Lifting the Security of NI-MAC Beyond Birthday Bound

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Abstract. In CRYPTO 1999, J. An and M. Bellare proposed a Merkle-Damgård iteration based MAC construction called NI-MAC in order to avoid constant re-keying on multiblock messages in NMAC and to ease the security proof. In CRYPTO 2014, Gazi et al. revisited the proof of NI-MAC in the view of structure graph introduced by Bellare et al. in CRYPTO 2005 and gave a tight bound of order $\frac{lq^2}{2^n}$, which is an improvement over the trivial bound of order $\frac{l^2q^2}{2^n}$, for q queries, each of length at most ℓ blocks. But this is again restricted to the birthday security. In order to prove the security of NI-MAC, Gazi et al. (CRYPTO 2014) introduced a variant of NI-MAC, called NI2-MAC and analyzed the advantage of NI2 MAC. Then he showed that the same proof technique will be applied to the security analysis of NI-MAC.
In this paper, we lift the birthday bound of NI2-MAC construction beyond birthday $O(q^2l^4/2^{2n})$ by a small change in the existing construction with one extra invocation of a independent keyed function. Finally, we argue how to lift the security of NI-MAC beyond birthday using the security proof for NI2-MAC.

Keywords: Beyond Birthday, MAC, NI, NI2, Structure-Graph.

1 Introduction

In symmetric key paradigm, MAC (Message Authentication Code) is used for preserving message integrity and message origin authentication. The design of a MAC should not only consider achieving security, but also target attaining efficiency. In the literature, three different approaches of designing a MAC exists: (a) universal hash function based MAC, a popular example of which is UMAC [6], (b) a compression function based MAC, like NMAC [2], HMAC [2], NI [1] etc. (c) Block cipher based MAC, such as CBC MAC [4], PMAC [7], OMAC [12]. etc.

Most of the popular MACs are block cipher based MACs, but each one of them suffers from the same problem - security is guaranteed up to the *birthday bound*. When the block length of the underlying block cipher is 128-bit, then birthday bound does not seem to be a problem, as we are guaranteed to have 64 bits of security which is well acceptable for many practical applications. But when we deal with 64-bit block cipher as used in many light weight crypto devices, then birthday bound problem becomes the main bottleneck. Throughout the paper, we use n to denote the block-length and q to denote the number of MAC-queries by the adversary.

Related Work on Beyond birthday Secure MAC. In recent researches, many MAC constructions have been proposed with security beyond the birthday barrier without degrading the performance. The first attempt was made in ISO 9797-1 [3] without security proof. But Algorithm 4 of ISO 9797-1 was attacked by Joux et al. [15] that falsified the security bound. Algorithm 6 of ISO 9797-1 was proven to be secure against $O(2^{2n/3})$ queries with restrictions on the message length [20]. In [20] Yasuda proved that the sum of two independent ECBC has beyond birthday bound. However, it requires four keys and it is rate 1/2 construction as it requires two block cipher calls for processing each message block. In 2011, he proposed PMAC_Plus Construction [21] that achieves beyond birthday security. In 2012, Zhang et al. [22] proposed a 3key version of f9 MAC that achieves BBB.

There is also another deterministic MAC mode provides security beyond the birthday bound. Given an n-bit to n-bit fixed-key blockcipher with MAC security ϵ against q queries, Dodis et al. [9] have designed a variable-length MAC achieving $O(\epsilon q \text{poly}(n))$ MAC security. However, this design requires even longer keys and more block cipher invocations. By parity method, Bellare et al. present MACRX [3] with BBB security, conditioned on the input parameters are random and distinct. In [13], Jaulmes et al. proposed a randomized MAC that provides BBB security based on the ideal model (or possibly based on tweakable block cipher). Another BBB secure randomized construction called generic enhanced hash then MAC has been proposed in [18] by Minematsu. Recently Datta et al. in [8] unify PMAC_Plus and 3kf9 in one key setting with beyond birthday security.

In CRYPTO 1999, J. An and M. Bellare [1] proposed a Merkle-Damgård iteration based MAC construction called NI-MAC. The construction of NI-MAC is similar to that of NMAC [2], the only difference is that in NI-MAC the compression function f takes an additional input key k at each invocation. The motivation of designing NI was to avoid constant re-keying on multi-block messages in NMAC and to allow for a security proof starting by the standard switch from a PRF to a random function, followed by information-theoretic analysis.

In CRYPTO 2014, Gazi et al. [10] revisited the proof of NI-MAC in the view of structure graph introduced by Bellare et al. in CRYPTO 2005 [5] and gave a tight bound of order $\frac{lq^2}{2^n}$, which is an improvement over trivial bound of order $\frac{l^2q^2}{2^n}$, for q queries, each of length at most ℓ blocks. But this is again restricted to the birthday security. In order to prove the security of NI-MAC, Gazi et al. [10] introduced a variant of NI-MAC, called NI2-MAC, and then derived the security of NI-MAC from the security analysis of NI2-MAC. In this paper, we propose an extension of NI2-MAC with a single invocation of an additional pseudo-random function and prove (Section 4) that it achieves beyond-birthday security. Furthermore, we make a remark at the end that if we extend the NI MAC in the same way as we did for NI2 MAC, then also we achieve beyond birthday bound security.

Organization: Section 2 revisits the definition of prf, mac, structure graph. Section 3 contains the construction of NI^+ and $\mathrm{NI2}^+$ MAC. Security analysis of $\mathrm{NI2}^+$ is shown from Section 4 to Section 6. We conclude the paper in Section 7.

2 Preliminaries

In this section, we briefly discuss the notations and definitions used in this paper. We also state some existing basic results.

2.1 PRF and Secure MAC

We denote $|S|$ as the cardinality of set S and S^c as the complement set of S. Let $x \xleftarrow{\$} S$ denote that x is chosen uniformly at random from S. Let $Func(A, B)$ denote the set of all functions from A to B A function $\rho : A \to B$ is said to be a random function, if ρ is chosen uniformly at random from the $Func(A, B)$.

We will specify a random function by performing *lazy sampling*. In lazy sampling initially the function ρ is undefined at every point of its domain. We maintain two sets that grows dynamically. One is domain, $\mathrm{Dom}(\rho)$ and another is Range, $\mathrm{Ran}(\rho)$, both initialized to be empty. $\mathrm{Dom}(\rho)$, $\mathrm{Ran}(\rho)$ keeps the record of already defined domain points and range points of function ρ respectively. Therefore, if $x \notin \mathrm{Dom}(\rho)$ then we will choose $y \xleftarrow{\$} B \setminus \mathrm{Ran}(\pi)$ and add y in $\mathrm{Ran}(\rho)$ and x in $\mathrm{Dom}(\rho)$. In this regard, x is said to be *fresh*.

We consider that an adversary $\mathcal{A}$ is an oracle machine with access to its oracle $\mathcal{O}(\cdot)$ and outputs either 1 or 0. Accordingly, we write $\mathcal{A}^{\mathcal{O}(\cdot)} = 1$ or 0. The resource of $\mathcal{A}$ is measured in terms of the time complexity $T(n)$ that it takes to interacts with its oracle $\mathcal{O}(\cdot)$ and the query complexity $q(n)$ which says the number of queries and replies exchanged between the adversary and its oracle. For practical purpose, we restrict to probabilistic polynomial time (PPT) adversaries only.

The PRF-advantage of a function $F_k : A \to B$ is defined as

$$\mathbf{Adv}^{\mathbf{PRF}}_{\mathbf{F_k}}(\mathcal{A}) = \Pr\left[\mathcal{A}^{F_k(\cdot)} = 1 : k \xleftarrow{\$} \mathcal{K}\right] - \Pr\left[\mathcal{A}^{f(\cdot)} = 1 : f \xleftarrow{\$} Func(A, B)\right].$$

If this advantage is negligible in the length of the input for all PPT adversaries, F is said to be a secure PRF. Note that the first probability is calculated over the internal coin tosses of the algorithm $\mathcal{A}$ and randomness of $k \xleftarrow{\$} \mathcal{K}$ and second probability is calculated over the randomness of $f \xleftarrow{\$} Func(A, B)$.

The length of M in bits is denoted by $len(M)$. When it is not a multiple of n, we append $10^{n-1-len(M) \mod n}$ to M to make $len(M)$ a multiple of n. We denote the maximum number of block in a query by l. We denote the partition of a message M as $M = M_1||M_2||\dots||M_l$ where each M_i is an n-bit block and the number of blocks of M is denoted by l.

An adversary attacking a MAC with q queries obtains q tags for q distinct messages and produces a valid tag of a fresh message that he has not queried earlier. It is known [11] that any secure PRF is a secure MAC. Thus, to show that a MAC construction is secure, one needs to show that the PRF-advantage (which is a function of q, l and n) of an adversary for the construction is negligible.

2.2 Structure Graphs

In this section, we briefly revisit the structure graph analysis of CBC-MAC [5] by Bellare et al. and that of NI-MAC [10] by Gazi et al.

Consider an iterated/cascaded construction with a function f, where f could be a random permutation or a random function, that works on a message $M = M_1||M_2||\dots||M_l$ of length l blocks as follows:

$$Y_0 = \mathbf{0}, \text{ and } Y_i = f(Y_{i-1}, M_i) \text{ for } i = 1, \dots, l.$$

Note that for CBC-MAC analysis, $f(\alpha, \beta)$ is taken as $\pi(\alpha \oplus \beta)$ and for the NI-MAC analysis, $f(\alpha, \beta)$ is taken as $\rho(\alpha||\beta)$, where π is a random permutation over n bits and ρ is a random function from $b + n$ bits to n bits, where b is the message block-length and n is the length of the chaining variable as well as the tag.

For a set of any two fixed distinct messages $\mathcal{M} = \{M^{(1)}, M^{(2)}\}$ and a function f, we construct the structure graph $\mathcal{G}^f(\mathcal{M})$ with $\{0,1\}^n$ as the set of nodes as follows. We follow the computations for $M^{(1)}$ followed by those of $M^{(2)}$ by creating nodes labelled by the values y_i of the intermediate chaining variables Y_i with the edge (Y_i, Y_{i+1}) labelled by the block M_{i+1}. In this process, if we arrive at a vertex already labelled, while not following an existing edge, we call this event an f-collision. An accident is an f-collision that does not close a cycle with alternating edge-directions such that the XOR of the labels of the cycle becomes 0.

More formally, let for two distinct messages $M^{(1)}$ and $M^{(2)}$ of l_1 and l_2 blocks respectively, where

$$M^{(1)} = M_1^{(1)}||M_2^{(1)}||\dots||M_{l_1}^{(1)} \text{ and } M^{(2)} = M_1^{(2)}||M_2^{(2)}||\dots||M_{l_2}^{(2)},$$

the corresponding Y-values be given by

$$Y_0^{(1)}, Y_1^{(1)}, Y_2^{(1)}, \dots, Y_{l_1}^{(1)} \text{ and } Y_0^{(2)}, Y_1^{(2)}, Y_2^{(2)}, \dots, Y_{l_2}^{(2)}$$

respectively. Let $\sigma = l_1 + l_2$. We use the notation M_i to refer to the block $M_i^{(1)}$, when $i < l_1$, otherwise to refer to the block $M_{i-l_1}^{(2)}$. Similarly, let Y_i to refer to $\mathbf{0}$ when $i = 0$; $Y_i^{(1)}$, when $1 \le i \le l_1$; and $Y_{i-l_1}^{(2)}$, when $l_1 + 1 \le i \le \sigma$. Now, consider the mappings

$$[[\cdot]] \text{ and } [[\cdot]' \text{ on } \{0, \dots, \sigma\}$$

so that $[[i]] = \min\{j : Y_i = Y_j\}$ and $[[i']] = [[i]]$ for $i \neq l_1$ except that $[[l_1]]' = 0$.

For any fixed f and any two distinct messages $\mathcal{M} = \{M^{(1)}, M^{(2)}\}$, we define the structure graph $\mathcal{G}^f(\mathcal{M})$ to be the triple $\mathcal{G}^f(\mathcal{M}) = (V, E, L)$, where

$$V = \{[[i]] : 0 \leq i \leq \sigma\}, \qquad E = \{([[i-1]]', [[i]]) : 1 \leq i \leq \sigma\}$$

and $L = E \rightarrow \{0,1\}^n$ is an edge-labeling function defined as

$$L((u, v)) = \{M_i : [[i-1]]' = u \text{ and } [[i]] = v\}.$$

Let (V_i, E_i, L_i) be the graph obtained after processing only the first i out of σ blocks of $\mathcal{M}$. We say that $(i, [[i]])$ is an f-collision if $[[i]] < i$ and $M_i \notin L_{i-1}([[i-1]]', [[i]])$. Note that the last condition on M_i implies that collision occurred due to parallel edges with the same message label is not considered.

In [5], a general collision is called a *true collision* (except the collision that occurs due to parallel edges with same label on the edges). Further, a true collision is called an *accident* if it is not followed from a cycle C with alternating edges with the sum of the labels of the edges involved in C to $\mathbf{0}$, otherwise it is called an *induced collision*. However, for NI2-MAC, all f-collisions are accidents. In our work, we need to consider the accidents in $\mathcal{G}^f(\mathcal{M})$. Let $\mathcal{G}(\mathcal{M})$ denote the set of all structure graphs corresponding to the set of messages $\mathcal{M}$ (by varying f over a function family). For a fixed graph G, let $Acc(G)$ denote the set of all accidents in G. We state the following known results.

Proposition 1. *[10, Lemma 2] For a fixed graph G, $\Pr_f[\mathcal{G}^f(\mathcal{M}) = G] \leq 2^{-n|Acc(G)|}$.*

Proposition 2. *[5, Lemma 7]* $\Pr[G \xleftarrow{\$} \mathcal{G}(\mathcal{M}) : |Acc(G)| \geq 2] \leq \frac{8l^4}{2^{2n}}$.

3 Proposed Construction of NI$^+$ and NI2$^+$ for Beyond-Birthday Secure MAC

We present the schematic diagram of NI$^+$ and NI2$^+$ in Fig. 3.1 and Fig. 3.2 followed by the description in Algorithm 1 and 2 respectively. Note that the

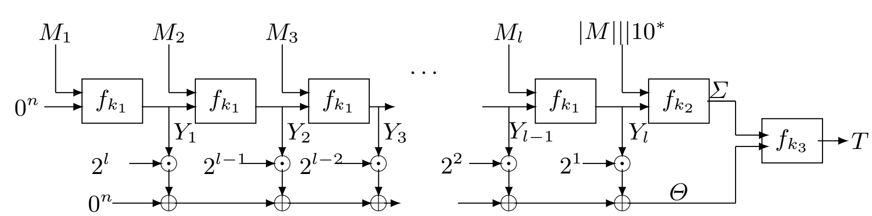

Fig. 3.1: Construction of NI$^+$ MAC

Input: $f_{K_1}, f_{K_2}, f_{K_3} : K_1, K_2, K_3 \xleftarrow{\$} \mathcal{K},\ M \leftarrow \{0,1\}^*$
Output: $T \in \{0,1\}^n$
$M_1 || M_2 || \ldots M_l \leftarrow M || 10^*$; //$l$ is the number of message blocks in M
$Z \leftarrow 0^n$;
$Y \leftarrow 0^n$;
for $i = 1$ *to* l **do**
    $Y \leftarrow f_{K_1}(M_i, Y)$;
    $Z \leftarrow 2 \cdot (Z \oplus Y)$;
end
$\Theta \leftarrow Z$;
$\Sigma \leftarrow f_{K_2}(|M| || 10^*, Y)$;
$T \leftarrow f_{K_3}(\Sigma, \Theta)$;
Return T;

Algorithm 1: Algorithm for NI^+ MAC

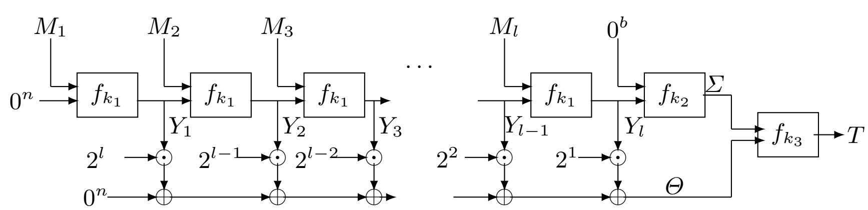

Fig. 3.2: Construction of $\mathrm{NI2}^+$ MAC

Input: $f_{K_1}, f_{K_2}, f_{K_3} : K_1, K_2, K_3 \xleftarrow{\$} \mathcal{K},\ M \leftarrow \{0,1\}^*$
Output: $T \in \{0,1\}^n$
$M_1 || M_2 || \ldots M_l \leftarrow M || 10^*$; //$l$ is the number of message blocks in M
$Z \leftarrow 0^n$;
$Y \leftarrow 0^n$;
for $i = 1$ *to* l **do**
    $Y \leftarrow f_{K_1}(M_i, Y)$;
    $Z \leftarrow 2 \cdot (Z \oplus Y)$;
end
$\Theta \leftarrow Z$;
$\Sigma \leftarrow f_{K_2}(0^b, Y)$;
$T \leftarrow f_{K_3}(\Sigma, \Theta)$;
Return T;

Algorithm 2: Algorithm for $\mathrm{NI2}^+$ MAC only difference between the two constructions is in the input of function f_{K_2} where f_{K_1}, f_{K_2} and f_{K_3} are three independently chosen keyed functions such that $f_{K_1}, f_{K_2} : \{0,1\}^{n+b} \rightarrow \{0,1\}^n$ and $f_{K_3} : \{0,1\}^{2n} \rightarrow \{0,1\}^n$. We denote

$$Casc^{f_{K_1}}(M) := f_{K_1}(\ldots(f_{K_1}(f_{K_1}(f_{K_1}(0, M_1), M_2), M_3), \ldots,)M_l)$$

to be the output of the last message block in the upper lane of the construction depicted in Fig.3.2.

For any message $M \in \{0,1\}^*$, NI2$^+$ MAC (after suitably padding with 10^* if the message length is not a multiple of the block length b) partitions M into l many blocks each of which is b bits long. Then the blocks are iteratively processed as depicted in Fig.3.2. Final output Y_l of $Casc^{f_{K_1}}(M)$ as depicted in Fig.3.2 and 0^b becomes the input of $f_{K_2}(\cdot,\cdot)$ and the output of $f_{K_2}(\cdot,\cdot)$ is denoted as Σ. This is the so-called NI2 construction which we extend as follows. A linear combination of the intermediate chaining value of $Casc^{f_{K_1}}(M)$ is denoted as Θ. The symbol '2' in the construction is the root of an irreducible polynomial of degree n. Σ and Θ are then fed into $f_{K_3}(\cdot,\cdot)$ and the output is returned as tag T.

Remark 1 *NI-MAC, as originally proposed by An and Bellare in [1] replaces the* 0 *block at the input of* f_{K_2} *with the bit length* $|M|$ *of the message* M*. We extend NI-MAC as depicted in Fig. 3.1 in the same way as we do for NI2-MAC and obtain NI*$^+$*-MAC.*

Note: In subsequent sections all the security proofs are done for NI2$^+$ MAC.

4 Security Analysis of NI2$^+$-MAC

Gazi et. al in [10] have shown that the advantage of distinguishing the output of NI-MAC from random output is bounded above by $\frac{q^2}{2^n}\left(l + \frac{64l^4}{2^n}\right)$ and that for NI2-MAC is $\frac{q^2}{2^n}\left(ld^{'}(l) + \frac{64l^4}{2^n}\right)$ where $d^{'}(l) = \max_{l^{'} \in \{1,\ldots,l\}} |\left\{d \in \mathbb{N} : d|l^{'}\right\}|$. In this section we analyze the advantage of our construction NI2$^+$-MAC and show that the advantage of our construction achieves beyond birthday bound security; better than that of NI-MAC or NI2-MAC. Finally, we make a remark on the achievability of BBB security of NI$^+$ MAC. Thus we have the following theorem.

Theorem 1. *Let* $f : \{0,1\}^n \times \{0,1\}^n \times \{0,1\}^b \rightarrow \{0,1\}^n$ *be a* (ϵ_1, t, q) *secure PRF and* (ϵ_2, t, lq) *secure PRF. Let* $h : \{0,1\}^n \times \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}^n$ *be a* (ϵ_3, t, q) *secure PRF. Then NI2*$^+$ *be a* $\left(\epsilon^{'}, t^{'}, q, l\right)$ *secure PRF, where*

$$\epsilon^{'} \leq \epsilon_1 + \epsilon_2 + \epsilon_3 + \frac{11q^2l^4}{2^{2n}},$$

such that $t = t^{'} + \tilde{O}(lq)$*.*

Proof. We give the sketch of the proof of Theorem 1 below. Let $\mathcal{A}$ be a adaptive PRF-adversary against NI2$^+$ running in time t and asking at most q queries, each of length at most l blocks. NI2$^+$ uses three independent keyed functions f_1, f_2 and h_3. Now if we replace f_1, f_2 and h_3 by three different random functions r_1, r_2 and r_3 respectively such that $r_1, r_2 \xleftarrow{\$} Func(\{0,1\}^n \times \{0,1\}^n \times \{0,1\}^b, \{0,1\}^n)$ and $r_3 \xleftarrow{\$} Func(\{0,1\}^n \times \{0,1\}^n \times \{0,1\}^n, \{0,1\}^n)$ and call the resulting construction NI2$_r^+$, then we have

$$\Delta^{\mathcal{A}}(\text{NI2}^+, R) \leq \epsilon_1 + \epsilon_2 + \epsilon_3 + \Delta^{\mathcal{A}}(\text{NI2}_r^+, R),$$

where ϵ_i is the PRF-advantage of f_i, $i = 1, 2$ and ϵ_3 is the PRF-advantage of h_3 and $R : \{0,1\}^* \rightarrow \{0,1\}^n$ be a uniform random function.

Therefore to prove Theorem 1, we only need to prove

$$\Delta^{\mathcal{A}}(\text{NI2}_r^+, R) \leq \tfrac{11q^2l^4}{2^{2n}}.$$

In the experiment where $\mathcal{A}$ interacts with NI2$_r^+$, let C_i denotes the event that during the first i queries, the inputs to r_3, i.e., (Σ, Θ) for any two distinct queries $M^{(j)}$ and $M^{(k)}$ are also distinct. That means $(\Sigma^{(j)}, \Theta^{(j)}) \neq (\Sigma^{(k)}, \Theta^{(k)})$, $\forall 1 \leq j, k \leq i$. Therefore, as long as the monotone condition [17] $C = C_0, C_1, \ldots$ remains satisfied, the distribution of the responses of NI2$_r^+$ to distinct queries will be exactly identical to the distribution of the outputs of r_3 on distinct inputs and thus to independent uniform random values. In other words, we have

$$\text{NI2}_r^+ | C \equiv R.$$

Thus, using Lemma 1 in [10] we have, $\Delta^{\mathcal{A}}(\text{NI2}_r^+, R)$ is upper-bounded by the probability that a distinguisher $\mathcal{A}$ issuing q queries to NI2$_r^+$ makes the monotone condition C fail. This probability is denoted by $\Pr_{\mathcal{A}}[\text{NI2}_r^+; \overline{C}]$. Thus,

$$\Delta^{\mathcal{A}}(\text{NI2}_r^+, R) \leq \Pr_{\mathcal{A}}[\text{NI2}_r^+; \overline{C}]. \tag{1}$$

Now we explain how to construct a non-adaptive PRF adversary $\mathcal{A}_{na}$ from the above adaptive PRF adversary $\mathcal{A}$.

Construction of Non-adaptive PRF Adversary. Let $\mathcal{A}_{na}$ be the non adaptive PRF adversary that we want to construct from the adaptive PRF adversary $\mathcal{A}$. $\mathcal{A}_{na}$ will simulate the adaptive PRF adversary $\mathcal{A}$ in the following way. At the time of i^{th} query, $M^{(i)}$, where $1 \leq i \leq q$, asked by adversary $\mathcal{A}$, $\mathcal{A}_{na}$ will return random string in response of i^{th} query to $\mathcal{A}$. After all the q queries are over, $\mathcal{A}_{na}$ will (non-adaptively) ask all the queries that $\mathcal{A}$ asked during simulated interaction.

Therefore, we have the following

$$\Pr_{\mathcal{A}}[\text{NI2}_r^+; \overline{C}] = \Pr_{\mathcal{A}_{na}}[\text{NI2}_r^+; \overline{C}]. \tag{2}$$

The maximum probability over all such non-adaptive distinguishers $\mathcal{A}_{na}$ is given by

$$\Pr[\text{NI2}_r^+; \overline{C}] = \max_{\mathcal{A}_{na}} \Pr_{\mathcal{A}_{na}}[\text{NI2}_r^+; \overline{C}] \tag{3}$$

With respect to the $\mathrm{NI2}_r^+$ construction, let $Coll(l)$ denotes the probability that for random choice of the compression function f_1 and f_2, results in a collision in Σ and Θ maximized over the choice of two distinct inputs $M^{(i)}, M^{(j)}$, each of which is at most l blocks long.

More formally, for $f_1, f_2 \xleftarrow{\$} Func\left(\{0,1\}^n \times \{0,1\}^n \times \{0,1\}^b \rightarrow \{0,1\}^n\right)$ we define,

$$Coll(l) := \max_{M^{(i)} \neq M^{(j)} ||M^{(i)}||,||M^{(j)}|| \leq l} \Pr^{f_1,f_2}[\left(\Sigma^{(i)}, \Theta^{(i)}\right) = \left(\Sigma^{(j)}, \Theta^{(j)}\right)]$$

Note that, $(\Sigma^{(i)}, \Theta^{(i)}) = (\Sigma^{(j)}, \Theta^{(j)})$ implies $\Sigma^{(i)} = \Sigma^{(j)}$ and $\Theta^{(i)} = \Theta^{(j)}$. Therefore, to bound the probability of occurrence of a collision in the input of r_3 necessarily implies to bound the probability of occurrence of a collision in Σ and a collision in Θ. That means

$$\Pr^{f_1,f_2}[\left(\Sigma^{(i)}, \Theta^{(i)}\right) = \left(\Sigma^{(j)}, \Theta^{(j)}\right)] = \Pr^{f_1,f_2}[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] \quad (4)$$

Note that, $\mathcal{A}_{na}$ violates the monotone condition C only when the collision occurs at the input of r_3. Therefore from Equation (1), (2) and (3), and using union bound we obtain,

$$\Delta^{\mathcal{A}}(\mathrm{NI2}_r^+, R) \leq \Pr[\mathrm{NI2}_r^+; \overline{C}] \leq \frac{q^2}{2} Coll(l). \quad (5)$$

In Lemma 1 of Section 4.1, we show that $Coll(l) \leq \frac{22l^4}{2^{2n}}$. Therefore, plugging in the bound of $Coll(l)$ into Equation (5), we get the result. □

4.1 Computation of $Coll(l)$

Recall that, $Coll(l)$ was defined as $\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}]$ maximized over the choice of pair of distinct inputs $M^{(i)}$ and $M^{(j)}$, each of length at most l blocks. Therefore, to establish the bound on $Coll(l)$, we derive the bound on $\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}]$

Lemma 1. *Given two fixed distinct messages $M^{(i)}, M^{(j)}$, each of length is at most l blocks, we have*

$$\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] \leq \frac{22l^4}{2^{2n}}.$$

Proof. Let $Z^{(i)} = Y^{(i)}_{l_i}$ denote the input to the function r_2 for message $M^{(i)}$(refer to Fig.3.2). Similarly, we set $Z^{(j)} = Y^{(j)}_{l_j}$. So, we have,

$$
\begin{aligned}
&\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] \\
&= \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge Z^{(i)} = Z^{(j)}] + \\
&\quad \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge Z^{(i)} \neq Z^{(j)}] \qquad (6) \\
&\leq \Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] + \\
&\quad \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge Z^{(i)} \neq Z^{(j)}] \qquad (7) \\
&\leq \Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] + \\
&\quad \left(\sum_{k=0}^{1} \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \cdot \Pr[Z^{(i)} \neq Z^{(j)}] \right) \\
&\quad + \Pr[NCOL \geq 2] \qquad (8) \\
&\leq \Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] + \\
&\quad \sum_{k=0}^{1} \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] + \Pr[NCOL \geq 2].
\end{aligned}
$$

Since the event $Z^{(i)} = Z^{(j)}$ is a subset of the event $\Sigma^{(i)} = \Sigma^{(j)}$, the first term of Equation (6) is equal to $\Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}]$.

According to Claim 1, we have $\Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] \leq \frac{ld'(l)}{2^{2n}} + \frac{8l^4}{2^{2n}}$. From Proposition 2 we have, $\Pr[NCOL \geq 2] \leq \frac{8l^4}{2^{2n}}$. From Claim 2, we have, $\sum_{k=0}^{1} \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \leq \frac{4l^2+1}{2^{2n}}$. Therefore,

$$
\begin{aligned}
\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] &\leq \frac{ld^{'}(l)}{2^{2n}} + \frac{8l^4}{2^{2n}} + \frac{4l^2+1}{2^{2n}} + \frac{8l^4}{2^{2n}} \\
&\leq \frac{ld^{'}(l)}{2^{2n}} + \frac{16l^4}{2^{2n}} + \frac{4l^2+1}{2^{2n}} \\
&\leq \tfrac{22l^4}{2^{2n}}
\end{aligned}
$$

In the next two sections, we state and prove the two claims above.

5 Details of the Proof of Claim 1

Claim 1 *Fix two distinct messages $M^{(i)}, M^{(j)}$ each of length at most l blocks. Then,*

$$\Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] \leq \frac{ld'(l)}{2^{2n}} + \frac{8l^4}{2^{2n}},$$

where $Z^{(i)} = Y^{(i)}_{l_i}, Z^{(j)} = Y^{(j)}_{l_j}$, and l_i, l_j are the number of blocks of $M^{(i)}, M^{(j)}$ respectively.

Proof. We prove the claim using the structure graph. After fixing two messages $M^{(i)}$ and $M^{(j)}$ and choosing a function f uniformly at random from the set of all functions over $\{0,1\}^b \times \{0,1\}^n \rightarrow \{0,1\}^n$, we analyze the structure graph $G := G^f(M^{(i)}, M^{(j)})$. In particular, we analyze the probability of the event $Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}$ in view of number of collisions (say, NCOL) occurred in the corresponding structure graph G. Therefore, we have,

$$Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)}] = Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 1] \\ + Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL \geq 2].$$

In Section 5.1, we show that

$$\Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 1] \leq \frac{ld'(l)}{2^{2n}}, \tag{9}$$

where $d'(l)$ is the maximum number of positive divisors of the integer l' from $[1, l]$.

When NCOL in the graph is at least 2, then using Proposition 2 we have,

$$\Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL \geq 2] \leq \Pr[NCOL \geq 2] \leq \frac{8l^4}{2^{2n}}. \tag{10}$$

Therefore, combining Equations (9) and (10), we get the result. □

Now the only part of the proof that remains is to prove Equation (9).

5.1 Proof of Equation (9)

We can write

$$Pr[Z^{(i)} = Z^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 1]$$
$$= Pr[Z^{(i)} = Z^{(j)} \wedge NCOL = 1] \cdot Pr[\Theta^{(i)} = \Theta^{(j)} \mid Z^{(i)} = Z^{(j)} \wedge NCOL = 1]. \tag{11}$$

In Equation (11), there are two probabilities that need to be computed. First, we compute $Pr[Z^{(i)} = Z^{(j)} \wedge NCOL = 1]$ by considering different structure graphs with $NCOL = 1$, corresponding to the construction $\mathrm{NI2}_r^+$. Let G denote the set of all structure graphs with $NCOL = 1$ and $Z^{(i)} = Z^{(j)}$. Without loss of generality, let l_i and l_j be the lengths of the messages $M^{(i)}$ and $M^{(j)}$ respectively, with $l_i \geq l_j$. Let $G_1 \subset G$ be the set of all structure graphs such that the $M^{(i)}$-path does not contain any loop. The $G_2 = G \setminus G_1$ is the set of the remaining structure graphs. For the ease of understanding blue colored path represents the $M^{(i)}$ path and red colored path represents the $M^{(j)}$ path.

Analysis of G_1. If $M^{(j)}$ is a proper prefix of $M^{(i)}$, then $|G_1| = 0$, since in that case $Z^{(i)}$ won't be equal to $Z^{(j)}$. So without loss of generality, lets assume that $M^{(j)}$ is not a prefix of $M^{(i)}$. Suppose the first p blocks constitute the common prefix. Define $t^* = \min\{t > l_i + p : [[t]] \leq l_i\}$. Thus, the edge $([[t^* - 1]]', [[t^*]])$ in G creates the collision and from that point onwards, $M^{(j)}$ path will follow the rest of $M^{(i)}$ path which is nothing but the common suffix part of $M^{(i)}$ and $M^{(j)}$.

The scenario is explained in Fig. 5.1. Since there are $\leq l$ choices for t^*, we have $|G_1| \leq l$.

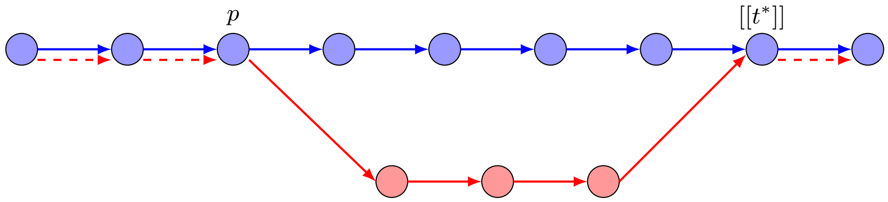

Fig. 5.1: Structure Graph of type G_1

Analysis of G_2. In graph G_2, $M^{(i)}$ path creates a collision by creating a self loop. We define $t^* = \min\{t : [[t]] \leq t\}$ and let $p^* = [[t^*]]$. Therefore, (t^*, p^*) denotes the collision in $M^{(i)}$ path. Now we can split $M^{(i)}$ into three mutual disjoint strings x, y, z such that $x := M_1^{(i)}||\ldots||M_{p^*}^{(i)}$, $y := M_{p^*+1}^{(i)}||\ldots M_{t^*}^{(i)}$ and some z chosen to be the smallest string so that we can write $M^{(i)} = x||y^a||z$ for some $a \geq 1$.

Note that to have $Z^{(i)} = Z^{(j)}$ and one collision has already been occurred in the loop, therefore, $M^{(j)}$-path must be a subpath of $M^{(i)}$-path and it cannot bifurcate from $M^{(i)}$ path and then collide with the last output block of $M^{(i)}$ as that would increases the number of collisions to 2. Thus, the $M^{(j)}$-path must be of the form $x||y^b||z$, where $b < a$ (since $l_i > l_j$ in this case). Hence, the number of blocks in y, i.e., $t^* - p^*$, in the diagram must divide $l_i - l_j$. This

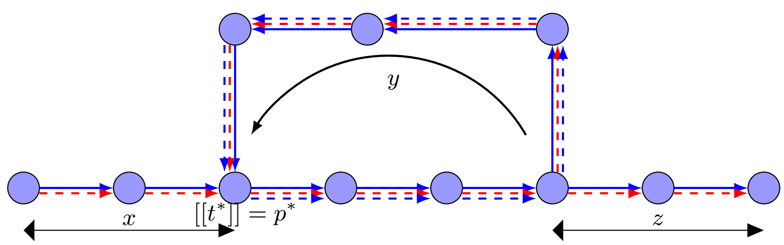

Fig. 5.2: Structure Graph of type G_2

scenario is explained in Fig. 5.2. There are at most l choices for such a t^* and $d'(l)$ choices for such a p^*. Hence, $|G_2| \leq ld'(l)$. In the special case, when $l_i = l_j$, then obviously, $|G_2| = 0$.

Therefore, considering G_1 and G_2 together, by Proposition 1, we have

$$\Pr[Z^{(i)} = Z^{(j)} \wedge NCOL = 1] \leq \frac{ld'(l)}{2^n}. \tag{12}$$

Now, we compute the second probability of Equation 11, i.e., $Pr[\Theta^{(i)} = \Theta^{(j)} \mid Z^{(i)} = Z^{(j)} \wedge NCOL = 1]$. Note that $\Theta^{(i)} = \Theta^{(j)}$ gives an equation of the form

$$2^{l_i}Y_1^{(i)} + 2^{l_i-1}Y_2^{(i)} + \cdots + 2Y_{l_i}^{(i)} = 2^{l_j}Y_1^{(j)} + 2^{l_i-1}Y_2^{(i)} + \cdots + 2Y_{l_j}^{(j)}. \tag{13}$$

The condition $Z^{(i)} = Z^{(j)}$ and $NCOL = 1$ is equivalent to the condition $Y_{l_i}^{(i)} = Y_{l_j}^{(j)}$ and $Y_a^{(i)} \neq Y_b^{(j)}$, whenever either $a < l_i$ or $b < l_j$. With this condition, Equation 13 becomes

$$2^{l_i} Y_1^{(i)} + 2^{l_i - 1} Y_2^{(i)} + \cdots + 2^2 Y_{l_i - 1}^{(i)} = 2^{l_j} Y_1^{(j)} + 2^{l_i - 1} Y_2^{(i)} + \cdots + 2^2 Y_{l_j - 1}^{(j)}. \quad (14)$$

Now, for both the graphs G_1 and G_2, we will be able to find at least one Y variable belonging to the part between p and t^*, such that Equation (14) becomes non-trivial for such variable Y, giving a probability of $\frac{1}{2^n}$ for the second term of Equation (11). When this along with Equation (12) is plugged in Equation (11), the probability in Equation (11), i.e., in Equation (9),becomes bounded by $\frac{ld'(l)}{2^{2n}}$.

6 Details of the Proof of Claim 2

Claim 2 *Fix two distinct messages $M^{(i)}, M^{(j)}$ each of length at most l blocks. Then,*

$$\sum_{k=0}^{1} \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \leq \frac{4l^2 + 1}{2^{2n}},$$

where $Z^{(i)} = Y_{l_i}^{(i)}, Z^{(j)} = Y_{l_j}^{(j)}$, l_i, l_j is the number of blocks of $M^{(i)}, M^{(j)}$ respectively.

Proof. It is to be noted that, under the condition $Z^{(i)} \neq Z^{(j)}$, $\Sigma^{(i)} = \Sigma^{(j)}$ is independent on $\Theta^{(i)} = \Theta^{(j)}$ & $NCOL = k$ for $k = 0, 1$. Therefore, we can write

$$\sum_{k=0}^{1} \Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}]$$

$$= \Pr[\Sigma^{(i)} = \Sigma^{(j)} | Z^{(i)} \neq Z^{(j)}] \left(\sum_{k=0}^{1} \Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \right).$$

Now, $\Pr[\Sigma^{(i)} = \Sigma^{(j)} | Z^{(i)} \neq Z^{(j)}] \leq \frac{1}{2^n}$ as f_3 is independent from f_1 and f_2; collision probability of a random function. Again from Claim 2 we have,

$$\sum_{k=0}^{1} \Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \leq \frac{4l^2 + 1}{2^n}. \quad (15)$$

Combining the collision probability of a random function and Equation (15), we get the result. □

.

Therefore, we are only left with the proof of Equation (15).

6.1 Proof of Equation (15)

To prove the equation, we separately bound the following $\Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 0 \mid Z^{(i)} \neq Z^{(j)}]$ and $\Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 1 \mid Z^{(i)} \neq Z^{(j)}]$ separately.

Again we consider two distinct messages $M^{(i)}$ and $M^{(j)}$ with lengths l_i and $l_j$respectively, with $l_i \geq l_j$. Since we are given the condition $Z^{(i)} \neq Z^{(j)}$, the structure graphs will have the common feature that the end-point $Y_{l_i}^{(i)}$ of $M^{(i)}$-path and the end-point $Y_{l_j}^{(j)}$ of $M^{(j)}$-path must be different, i.e., from Equation (13), we have $Y_{l_i}^{(i)} \oplus Y_{l_j}^{(j)} = c \neq 0$. Thus, Equation (13) becomes non-trivial, with probability $\frac{1}{2^n}$.

Now, we need to count the number of distinct structure graphs for each of the cases $NCOL = 0$ and $NCOL = 1$.

Clearly, when $NCOL = 0$, only such structure graph is possible, as shown in Fig. 6.1. Thus, we have

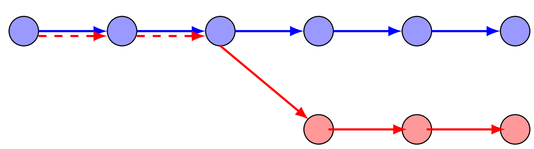

Fig. 6.1: Structure Graph of accident 0

$$\Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 0 \mid Z^{(i)} \neq Z^{(j)}] \leq \frac{1}{2^n}. \tag{16}$$

Now, let us consider the case $NCOL = 1$. Let G be the set of all structure graphs with $NCOL = 1$ with $Z^{(i)} \neq Z^{(j)}$. Let $G_1 \subset G$ be the set of all structure graphs such that the $M^{(i)}$-path does not contain any loop. The $G_2 = G \setminus G_1$ is the set of remaining structure graphs.

Analysis of G_1. For G_1, the $M^{(j)}$ path can either intersect with $M^{(i)}$ exactly once or $M^{(j)}$ path does not intersect with $M^{(i)}$ but it creates a loop with itself. In the first case, $M^{(j)}$-path cannot have any loop as shown in Fig. 6.2 as that would increase the number of collision to 2, and in the second case, the $M^{(j)}$ path cannot intersect $M^{(i)}$-path at all as that would again increase the number of collision to 2 as shown in Fig. 6.3. In either case, the number of such graphs is at most l^2.

Analysis of G_2. For G_2, note that $M^{(i)}$ path contains a loop. Now the $M^{(j)}$ path may or may not intersects $M^{(i)}$ path. If it does, then it must follow the same loop as $M^{(i)}$ and then exit either from the loop or afterwards, as shown

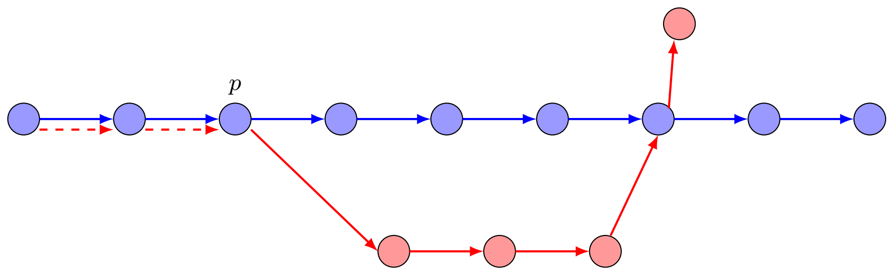

Fig. 6.2: Structure Graph of type G_1; $M^{(i)}$ path has no loop

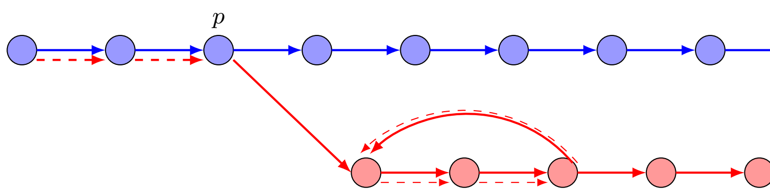

Fig. 6.3: Structure Graph of type G_1; $M^{(i)}$ path has no loop, $M^{(j)}$ path has loop

in Fig. 6.4. $M^{(j)}$ path may also bifurcate from $M^{(i)}$ path before the loop and then it should not intersect with $M^{(i)}$ path again or it should not make any self loop with itself as both of the cases would increases the number of collision to 2. Note that $M^{(j)}$ path cannot intersect $M^{(i)}$ path before the loop as that would increase the number of collision to 2.

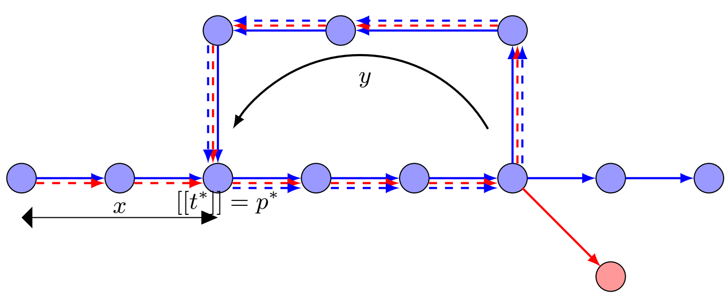

Fig. 6.4: Structure Graph of type G_2; $M^{(i)}, M^{(j)}$ both path contain a loop

If $M^{(j)}$ path does not intersect $M^{(i)}$ path, then $M^{(j)}$ path cannot make a loop with itself as that would increase the number of collision to 2. Therefore, again the case is similar to Fig. 6.3 where the blue colored path will then represent the $M^{(j)}$ path and red colored path will represent $M^{(i)}$ path. In either case, the number of such graphs is at most l^2.

Thus, for the above $4l^2$ graphs (combined G_1 and G_2),

$$\Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = 1 \mid Z^{(i)} \neq Z^{(j)}] \leq \frac{4l^2}{2^n}. \tag{17}$$

Therefore, from Equation (16) and (17), we get

$$\sum_{k=0}^{1} \Pr[\Theta^{(i)} = \Theta^{(j)} \wedge NCOL = k | Z^{(i)} \neq Z^{(j)}] \leq \frac{4l^2 + 1}{2^n}.$$

Remark 2 *We have achieved BBB security for the NI2$^+$ MAC which is the extended version of NI2 MAC. Note that NI2 MAC is a variant of NI MAC. One can easily show that same modification on NI MAC gives BBB security. It is to be noted that in case of NI$^+$ MAC when we calculate*

$$\Pr[\Sigma^{(i)} = \Sigma^{(j)} \wedge \Theta^{(i)} = \Theta^{(j)} \wedge Z^{(i)} = Z^{(j)}],$$

then we should consider only the structure graph that does not contain any loop as we need to consider the i^{th} and j^{th} message having same length.

7 Conclusion and Future Work

Rcently, NI2-MAC was introduced in order to prove the security of NI-MAC. In this paper, we show a modified construction of NI2-MAC and NI-MAC and prove its security to be beyond birthday. While we use we use an extra keyed function (f_{K_3}) in both the constructions, an interesting research problem would be to avoid the usage of this extra keyed function and achieves beyond birthday security.

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