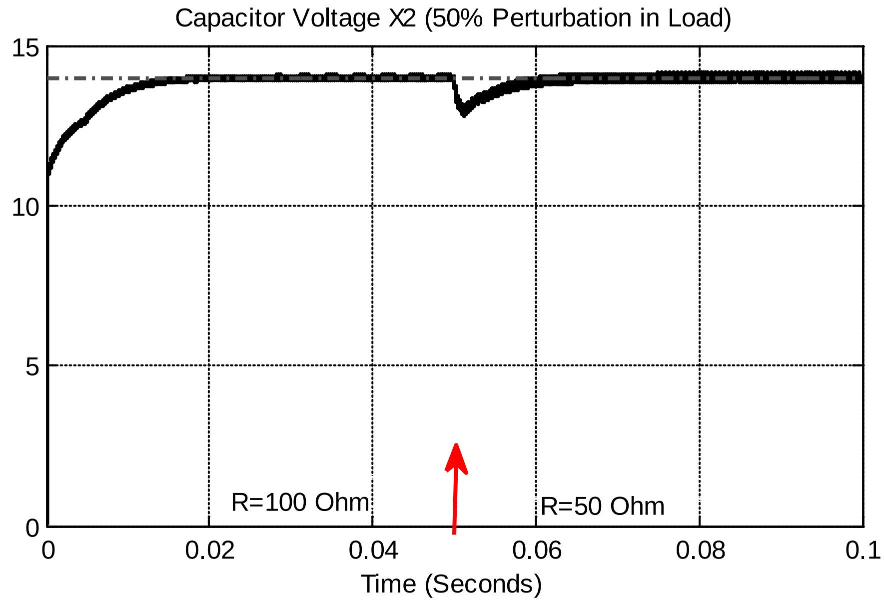

Figure 5.7 Response of the converter in the presence of 50% increase in load

Hence, it can be said that FFEP control approach results in a satisfactory performance when the load varies significantly.

5.3 Comparison of FFEP Control with Other Techniques

In this section FFEP control is compared with different control approaches. In Figure 5.8 a comparison between FFEP and a modified PI control is shown. It can be observed that even when the gains are fine-tuned, overshoot is observed when a negative step reference is applied. This attribute cannot be fully eliminated with a PI, as it is due to the peaking phenomenon [24] inherent in a non-minimum phase system. Therefore, in FFEP control as the non-minimum phase behavior is compensated to minimum phase by feedforward passivation, the system behaves close to a minimum phase system. As a result peaking is very much reduced as can be seen from Figure 5.8.

The FFEP is also compared with other existing passivity based approaches [40, 41]. In these techniques the energy of the system has been modeled by Hamiltonian [40] and Brayton - Moser (BM) [41] equations. They constitute the total energy in the system from the internal and external interconnections, and from the damping associated with the system. In Hamiltonian technique, a closed-loop energy function, the aggregate of the system, and controller energies is considered. By proper choice of nonlinear control action and damping injected into the inductor current, [40] achieves the voltage regulation by driving the closed-loop energy function to a minimum. This technique uses a state-modulated feedback and a non-quadratic Lyapunov energy function to resolve the non-minimum phase voltage control. In BM a closed-loop error dynamic function is considered instead of a closed-loop energy function, and the control task is accomplished by injecting damping into the capacitor voltage.

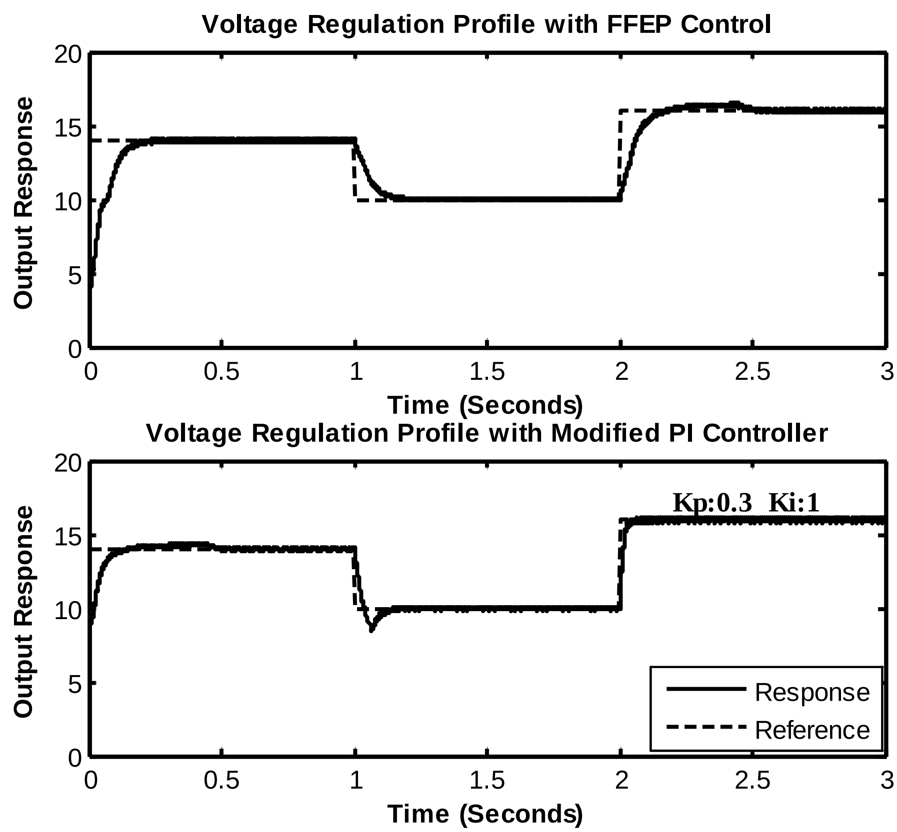

Figure 5.8 Comparison of FFEP control with modified PI controller

The principal investigation in [40] and [41] focuses on characterizing the energy of the system based on the physical structure and makes use of error dynamic and energy dynamic description to draw conclusions about the degree of passivity i.e. damping in the system. As a result, the system description is complex and the control tends to be rather involved because of the use of non-linear controllers.

FFEP control approaches the problem by characterizing the degree of passivity in the system from passivity indices rather than from the system's energy function to avoid circuit level energy analyses. Hence, FFEP control is a complementary system level approach to [40] and [41] and uses a simple linear controller to enhance the output voltage profile and attains robustness against load variations.

Passivity based control using the principles of [40, 41] is implemented on the buck-boost converter using Simulations and the damping is injected into the inductor current. The performance of the controller is as shown in Figure 5.9. From Figure 5.9, it can be observed there is a steady state error in the system performance. These controllers usually require additional controller like PI controllers to overcome steady state error [57].

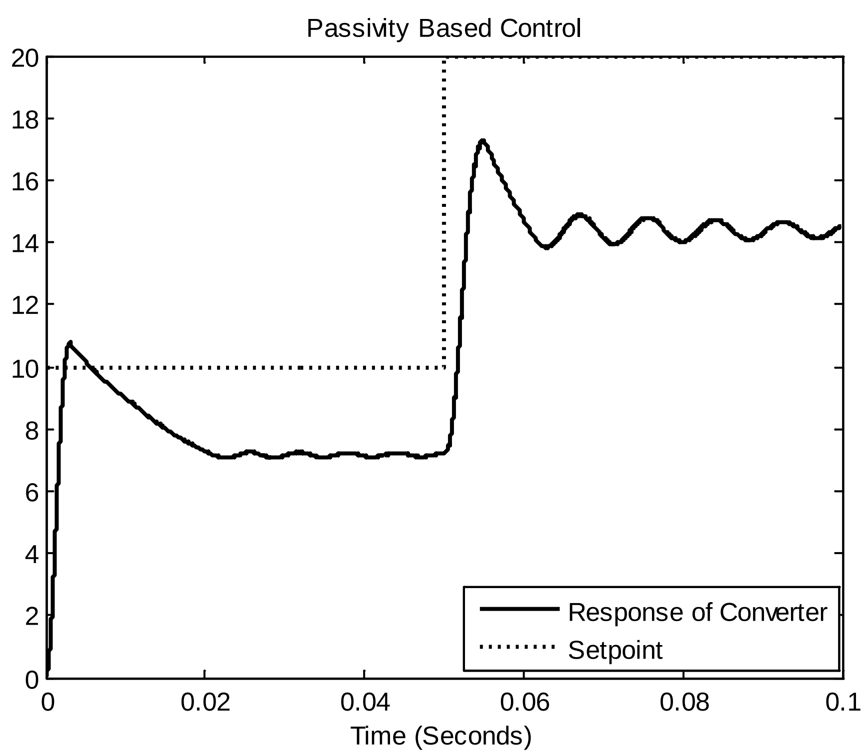

Figure 5.9 Passivity based control

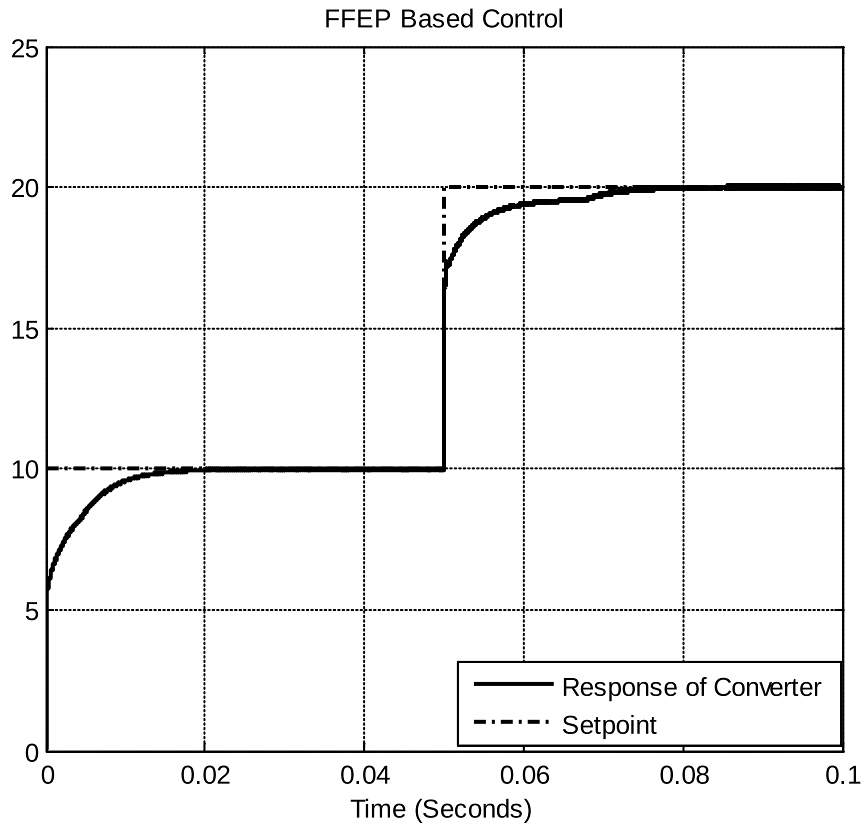

Figure 5.10 FFEP control performance

Passivity based controllers stabilize the converter but require additional controllers like PI and sliding mode to achieve satisfactory performance. Moreover, the control procedure is quite complex as it involves the use of circuit level descriptions like Hamiltonian and Brayton Moser equations for modeling. Figure 5.10 shows the performance of FFEP control for the same step change in reference. It can be observed that the system performance is satisfactory without steady state errors and that FFEP control is a much simpler and effective control approach.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

This thesis provided a solution to the problem of direct regulation of non-minimum phase voltage for second-order DC-DC power converters. Two different methodologies were introduced depending on the modeling of the converter. For linear converter models, 'Parallel Compensation Approach [56]' had been suggested and for bilinear models, 'Feedforward Excess Passivity (FFEP) based Control' had been proposed. The main idea behind these two control approaches is to compensate the non-minimum phase behavior of the converter such that delay in the system is compensated and a satisfactory performance of the converter is assured even in the presence of load variations.

In parallel compensation approach, the concept of strictly positive real form of the system was exploited to achieve a good performance. It has been observed from the simulation results that the parallel compensation approach accelerates the transient response of the converter, removes the undershoot and overshoot, considerably reduces the transient response oscillations and directly controls the voltage. It provided an alternative solution to the industry standard indirect current mode control. The efficiency of the control approach has been compared with the conventional proportional integral controller.

In FFEP based control, a novel system level approach to characterize the damping in the system has been adopted. The concept of Passivity was used to stabilize the unstable zero dynamics of the non-minimum phase voltage in the converter. The simulation and experimental results validate the claim. FFEP based control was also compared with the existing Passivity based control techniques.

Several applications to these control approaches are suggested in the next section, especially FFEP based control.

6.2 Future Recommendations

This section identifies different problems where FFEP based control can be used as an effective solution. They are given as:

- Stabilization of constant power loads (CPL) in DC-DC converter systems
- Framework to analyze DC based grid systems
- Model reference adaptive control for non-minimum phase systems
- Solution to overcome peaking effect in non-minimum phase systems

6.2.1 Constant Power Loads

Loads are basically classified into two types: positive incremental loads and negative incremental loads i.e. constant power loads (CPL). The conventional loads are positive incremental loads and are usually modeled as resistors, constant current sources, or series combination of voltage sources, resistors and inductors. Power electronic converters in multi-converter systems and motor drives exhibit CPL behavior at their input terminals when they are tightly regulated, as a result the power quality and system stability is affected. In DC systems, a point of load converters in a multi-converter system behaves as CPLs and affects the stability of the system.

The voltage and current characteristics of a CPL is as shown in Figure 6.1.and the DC and small-signal model are given as in Figure 6.2 [54]. The following analysis of buck-boost converter in the presence of CPL is carried out to show how a stable buck-boost converter (Equations 2.6 - 2.8) becomes unstable in the presence of a CPL.

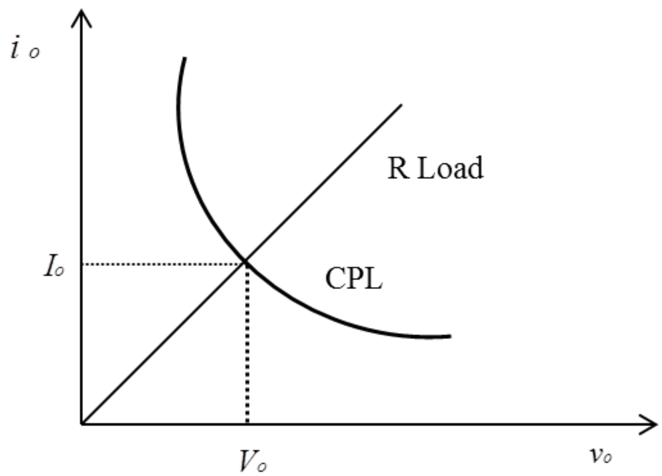

Figure 6.1 Behavior of constant power loads [CPL]

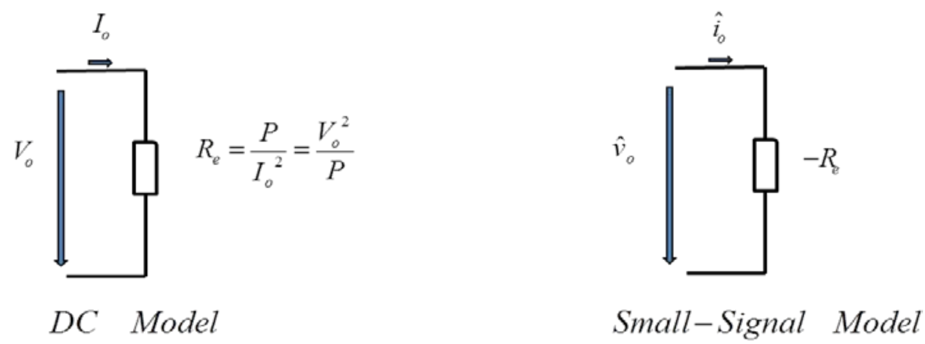

Figure 6.2 DC and small-signal model of CPL

This scenario usually arises when tightly regulated DC-DC converters acts as loads in multi-converter system.

The small signal model of the buck-boost converter with CPL is given as:

$$\tilde{v}_o(s) = \frac{-D}{D'} \frac{\tilde{v}_i(s)}{\left(1 - \frac{L}{D'^2 R_e} s + s^2 \frac{LC}{D'^2}\right)} + \frac{1}{D'} \frac{\left[(V_o - V_i) - \frac{LI_L}{D'} s\right]}{\left(1 - \frac{L}{D'^2 R_e} s + s^2 \frac{LC}{D'^2}\right)} \tilde{d}(s) \qquad (6.1)$$

The converter line-to-output transfer function is given as

$$\left.\frac{\tilde{v}_o(s)}{\tilde{v}_i(s)}\right|_{\tilde{d}(s)=0} = \frac{-D}{D'}\frac{1}{\left(1 - s\frac{L}{D'^2 R_e} + s^2\frac{LC}{D'^2}\right)} \tag{6.2}$$

The converter control-to-output transfer function is given as

$$\left.\frac{\tilde{v}_o(s)}{\tilde{d}(s)}\right|_{\tilde{v}_i(s)=0} = \frac{1}{D'}\frac{\left[(V_o - V_I) - s\frac{LI_L}{D'}\right]}{\left(1 - s\frac{L}{D'^2 R_e} + s^2\frac{LC}{D'^2}\right)} \tag{6.3}$$

As coefficients of the denominator polynomial of (6.2) and (6.3) are not positive, i.e. non-Hurwitz, the system is unstable in the presence of CPL.

In the above analysis the converter is modeled as a linear system to show the effect of CPL on the transfer functions. The above stabilization problem can be addressed by using FFEP control when the converter is modeled as a non-linear system The gains of the linear compensator will be constrained differently i.e. damping structure can be modified. This approach can be further pursued to address the stability of DC-DC converter system in the presence of CPLs.

6.2.2 Analysis of Distributed Power Systems

With the advent of Distributed Power Systems, different power systems with new architectures are coming into existence. A lot of research is being done with regard to standardization of architectures and development of system level tools for black box stability analysis and to analyze dynamic interactions between subsystems. Current research [55] uses Hammerstein models for power converters to study the two-port network behavioral modeling of a DC grid system such as shown in Figure 6.3.

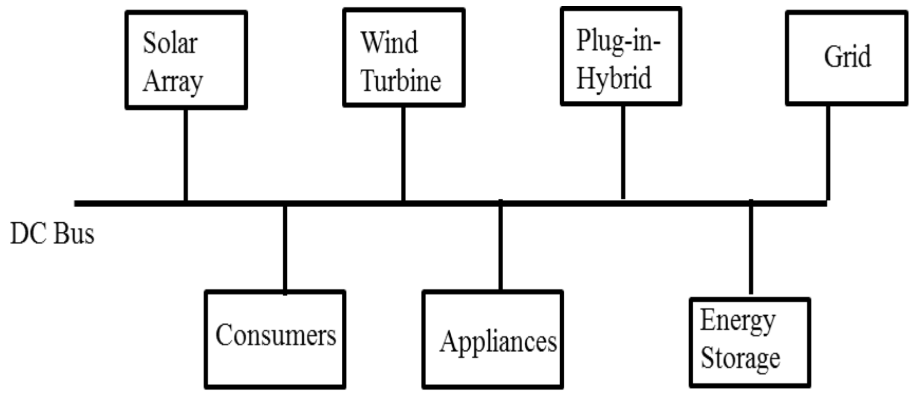

Figure 6.3 DC distributed power system [55]

In this context, it can pointed out that the system level approach to characterize the amount of dissipation i.e. passivity in a system used in FFEP control can be applied to this scenario. The inspiration for FFEP control is originally drawn from networked control systems [43] and from operability analysis of different processes [44], and can be extended to analyze large interconnected systems in DC distributed Power Systems. Further research can be done in this direction to identify efficient procedures to model the subsystems in DC distributed Power systems using Passivity rather than using converter level Hammerstein models which tend to complicate the analysis.

6.2.3 In Model Reference Adaptive Control

Simple Adaptive Control (SAC) [51] uses direct model reference control. In SAC there are no restrictions on the order of the reference models, such that the reference model can be of a much small order than the plant. The only criteria for the stability of SAC is that the plant must be strictly positive real (SPR) or almost strictly positive real (ASPR). Therefore SAC for non-minimum power converters is not applicable.

By the use of parallel compensation approach [56] in Chapter 3 or the FFEP control in Chapter 4, the plant is changed into an SPR and passive system respectively. Hence, depending on how the converter is modeled either approach can be carried out to facilitate the applicability of SAC to non-minimum phase power converters.

6.2.4 In Peaking Phenomenon

From [24] it is said that non-minimum phase systems are peaking systems i.e. they cannot be stabilized without peaking phenomenon. Such systems when connected in cascade, in multi-converter system would result in high transients and the system performance degrades. This can be avoided by the use of FFEP control, as in FFEP control the system is compensated to make it passive by dynamic feedforward passivation. Therefore, the system behavior approaches a minimum phase system and the peaking can be decreased to a considerable effect

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APPENDIX

APPENDIX

The following figure shows the test bed equipment for buck-boost converter. Figure A.1 shows the dSPACE rapid prototyping device, buck-boost converter and a programmable load.

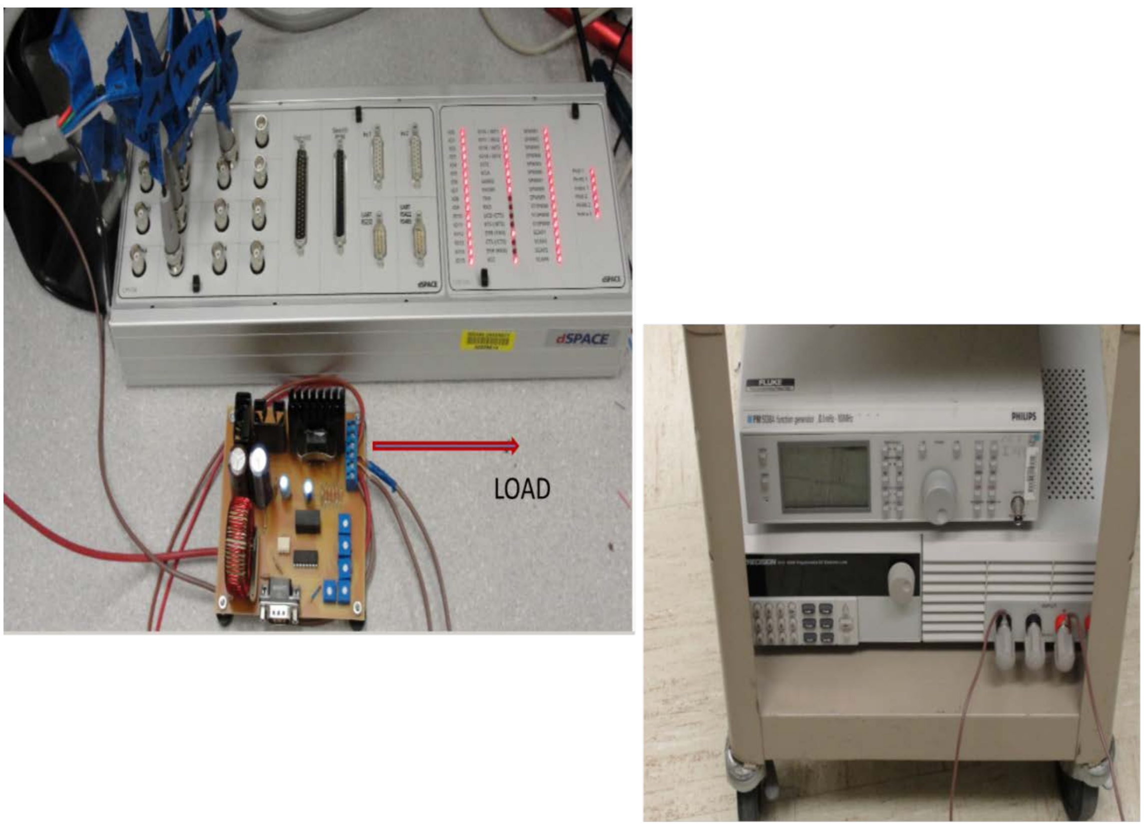

Figure A Buck-Boost Converter Experimental Setup

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