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# Commitment and Oblivious Transfer in the Bounded Storage Model with Errors

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## Abstract

In the bounded storage model the memory of the adversary is restricted, instead of its computational power. With this different restriction it is possible to design protocols with information-theoretical (instead of only computational) security. We present the first protocols for commitment and oblivious transfer in the bounded storage model with errors, i.e., the model where the public random sources available to the two parties are not exactly the same, but instead are only required to have a small Hamming distance between themselves. Commitment and oblivious transfer protocols were known previously only for the error-free variant of the bounded storage model, which is harder to realize.

**Keywords:** Bounded storage model, error correction, commitment, oblivious transfer, unconditional security.

# 1 Introduction

*Commitment schemes* are fundamental building blocks of modern cryptography. They are important in the construction of protocols such as identification protocols [27], contract signing [26], zero-knowledge proofs [29], coin flipping over the phone [6], and more generally in two- and multi-party computation protocols [28, 10]. A commitment scheme is a two-stage protocol between two parties, Alice and Bob. First they execute the *commit stage*, in which Alice chooses a value  $v$  as input and commits to it. Later, they execute the *open stage*, in which Alice reveals  $v$  to Bob. For the protocol to be secure, it must satisfy two conditions: the *hiding property*, which means that Bob cannot learn any information about  $v$  before the open stage, and the *binding property*, which means that after the commit phase, Alice cannot change  $v$  without that being detected by Bob.

Another essential primitive for two- and multi-party computation is *oblivious transfer* (OT). It is a two-party protocol in which Alice inputs two strings  $s_0$  and  $s_1$ , and Bob inputs a bit  $c$ . Bob's output is the string  $s_c$ . The protocol is called secure if Alice never learns the choice bit  $c$  and Bob does not learn any information about  $s_{1-c}$ . Oblivious transfer is a fundamental building block for multi-party computation and can be used to realize *any* secure two-party computation [38, 35].

In the setting where the parties only communicate through noiseless channels, unconditionally secure commitment and oblivious transfer are impossible (even if quantum channels are available [40]). However, both of them are possible in the context of computational security (in which the adversaries are restricted to be polynomial-time Turing machines), as long as computational hardness assumptions are made. Commitment can be obtained using generic assumptions such as the existence of pseudorandom generator [41] or (more efficiently) assuming the hardness of various specific computational problems [25, 6, 45]. Oblivious transfer can be obtained from dense trapdoor permutations [31] (which is conjectured to be stronger than pseudorandom generators) or assuming the hardness of many specific computational problems [48, 4, 36, 46, 23, 16].

If one wants to obtain unconditional security for commitment and oblivious transfer protocols, then one possibility is resorting to physical assumptions such as the existence of noisy channels. In this scenario the problem was studied from both the theoretical [15, 56, 34, 1, 43, 47, 22] and the efficient protocol designing [12, 11, 53, 13] points of view.

In this paper a different setting is considered, the so called *bounded storage model* (BSM) [39], in which the adversary is assumed to have bounded memory.

## 1.1 The Bounded Storage Model

In the bounded storage model, it is assumed that both parties have access to a public random string, and that a dishonest party cannot store the whole string. This string can be obtained from a natural source, from a trusted third party, or, in some cases even generated by one of the parties.

A variety of cryptographic tasks can be implemented in the bounded storage model. Cachin and Maurer [8] proposed a key agreement protocol in the bounded storage model in which the parties have a small pre-shared key, and use it to select bits from a public random source of size  $n$ . It was shown that key agreement in this setting is always possible if the pre-shared key has size proportional to  $\log n$ , as long as the adversary has bounded memory. They also proposed a protocol for key agreement by public discussion (that is, without a pre-shared key) that requires  $\sqrt{n}$  samples from the random source and is thus less practical. Later, Dziembowski and Maurer [24] showed that this protocol is optimal, in the sense that one cannot have key

agreement by public discussion in the bounded storage model with less than  $\sqrt{n}$  samples.

The first oblivious transfer protocol in the bounded storage model was introduced by Cachin et al. [7]. Improvements (in a slight different model) were presented by Ding [17] and Hong et al. [33]. Ding et al. [19] obtained the first constant-round protocol.

Recently, Shikata and Yamanaka [52] and independently Alves [2] studied the problem of commitment in the bounded storage model and provided protocols for it. Both constructions are based on the work of Ding et al. [19].

Unfortunately the bounded storage model assumes that there exists a random source that can be reliably broadcasted to all parties, without errors in the transmission, and this is hard to realize in practice. Our goal with this work is to study two-party protocols under more realistic assumptions.

## 1.2 Our contribution

In this work, a more general variant of the BSM is considered, in which errors can be introduced in the public random source in arbitrary positions. This setting captures the situation in which the source is controlled by an adversary, and also the situation in which there are errors due to noise in the channel. It is only assumed that the fraction of errors, relative to the length of the public string, is not too large. This model has been previously studied by Ding [18] in the context of secret key agreement protocols. He defined a general paradigm for BSM randomness extraction schemes and also showed how to incorporate error correction in key agreement protocols by using fuzzy extractors [20].

Our contribution is two-fold: we introduce the first protocols for bit commitment and oblivious transfer in the BSM with errors, thus extending the results of [18] to the case of two-party secure protocols. The basic idea of our commitment protocol is to use the public source to extract a completely random string, even from the point of view of the adversary, and use it as a one-time pad. This assures the hiding property. The binding property is guaranteed by the correlation between the strings sampled by Alice and Bob, along with the output of a hash function which is sent by Alice. No explicit error-correction happens, in contrast to [18].

We also propose the first protocol for oblivious transfer in this model, thus showing that any multi-party computation protocol can be realized. In the case of oblivious transfer it is necessary to use error correction techniques to ensure correctness.

Interestingly, how much noise can be handled by our protocols differs for the oblivious transfer and commitment schemes. We conjecture that there exists an intrinsic difference in the amount of noise that can be tolerated by commitment schemes and oblivious transfer schemes.

## 2 Preliminaries

The probability distribution of a random variable  $X$  will be denoted by  $P_X$ . The set  $\{1, \dots, n\}$  will be written as  $[n]$ . If  $x = (x_1, \dots, x_n)$  is a sequence and  $S = \{s_1, \dots, s_t\} \subseteq [n]$ ,  $x^S$  denotes the sequence  $(x_{s_1}, \dots, x_{s_t})$ .  $u \stackrel{\$}{\leftarrow} U$  denotes that  $u$  is drawn from the uniform distribution over the set  $U$  and  $U_r$  is the uniformly-distributed  $r$ -bit random variable.  $y \stackrel{\$}{\leftarrow} \mathcal{F}(x)$  denotes the act of running the probabilistic algorithm  $\mathcal{F}$  with input  $x$  and obtaining the output  $y$ .  $y \leftarrow \mathcal{F}(x)$  is similarly used for deterministic algorithms.

If  $X$  and  $Y$  are strings,  $\text{HD}(X, Y)$  denotes their Hamming distance (that is, the number of positions in which they differ) and  $X \oplus Y$  their bitwise exclusive or. Let  $\log x$  denote the logarithm of  $x$  in base 2. The binary entropy function is denoted by  $h$ : for  $0 \leq x \leq 1$ ,

$h(x) = -x \log x - (1-x) \log(1-x)$ . By convention,  $0 \log 0 = 0$ .  $H(X)$  denotes the entropy of  $X$  and  $I(X; Y)$  the mutual information between  $X$  and  $Y$ .

The *statistical distance* is a measure of the distance between two probability distributions:

**Definition 2.1 (Statistical distance)** *The statistical distance  $\|P_X - P_Y\|$  between two probability distributions  $P_X, P_Y$  over an alphabet  $\mathcal{X}$  is defined as*

$$\|P_X - P_Y\| = \max_{A \subseteq \mathcal{X}} \left| \sum_{x \in A} P_X(x) - P_Y(x) \right|.$$

We say  $P_X$  and  $P_Y$  are  $\varepsilon$ -close if  $\|P_X - P_Y\| \leq \varepsilon$ .

## 2.1 Entropy Measures

The main entropy measure in this work is the *min-entropy*, which captures the notion of unpredictability of a random variable.

**Definition 2.2 (Min-entropy)** *Let  $P_{XY}$  be a probability distribution over  $\mathcal{X} \times \mathcal{Y}$ . The min-entropy of  $X$ , denoted by  $H_\infty(X)$ , and the conditional min-entropy of  $X$  given  $Y$ , denoted by  $H_\infty(X | Y)$ , are respectively defined as*

$$H_\infty(X) = \min_{x \in \mathcal{X}} (-\log P_X(x))$$

$$H_\infty(X | Y) = \min_{y \in \mathcal{Y}} \min_{x \in \mathcal{X}} (-\log P_{X|Y=y}(x))$$

$X$  is called a  $k$ -source if  $H_\infty(X) \geq k$ .

The conditional min-entropy  $H_\infty(X | Y)$  measures the extractable private randomness from the variable  $X$ , given the correlated random variable  $Y$  possessed by an adversary. The min-entropy has the problem of being sensitive to small changes in the probability distribution. Due to this fact the notion of *smooth min-entropy* [50] will be used.

**Definition 2.3 (Smooth min-entropy)** *Let  $\varepsilon > 0$  and  $P_{XY}$  be a probability distribution. The  $\varepsilon$ -smooth min-entropy of  $X$  given  $Y$  is defined by*

$$H_\infty^\varepsilon(X|Y) = \max_{X'Y': \|P_{X'Y'} - P_{XY}\| \leq \varepsilon} H_\infty(X'|Y')$$

Intuitively, the smooth min-entropy is the maximum min-entropy in the neighborhood of the probability distribution. Similarly, we also define the max-entropy and its smooth version.

**Definition 2.4 ((Smooth) Max-entropy)** *The max-entropy is defined as*

$$H_0(X) = \log |\{x \in X | P_X(x) > 0\}|$$

and its conditional version is given by

$$H_0(X|Y) = \max_y H_0(X|Y=y).$$

The smooth variants are defined as

$$H_0^\varepsilon(X) = \min_{X': \|P_{X'} - P_X\| \leq \varepsilon} H_0(X'),$$

$$H_0^\varepsilon(X|Y) = \min_{X'Y': \|P_{X'Y'} - P_{XY}\| \leq \varepsilon} H_0(X'|Y').$$

The following inequalities are smooth min-entropy analogues of the chain rule for conditional Shannon entropy [50].

**Lemma 2.5** *Let  $\varepsilon, \varepsilon', \varepsilon'' > 0$  and  $P_{XYZ}$  be a tripartite probability distribution. Then*

$$\begin{aligned} H_{\infty}^{\varepsilon+\varepsilon'}(X, Y | Z) &\geq H_{\infty}^{\varepsilon}(X | Y, Z) + H_{\infty}^{\varepsilon'}(Y | Z) \\ H_{\infty}^{\varepsilon}(X, Y | Z) &< H_{\infty}^{\varepsilon+\varepsilon'+\varepsilon''}(X | Y, Z) + H_0^{\varepsilon''}(Y | Z) + \log(1/\varepsilon') \end{aligned}$$

The notion of *min-entropy rate* and a few results regarding its preservation will be used in the subsequent parts of this work.

**Definition 2.6 (Min-entropy rate)** *Let  $X$  be a random variable with an alphabet  $\mathcal{X}$ ,  $E$  be an arbitrary random variable, and  $\varepsilon \geq 0$ . The min-entropy rate  $R_{\infty}^{\varepsilon}(X | E)$  is defined as*

$$R_{\infty}^{\varepsilon}(X | E) = \frac{H_{\infty}^{\varepsilon}(X | E)}{\log |\mathcal{X}|}.$$

The following lemma is a restatement of a lemma in [19]. It says that a source with high min-entropy also has high min-entropy when conditioned on a correlated short string. This lemma is what makes the bounded storage assumption useful: it implies that the information that a memory bounded adversary has about a string sampled from the public random string is limited.

**Lemma 2.7** *Let  $X \in \{0, 1\}^n$  such that  $R_{\infty}^{\varepsilon}(X) \geq \rho$  and  $Y$  be a random variable over  $\{0, 1\}^{\phi n}$ . Fix  $\varepsilon' > 0$ . Then*

$$R_{\infty}^{\varepsilon'+\sqrt{8\varepsilon}}(X | Y) \geq \rho - \phi - \frac{1 + \log(1/\varepsilon')}{n}.$$

**Proof:** Let  $\psi = \rho - \phi - \frac{1+\log(1/\varepsilon')}{n}$ . By lemma 3.16 in [19] we have that if  $R_{\infty}^{\varepsilon}(X) \geq \rho$  then

$$\Pr_{y \leftarrow Y} \left[ R_{\infty}^{\sqrt{2\varepsilon}}(X | Y = y) \geq \psi \right] \geq 1 - \varepsilon' - \sqrt{2\varepsilon}.$$

To get the desired result, let  $\mathcal{G} = \{y \in \mathcal{Y} \mid R_{\infty}^{\sqrt{2\varepsilon}}(X | Y = y) \geq \psi\}$  and  $P_{XY}$  be the joint probability distribution of  $X$  and  $Y$ . Let  $P'_{XY}$  be the distribution that is  $\sqrt{2\varepsilon}$ -close to  $P_{XY}$  and is such that  $P'(X = x | Y = y) \leq 2^{-\psi n}$  for any  $x \in \mathcal{X}, y \in \mathcal{G}$ . Let  $P''_{XY}$  be obtained by letting  $P''(X | Y = y) = P'(X | Y = y)$  for  $y \in \mathcal{G}$  and defining  $P''(X = x | Y = y) = 2^{-n}$  for any  $x \in \mathcal{X}, y \notin \mathcal{G}$ . As  $\Pr[\mathcal{G}] \geq 1 - \varepsilon' - \sqrt{2\varepsilon}$ , it holds that  $\|P''_{XY} - P'_{XY}\| \leq \varepsilon' + \sqrt{2\varepsilon}$  and so  $\|P''_{XY} - P_{XY}\| \leq \varepsilon' + 2\sqrt{2\varepsilon}$ . Since  $P''(X = x | Y = y) \leq 2^{-\psi n}$  for every  $x \in \mathcal{X}, y \in \mathcal{Y}$ , the lemma follows.  $\blacksquare$

## 2.2 Averaging Samplers and Randomness Extractors

In the bounded storage model a typical approach for the usage of the source is the sample-then-extract paradigm, in which first some positions of the source are sampled and then an extractor is applied on these positions. Note that due to the assumption that it is infeasible to store the whole source string, it is not possible to apply an extractor to the complete string, the extractor need to be locally computable [54]. In this context, *averaging samplers* [5, 9, 57] are a fundamental tool. Intuitively, averaging samplers produce samples such that the average value of any function applied to the sampled string is roughly the same as the average when taken over the original string.

**Definition 2.8 (Averaging sampler)** A function  $\text{Samp}: \{0, 1\}^r \rightarrow [n]^t$  is an  $(\mu, \nu, \varepsilon)$ -averaging sampler if for every function  $f: [n] \rightarrow [0, 1]$  with average  $\frac{\sum_{i=1}^n f(i)}{n} \geq \mu$  it holds that

$$\Pr_{\mathcal{S} \stackrel{\$}{\leftarrow} \text{Samp}(U_r)} \left[ \frac{1}{t} \sum_{i \in \mathcal{S}} f(i) \leq \mu - \nu \right] \leq \varepsilon. \quad (1)$$

Averaging samplers enjoy several useful properties. Particularly important to this work is the fact that averaging samplers roughly preserve the *min-entropy rate*.

**Lemma 2.9 ([54])** Let  $X \in \{0, 1\}^n$  be such that  $R_\infty(X | E) \geq \rho$ . Let  $\tau$  be such that  $1 \geq \rho \geq 3\tau > 0$  and  $\text{Samp}: \{0, 1\}^r \rightarrow [n]^t$  be an  $(\mu, \nu, \varepsilon)$ -averaging sampler with distinct samples for  $\mu = (\rho - 2\tau)/\log(1/\tau)$  and  $\nu = \tau/\log(1/\tau)$ . Then for  $\mathcal{S} \stackrel{\$}{\leftarrow} \text{Samp}(U_r)$

$$R_\infty^{\varepsilon'}(X^{\mathcal{S}} | \mathcal{S}, E) \geq \rho - 3\tau$$

where  $\varepsilon' = \varepsilon + 2^{-\Omega(\tau n)}$ .

For  $t < n$ , the uniform distribution over subsets of  $[n]$  of size  $t$  is an averaging sampler, also called the  $(n, t)$ -random subset sampler.

**Lemma 2.10** Let  $0 < t < n$ . For any  $\mu, \nu > 0$ , the  $(n, t)$ -random subset sampler is a  $(\mu, \nu, e^{-t\nu^2/2})$ -averaging sampler.

**Proof:** It is just a restatement of Lemma 5.5 in [3]. **■**

The following is a result from [54] about explicit constructions of average samplers using less random coins.

**Lemma 2.11 ([54])** For every  $0 < \nu < \mu < 1$ ,  $\varepsilon > 0$ , there is an explicit construction of a  $(\mu, \nu, \varepsilon)$ -averaging sampler  $\text{Samp}: \{0, 1\}^r \rightarrow [n]^t$  that uses  $t$  distinct samples for any  $t \in [t_0, n]$ , where  $t_0 = O(\log(1/\varepsilon)/\nu^2)$ , and  $r = \log n + O(t \log(1/\nu))$  random bits.

A *randomness extractor* is a function that takes a string with high min-entropy as an input and outputs a string that is close (in the statistical distance sense) to a uniformly distributed string.

**Definition 2.12 (Strong extractor)** A function  $\text{Ext}: \{0, 1\}^n \times \{0, 1\}^r \rightarrow \{0, 1\}^m$  is a  $(k, \varepsilon)$ -strong extractor if for every  $k$ -source  $X$ , we have

$$\|P_{\text{Ext}(X, U_r), U_r} - P_{U_m, U_r}\| \leq \varepsilon.$$

The following lemma specifies the parameters of an explicit strong extractor construction [57].

**Lemma 2.13 ([57])** Let  $\rho, \psi > 0$  be arbitrary constants. For every  $n \in \mathbb{N}$  and every  $\varepsilon > e^{-n/2^{O(\log^* n)}}$ , there is an explicit construction of a  $(\rho n, \varepsilon)$ -strong extractor  $\text{Ext}: \{0, 1\}^n \times \{0, 1\}^r \rightarrow \{0, 1\}^m$  with  $m = (1 - \psi)\rho n$  and  $r = O(\log n + \log(1/\varepsilon))$ .

The oblivious transfer protocol presented in this work will use a variant of a strong extractor, called a *fuzzy extractor* [20]. Intuitively, fuzzy extractors are noise-resilient extractors, that is, extractors such that the extracted string can be reproduced by any party with a string that is close (in the Hamming distance sense) to the original source.

**Definition 2.14 (Fuzzy extractor)** A pair of functions  $\text{Ext}: \{0, 1\}^n \times \{0, 1\}^r \rightarrow \{0, 1\}^m \times \{0, 1\}^p$ ,  $\text{Rec}: \{0, 1\}^n \times \{0, 1\}^r \times \{0, 1\}^p \rightarrow \{0, 1\}^m$  is an  $(k, \varepsilon, \delta, \beta)$ -fuzzy extractor if:

- (Security) For every  $k$ -source  $X \in \{0, 1\}^n$ , let  $R \stackrel{\$}{\leftarrow} U_r$ ,  $(Y, P) \leftarrow \text{Ext}(X, R)$ . Then  $\|P_{YRP} - U_m \times P_{RP}\| \leq \varepsilon$ .
- (Recovery) For every  $X, X' \in \{0, 1\}^n$  such that  $\text{HD}(X, X') \leq \delta n$ , let  $R \stackrel{\$}{\leftarrow} U_r$ ,  $(Y, P) \leftarrow \text{Ext}(X, R)$ . Then it should hold that  $\Pr[\text{Rec}(X', R, P) = Y] \geq 1 - \beta$ .

Fuzzy extractors are a special case of *one-way key-agreement schemes* [32, 37]. Ultimately they are equivalent to performing information reconciliation followed by privacy amplification [49]. Since there is a restriction to close strings with respect to the Hamming distance, syndrome-based fuzzy extractors can be used, as summarized in the following lemma from Ding [18].

**Lemma 2.15 ([18])** Let  $1 \geq \rho, \psi > 0$  and  $1/4 > \delta > 0$  be arbitrary constants. There is a constant  $R$ , depending on  $\delta$ , such that for every sufficiently large  $n \in \mathbb{N}$ , and every  $\varepsilon > e^{-n/2^{O(\log^* n)}}$ , there exists an explicit construction of a  $(\rho n, \varepsilon, \delta, 0)$ -fuzzy extractor  $(\text{Ext}, \text{Rec})$ , where  $\text{Ext}$  is of the form  $\text{Ext}: \{0, 1\}^n \times \{0, 1\}^r \rightarrow \{0, 1\}^m \times \{0, 1\}^p$  with

$$\begin{aligned} m &= (1 - \psi)\rho n, \\ r &= O\left(\log n + \log \frac{1}{\varepsilon}\right), \\ p &\leq \frac{1 - R}{(1 - \nu)\alpha} m \end{aligned}$$

**Remark 2.16** The parameters  $R, \delta$  refer to the error-correcting code used in the construction, specifically, a code of size  $n$  with rate  $R$  that can correct  $\delta n$  errors. It is known [55] that, for a given  $\gamma$  with  $0 \leq \gamma \leq 1/2$  and  $0 \leq \varepsilon \leq 1 - h(\gamma)$ , there exists a random linear code with minimum distance  $\gamma n$  and  $R \geq 1 - h(\gamma) - \varepsilon$  (i.e., it matches the Gilbert-Varshamov bound). However this construction has no efficient decoding. We can instead use the concatenated solution in Theorem 4 of [30], which achieves the Zyablov bound. The construction provides a code with linear-time encoding and decoding such that, for a given  $0 < R < 1$  and  $\varepsilon > 0$ , can correct  $\delta n$  errors, where

$$\delta \geq \max_{R < r < 1} \frac{(1 - r - \varepsilon)y}{2} \tag{2}$$

and  $y$  is the unique number in  $[0, 1/2]$  with  $h(y) = 1 - R/r$ .

### 2.3 Interactive Hashing and Binary Encoding of Subsets

The oblivious transfer protocol proposed in this paper uses *interactive hashing* as a subprotocol. Initially introduced in the context of computationally-secure cryptography [44], interactive hashing was later generalized to the information-theoretic setting, and is particularly useful in the context of oblivious transfer protocols [7, 19, 14, 51, 47]. It is a protocol where Bob inputs a string  $W$  and Alice and Bob output two strings  $W_0, W_1$ , in such a way that one of the output strings is equal to  $W$ , and the other string is completely random from Bob's point of view, even if he is dishonest. That is, if  $W_d = W$ , then the protocol ensures that Bob cannot choose  $W_{1-d}$ . On the other hand, Alice does not know  $d$ .

A variety of protocols for realizing interactive hashing have been proposed [7, 19, 42]. In this work interactive hashing is used as a black box since the security of the oblivious transfer protocol does not depend on particular features of the interactive hashing protocol used, but only on its security properties.

**Definition 2.17 (Interactive hashing)** *Interactive hashing is a protocol between two parties, Alice and Bob, in which Bob inputs  $W \in \{0, 1\}^m$  and Alice inputs nothing, and both parties output  $W_0, W_1 \in \{0, 1\}^m$ , in lexicographic order, such that  $W_d = W$  for some  $d \in \{0, 1\}$ . The protocol is called an  $\eta$ -uniform  $(t, \theta)$ -secure interactive hashing protocol if:*

1. *Alice's view of the protocol is independent of  $d$ . That is, let  $A^*$  be a strategy for Alice and  $\text{view}_{A^*}^{\langle A^*, B \rangle}(W)$  be Alice's view of the protocol with input  $W$ . Then  $\left\{ \text{view}_{A^*}^{\langle A^*, B \rangle}(W) \mid W = W_0 \right\}$  and  $\left\{ \text{view}_{A^*}^{\langle A^*, B \rangle}(W) \mid W = W_1 \right\}$  are equal distributions.*
2. *If both parties are honest, then  $W_{1-d}$  is close to completely random. That is,  $W_{1-d}$  is  $\eta$ -close to the uniform distribution on the  $2^m - 1$  strings different from  $W_d$ .*
3. *Let  $T \subset \{0, 1\}^m$  be such that  $|T| \leq 2^t$ . Then after the protocol execution between an honest Alice and a possibly malicious Bob, it should hold that  $\Pr[W_0, W_1 \in T] \leq \theta$ , where the probability is taken over the randomness used by Alice and Bob.*

By allowing  $W_{1-d}$  to be distributed only closely to uniform, this definition is weaker than the one usually given in the literature [51]. It is, however, enough to prove security of our oblivious transfer protocol. This more general definition allows for the possibility of using the constant-round protocol of Ding et al. [19] for interactive hashing.

**Lemma 2.18 ([19])** *Let  $t, m$  be positive integers such that  $t \geq \log m + 2$ . Then there exists a four-message  $(2^{-m})$ -uniform  $(t, 2^{-(m-t)+O(\log m)})$ -secure interactive hashing protocol.*

The following lemma is a result by [42]. It is 0-uniform (that is,  $W_{1-d}$  is distributed uniformly), and achieves near-optimal security [51], but has the disadvantage of taking  $m - 1$  rounds to execute.

**Lemma 2.19 ([42])** *There exists a 0-uniform  $(t, a \cdot 2^{-(m-t)})$ -secure interactive hashing protocol for some constant  $a > 0$ .*

A secure interactive hashing scheme guarantees that one of the outputs is random; however, in the oblivious transfer protocols, the two binary strings are not used directly, but as encodings of subsets. Thus for the protocol to succeed, both outputs need to be valid encodings of subsets of  $\binom{[n]}{\ell}$ . The original protocol of Cachin et al. [7] for oblivious transfer used an encoding scheme that has probability of success  $1/2$ , thus requiring that the protocol be repeated several times to guarantee correctness. Later, Ding et al. [19] proposed a “dense” encoding of subsets, ensuring that most  $m$ -bit strings are valid encodings. More precisely, they showed the following result.

**Lemma 2.20** *Let  $\ell \leq n$ ,  $m \geq \lceil \log \binom{n}{\ell} \rceil$ ,  $t_m = \lfloor 2^m / \binom{n}{\ell} \rfloor$ . Then there exists an injective mapping  $F_m: \binom{[n]}{\ell} \times [t_m] \rightarrow [2^m]$  with  $|\text{Im}(F_m)| > 2^m - \binom{n}{\ell}$ .*



## 2.4 Miscellaneous

**Definition 2.21 (2-universal hash functions)** A family of functions  $G = \{g: \mathcal{H} \rightarrow \mathcal{L}\}$  is called a family of 2-universal hash functions if for any  $x, y \in \mathcal{H}$

$$\Pr_{g \stackrel{\$}{\leftarrow} G} [g(x) = g(y)] \leq \frac{1}{|\mathcal{L}|}$$

The following is a basic fact that follows from simple counting.

**Lemma 2.22** Let  $0 \leq \delta < 1/2$  and let  $X, Y \in \{0, 1\}^n$  such that  $\text{HD}(X, Y) \leq \delta n$  and  $H_\infty(X) \geq \alpha n$  where  $0 < \alpha < 1$ . Then  $H_\infty(Y) \geq (\alpha - h(\delta))n$ .

The following lemma shows that random subsets of two sets  $X$  and  $Y$  have relative Hamming distances that are close to the one between  $X$  and  $Y$ .

**Lemma 2.23** Let  $X, Y \in \{0, 1\}^n$ ,  $\mathcal{S}$  be a random subset of  $[n]$  of size  $r$  and consider any  $\nu \in [0, 1]$ . On one hand, if  $\text{HD}(X, Y) \leq \delta n$ , then  $\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}}) < (\delta + \nu)r$  except with probability  $e^{-r\nu^2/2}$ . On the other hand, if  $\text{HD}(X, Y) \geq \delta n$ , then  $\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}}) > (\delta - \nu)r$  except with probability  $e^{-r\nu^2/2}$ .

**Proof:** Lets begin with the first part of the Lemma. By Lemma 2.10, a random subset sampler is an  $(\mu, \nu, e^{-r\nu^2/2})$ -averaging sampler for any  $\mu, \nu > 0$ . Hence for any  $f: [n] \rightarrow [0, 1]$  with  $\frac{1}{n} \sum_{i=1}^n f(i) \geq \mu$

$$\Pr \left[ \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} f(i) \leq \mu - \nu \right] \leq e^{-r\nu^2/2}, \quad (3)$$

Let

$$f(i) = \begin{cases} 0, & \text{if } X_i \neq Y_i, \\ 1, & \text{otherwise.} \end{cases}$$

Fix  $\mu = 1 - \delta$ . Note that  $\frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} f(i) = 1 - \frac{\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}})}{r}$  and  $\frac{1}{n} \sum_{i=1}^n f(i) = 1 - \frac{\text{HD}(X, Y)}{n} \geq \mu$ . Thus by Equation (3)

$$\begin{aligned} e^{-r\nu^2/2} &\geq \Pr \left[ \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} f(i) \leq \mu - \nu \right] \\ &= \Pr \left[ 1 - \frac{\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}})}{r} \leq 1 - \delta - \nu \right] \\ &= \Pr [\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}}) \geq (\delta + \nu)r] \end{aligned}$$

which proves the first part of the Lemma.

The second part of the Lemma uses the same idea, but now the function  $f$  is

$$f(i) = \begin{cases} 0, & \text{if } X_i = Y_i, \\ 1, & \text{otherwise.} \end{cases}$$

Fixing  $\mu = \delta$  it holds that  $\frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} f(i) = \frac{\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}})}{r}$  and  $\frac{1}{n} \sum_{i=1}^n f(i) = \frac{\text{HD}(X, Y)}{n} \geq \mu$  and hence

$$\begin{aligned} e^{-r\nu^2/2} &\geq \Pr \left[ \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} f(i) \leq \mu - \nu \right] \\ &= \Pr \left[ \frac{\text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}})}{r} \leq \delta - \nu \right] \\ &= \Pr \left[ \text{HD}(X^{\mathcal{S}}, Y^{\mathcal{S}}) \leq (\delta - \nu)r \right] \end{aligned}$$

which finishes the proof of the lemma.  $\blacksquare$

The following statement of the birthday paradox is standard.

**Lemma 2.24** *Let  $A, B \subset [n]$ , chosen independently at random, with  $|A| = |B| = 2\sqrt{\ell n}$ . Then*

$$\Pr[|A \cap B| < \ell] < e^{-\ell/4}$$

**Proof:** See corollary 3 in [17].  $\blacksquare$

The following useful lemma will also be needed in the subsequent sections.

**Lemma 2.25** *Let  $0 < \sigma < 1/2$ . Then*

$$\sum_{i=1}^{\sigma k} \binom{k}{i} \leq 2^{h(\sigma)k}.$$

**Proof:** It holds that

$$\begin{aligned} 2^{-h(\sigma)k} &= 2^{(\sigma \log \sigma + (1-\sigma) \log(1-\sigma))k} \\ &= \sigma^{\sigma k} (1-\sigma)^{(1-\sigma)k} \\ &\leq \sigma^i (1-\sigma)^{k-i} \quad \text{for } i = 1, \dots, \sigma k. \end{aligned}$$

where the last inequality is valid for  $\sigma < 1/2$ .

Hence

$$\begin{aligned} 2^{-h(\sigma)k} \sum_{i=1}^{\sigma k} \binom{k}{i} &\leq \sum_{i=1}^{\sigma k} \binom{k}{i} \sigma^i (1-\sigma)^{k-i} \\ &\leq \sum_{i=0}^k \binom{k}{i} \sigma^i (1-\sigma)^{k-i} = 1. \end{aligned}$$

This proves the lemma.  $\blacksquare$

### 3 Commitment in the Bounded Storage Model

#### 3.1 Security Model

The following model for the protocol execution is considered.

**TRANSMISSION PHASE.** Prior to the realization of the protocols' main part, a transmission phase is executed. In this phase, the sender (Alice) has access to an  $\alpha n$ -source  $X \in \{0, 1\}^n$ , where  $0 < \alpha < 1$ , and the receiver (Bob) to  $\tilde{X} \in \{0, 1\}^n$  such that  $\text{HD}(X, \tilde{X}) \leq \delta n$ . Note that this captures both the situation where the source is noisy and the situation where Alice controls part of the source. Bob then computes a randomized function  $f(\tilde{X})$  with output size smaller or equal to  $\gamma n$  for  $\gamma < \alpha$ , stores its output and discards  $\tilde{X}$ .<sup>1</sup> This is the so called bounded storage assumption. After that, Alice and Bob output bits  $A_T$  and  $B_T$ , respectively, with the value 1 if they want to continue the protocol and 0 if they want to abort it.

**COMMIT PHASE.** Alice has an input string  $v \in \{0, 1\}^m$  (which is a realization of a random variable  $V$ ) that she wants to commit to. The parties exchange messages, possibly in several rounds. Let  $M$  be the random variable denoting all the communication in this phase. This random variable  $M$  is a function of  $v$ ,  $X$ ,  $f(\tilde{X})$  and the randomness  $R_A$  and  $R_B$  used by Alice and Bob, respectively.

**OPEN PHASE.** Alice sends Bob the string  $\tilde{v}$  that she claims she committed to. The parties can then exchange messages in several rounds. Let  $N$  be the random variable denoting all the communication in this phase. Finally, Bob outputs a bit  $T(\tilde{v}, f(\tilde{X}), R_B, M, N)$ , which is 1 if Bob accepts Alice's commitment, and 0 otherwise.

**SECURITY.** A protocol is called  $(\lambda_C, \lambda_H, \lambda_B)$ -secure if it satisfies the following properties:

1.  $\lambda_C$ -correct: if Alice and Bob are honest, then

$$\Pr[A_T = B_T = T(v, f(\tilde{X}), R_B, M, N) = 1] \geq 1 - \lambda_C.$$

2.  $\lambda_H$ -hiding: if Alice is honest then

$$I(V; R_B, M | \tilde{X}) \leq \lambda_H.$$

3.  $\lambda_B$ -binding: if Bob is honest, then there are no  $\tilde{v} \neq v$ ,  $N$  and  $\tilde{N}$  such that

$$\Pr[T(v, f(\tilde{X}), R_B, M, N) = 1] \geq \lambda_B$$

and

$$\Pr[T(\tilde{v}, f(\tilde{X}), R_B, M, \tilde{N}) = 1] \geq \lambda_B.$$

It should be pointed out that even if Bob gains infinite storage power after the transmission phase is over and the source is not available anymore, this does not affect the security of the protocol, i.e., it has everlasting security.

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<sup>1</sup>In order to achieve security it is not necessary to impose any storage bound on Alice, but in the proposed protocol an honest Alice stores the same amount of information as an honest Bob.

### 3.2 A Commitment Protocol

The scheme works as follows. First, both parties sample a number of bits from the public source. Alice then extracts the randomness of her sample and uses it to conceal her commitment before sending it to Bob. This guarantees the hiding condition. She also computes a hash of her sample, where the hash function is chosen by Bob. Alice sends Bob the concealed commitment along with the hash value. In the open phase, Alice sends her committed value and her sampled string. Bob then performs a number of checks for consistency. These checks enforce binding. The details of the protocol are presented below.

As remarked in Section 3.1, it is assumed that in the transmission phase, an  $\alpha n$ -source  $X \in \{0, 1\}^n$  for a fixed  $0 < \alpha < 1$ , is available to Alice, and  $\tilde{X} \in \{0, 1\}^n$  with  $\text{HD}(X, \tilde{X}) \leq \delta n$  is available to Bob. The security parameter is  $\ell$  and  $k$  is set as  $k = 2\sqrt{\ell n}$ . Fix  $\varepsilon' > 0$  and let  $\rho = \alpha - \gamma - \frac{1 + \log(1/\varepsilon')}{n}$ . Fix  $\tau$  such that  $\frac{\rho}{3} \geq \tau > 0$ , and  $\omega, \zeta > 0$  such that  $\rho - 3\tau > \omega > 2h(\delta + \zeta)$  and  $\delta + \zeta < 1/2$ . It is assumed that the following functionalities, which are possible due to the lemmas in Section 2, are available to the parties:

- A family  $G$  of 2-universal hash functions  $g: \{0, 1\}^k \rightarrow \{0, 1\}^{\omega k}$ .
- A  $(k_E, \varepsilon_E)$ -strong extractor  $\text{Ext}: \{0, 1\}^k \times \{0, 1\}^r \rightarrow \{0, 1\}^m$ , where  $k_E = (\rho - 3\tau - \omega)k$  and  $m = (1 - \psi)k_E$  for any  $\psi > 0$ ,  $\varepsilon_E > e^{-k/2^{O(\log^* k)}}$ .

**Remark 3.1** Note that it should hold that  $2h(\delta) < \omega + 3\tau < \rho < \alpha - \gamma$ , so the protocol is only possible if  $2h(\delta) < \alpha - \gamma$ .

#### Transmission phase:

1. Alice chooses uniformly  $k$  positions from  $X$ . Similarly, Bob samples  $k$  positions from  $\tilde{X}$ . We call their sets of positions  $A$  and  $B$ , respectively.

#### Commit phase:

1. Bob chooses  $g \xleftarrow{\$} G$  and sends its description to Alice.
2. Alice computes  $h \leftarrow g(X^A)$ ,  $u \xleftarrow{\$} \{0, 1\}^r$ , and  $y \leftarrow \text{Ext}(X^A, u)$ . She then computes  $\Omega = v \oplus y$  and sends  $(\Omega, h, A, u)$  to Bob.

#### Open phase:

1. Alice sends  $v'$  and  $W$  to Bob, which are defined as  $v' = v$  and  $W = X^A$  in the case that she is honest.
2. Let  $C = A \cap B$ ,  $c = |C|$  and  $W^C$  be the restriction of  $W$  to the positions corresponding to the set  $C$ . Bob verifies whether  $c \geq \ell$ ,  $\text{HD}(W^C, \tilde{X}^C) \leq (\delta + \zeta)c$ ,  $h = g(W)$  and  $v' = \text{Ext}(W, u) \oplus \Omega$ . If any verification fails Bob outputs 0, otherwise he outputs 1.

### 3.3 Proof of Security for the Commitment Protocol

In this section it is proved that the above protocol for commitment is  $(\lambda_C, \lambda_H, \lambda_B)$ -secure for  $\lambda_C, \lambda_H$  and  $\lambda_B$  negligible in  $\ell$ .

**Lemma 3.2** *The protocol is  $\lambda_C$ -correct for  $\lambda_C$  negligible in  $\ell$ .*

**Proof:** It is clear that if both Alice and Bob are honest, the protocol will fail only in the case that  $c < \ell$  or  $\text{HD}(X^C, \tilde{X}^C) > (\delta + \zeta)c$ . By Lemma 2.24,  $c \geq \ell$  except with probability  $e^{-\ell/4}$ . By Lemma 2.23,  $\text{HD}(X^C, \tilde{X}^C) \leq (\delta + \zeta)c$  except with probability  $e^{-c\zeta^2/2}$ , which is negligible in  $\ell$  if  $c \geq \ell$ . ■

**Lemma 3.3** *The protocol is  $\lambda_H$ -hiding, for  $\lambda_H$  negligible in  $\ell$ .*

**Proof:** After the commit phase, (a possibly malicious) Bob possesses  $(\Omega, h, A, u)$ ,  $g$  and the output of a function  $f(\cdot)$  of  $\tilde{X}$ , where  $|f(\tilde{X})| \leq \gamma n$  with  $\gamma < \alpha$ . The only random variable that can provide mutual information about  $V$  when conditioned on  $\tilde{X}$  is  $\Omega$ , but we prove below that  $\Omega$  is almost uniform from Bob's point of view, and so it works as an one-time pad and only negligible information can be leaked.

By Lemma 2.7,

$$R_\infty^{\varepsilon'}(X | f(\tilde{X})) \geq \alpha - \gamma - \frac{1 + \log(1/\varepsilon')}{n} = \rho.$$

Since Alice chooses  $A$  randomly and this is an  $(\mu, \nu, e^{-k\nu^2/2})$ -averaging sampler for any  $\mu, \nu > 0$  according to Lemma 2.10, by setting  $\mu = \frac{\rho - 2\tau}{\log(1/\tau)}$ ,  $\nu = \frac{\tau}{\log(1/\tau)}$ , we have by Lemma 2.9 that

$$R_\infty^{\varepsilon'' + \varepsilon'}(X^A | A, f(\tilde{X})) \geq \rho - 3\tau$$

where  $\varepsilon''$  is a negligible function of  $k$ .

It holds that

$$\begin{aligned} H_\infty^{\varepsilon'' + \varepsilon'}(X^A | h, A, u, g, f(\tilde{X})) &= H_\infty^{\varepsilon'' + \varepsilon'}(X^A | h, A, f(\tilde{X})) \\ &\geq H_\infty^{\varepsilon'' + \varepsilon'}(X^A | A, f(\tilde{X})) - H_0(h) \\ &\geq (\rho - 3\tau - \omega)k \\ &= k_E. \end{aligned}$$

Therefore, setting  $\varepsilon'$  and  $\varepsilon_E$  to be negligible in  $\ell$ , the use of the strong extractor to obtain  $y$  (and of  $y$  to xor the message) guarantees that only negligible information about the committed message can be leaked. ■

**Lemma 3.4** *The protocol is  $\lambda_B$ -binding, for  $\lambda_B$  negligible in  $\ell$ .*

**Proof:** The protocol is binding if, after the commit phase, Alice cannot choose between two different values to open without being caught. Let  $\sigma = \delta + \zeta$ . The only way Alice can cheat is if she can come up with two strings  $W, W'$  such that  $g(W) = g(W')$ ,  $\text{HD}(W^C, \tilde{X}^C) \leq \sigma c$  and  $\text{HD}(W'^C, \tilde{X}^C) \leq \sigma c$  (with  $c \geq \ell$ ). If this happens, it holds that either there are two strings  $W, W'$  such that  $g(W) = g(W')$ ,  $\text{HD}(W, \tilde{X}^A) \leq \sigma k$  and  $\text{HD}(W', \tilde{X}^A) \leq \sigma k$ ; or Alice can compute  $W$  (without knowing the set  $B$  that together with  $A$  determines  $C$ ) such that  $\text{HD}(W, \tilde{X}^A) > \sigma k$  and  $\text{HD}(W^C, \tilde{X}^C) \leq \sigma c$ . It is proven below that the probability that Alice succeeds in cheating decreases exponentially with the security parameter  $\ell$  (or, equivalently in

$k, c$ ). First the probability that there exists two different strings  $W, W'$  both within Hamming distance  $\sigma k$  from  $\tilde{X}^A$  and such that  $g(W) = g(W')$  is upper bounded by

$$\Pr \left[ \exists W, W' \text{ s.t. } \begin{cases} W \neq W' \\ g(W) = g(W') \\ \text{HD}(W, \tilde{X}^A) \leq \sigma k \\ \text{HD}(W', \tilde{X}^A) \leq \sigma k \end{cases} \right] = \sum_{W : \text{HD}(W, \tilde{X}^A) \leq \sigma k} \left( \sum_{W' \neq W : \text{HD}(W', \tilde{X}^A) \leq \sigma k} 2^{-\omega k} \right) \leq 2^{-(\omega - 2h(\sigma))k}$$

where Lemma 2.25 was used to obtain the inequality. By design, it holds that  $\omega > 2h(\sigma)$ , therefore the probability that Alice successfully cheats by finding two strings that are at distance at most  $\sigma k$  from  $\tilde{X}^A$  and hash to the same value is negligible in  $k$ .

Now considering the second case, by assumption  $W$  has Hamming distance  $(\sigma + \psi)k$  from  $\tilde{X}^A$  for some  $\psi > 0$ . Since Bob is honest,  $B$  is chosen randomly. Hence Lemma 2.23 can be applied and thus the probability that  $\text{HD}(W^C, \tilde{X}^C) \leq \sigma c$  is smaller than  $e^{-c\psi^2/2}$ . ■

## 4 Oblivious Transfer in the Bounded Storage Model

### 4.1 Security Model

The initial transmission phase is the same as in the case of the commitment protocols (see Section 3.1) and is executed prior to the realization of the main part of the protocol. The definition of oblivious transfer used is the one presented in [19]. An oblivious transfer protocol is a protocol between two players, Alice and Bob, in which Alice inputs two strings  $s_0, s_1 \in \{0, 1\}^m$  and outputs nothing, and Bob inputs  $c \in \{0, 1\}$  and outputs  $s \in \{\perp, s_c\}$ . In the following,  $\text{view}_{A^*}(s_0, s_1; c)$  denotes the view of Alice using a strategy  $A^*$  with honest Bob, and  $\text{view}_{B^*}(s_0, s_1; c)$  denotes the view of Bob using a strategy  $B^*$  with honest Alice. A strategy used by Bob has bounded storage, but a strategy used by Alice can be unbounded.

Intuitively, the protocol will be secure for Bob if the view of Alice does not depend on the choice bit  $c$ , and secure for Alice if Bob cannot obtain any information about  $s_{1-c}$ . However this is tricky to formalize, because a malicious Bob could choose to play with a different bit, depending on the public random source and the messages exchanged before any secret is used by Alice.

In order to have a general definition of security, the main part of the oblivious transfer protocols is further divided into two phases: the *setup phase*, consisting of all the communication before Alice uses her secrets, and the *transfer phase*, which goes up until the point where Bob outputs  $s$ . By the end of the setup phase, Bob must have chosen a bit  $i$ , which may be different from  $c$  and can depend on all the messages exchanged thus far. To guarantee Alice's security it is thus required that there is an index  $i$ , determined at the setup phase, such that for any two pairs  $(s_0, s_1), (s'_0, s'_1)$  with  $s_i = s'_i$  the distributions of  $s_{1-i}$  and  $s'_{1-i}$  are close given Bob's view. Following the terminology of [19], pairs  $(s_0, s_1), (s'_0, s'_1)$  satisfying  $s_i = s'_i$  will be called  *$i$ -consistent*. To account for aborts, it is assumed that at the end of the setup phase, Alice and Bob output bits  $A_S, B_S$ , respectively, which are 1 if they want to continue and 0 if they abort.

**SECURITY.** A protocol is called  $(\lambda_C, \lambda_B, \lambda_A)$ -secure if it satisfies the following properties:

1.  $\lambda_C$ -correct: if Alice and Bob are honest, then

$$\Pr[A_T = B_T = A_S = B_S = 1 \wedge s = s_c] \geq 1 - \lambda_C$$

2.  $\lambda_B$ -secure for Bob: for any strategy  $A^*$  used by Alice,

$$\|\{\text{view}_{A^*}(s_0, s_1; 0)\} - \{\text{view}_{A^*}(s_0, s_1; 1)\}\| \leq \lambda_B$$

3.  $\lambda_A$ -secure for Alice: for any strategy  $B^*$  used by Bob with input  $c$ , there exists a random variable  $i$ , defined at the end of the setup stage, such that for every two  $i$ -consistent pairs  $(s_0, s_1), (s'_0, s'_1)$ , we have

$$\|\{\text{view}_{B^*}(s_0, s_1; c)\} - \{\text{view}_{B^*}(s'_0, s'_1; c)\}\| \leq \lambda_A$$

## 4.2 An Oblivious Transfer Protocol

The idea of the protocol is that initially both parties samples some positions from the public random source. Then an interactive hashing protocol (with an associated dense encoding) is used to select two subsets of the positions sampled by Alice. The input of Bob to the interactive hashing is one subset for which he has also sampled the public random source in that positions. The other subset is out of Bob's control due to the properties of the interactive hashing protocol. Finally the two subsets are used as input to a fuzzy extractor in order to obtain one-time pads. Bob sends one bit indicating which input string should be xored with which one-time pad. The security for Alice is guarantee by the fact that one of the subsets is out of Bob's control and will have high min-entropy given his view, thus resulting in a good one-time pad. The security for Bob follows from the security of the interactive hashing. The correctness follows from the correctness of the fuzzy extractor.

The protocol is defined below. We assume that in the transmission phase, an  $\alpha n$ -source  $X \in \{0, 1\}^n$  is available to Alice, and  $\tilde{X} \in \{0, 1\}^n$  with  $\text{HD}(X, \tilde{X}) \leq \delta n$  is available to Bob. The security parameter is  $\ell$  and  $k$  is set as  $k = 2\sqrt{\ell n}$ . Fix  $\varepsilon', \hat{\varepsilon}, \xi > 0$  and let  $\rho = \alpha - \gamma - \frac{1 + \log(1/\varepsilon')}{n}$ . Fix  $0 < \zeta < 1$  and  $\tau$  such that  $\frac{\rho}{3} \geq \tau > 0$ . Let  $\varepsilon'' = e^{-\ell \nu^2/2} - 2^{-\Omega(\tau n)}$ , where the last term comes from Lemma 2.9, and let  $\tilde{\varepsilon} = (\varepsilon' + \varepsilon'')^{1-\zeta}$ . It is assumed that the following functionalities, which are possible due to the lemmas in Section 2, are available to the parties:

- A pair of functions  $\text{Ext}: \{0, 1\}^\ell \times \{0, 1\}^r \rightarrow \{0, 1\}^{m_F} \times \{0, 1\}^p$  and  $\text{Rec}: \{0, 1\}^\ell \times \{0, 1\}^r \times \{0, 1\}^p \rightarrow \{0, 1\}^{m_F}$  that constitutes an  $(k_F \ell, \varepsilon_F, \delta + \xi, 0)$ -fuzzy extractor where  $k_F = \rho + R - 3\tau - 2m_F - 1 - \frac{1 + \log(1/\tilde{\varepsilon})}{\ell}$ ,  $m_F$  is an arbitrary number with  $0 < m_F < 1$ ,  $p = (1 - R)\ell$ , and  $\delta + \xi$  depends on  $R$ .
- An  $2^{-m}$ -uniform  $(t, \theta)$ -secure interactive hashing protocol where  $\theta$  is negligible in  $\ell$ ,  $t \geq m - \zeta \log(1/(\varepsilon' + \varepsilon''))$  and  $m \geq 2\ell \log k$ . Let  $F_m$  be a dense encoding of the subsets of size  $\ell$  of a set of size  $k$ .

Recall (Remark 2.16) that there is a tradeoff between the fraction of errors  $\delta + \xi$  that the fuzzy extractor can tolerate and the rate  $R$  of the code used in the construction. The construction given in Theorem 4 of [30] has linear-time encoding and decoding and achieves the Zyablov bound: for given  $1 > R, \theta > 0$ , the code has rate  $R$  and

$$\delta + \theta \geq \max_{R < r < 1} \frac{(1 - r - \theta)y}{2} \quad (4)$$

where  $y$  is the unique number in  $[0, 1/2]$  with  $h(y) = 1 - R/r$  and  $\delta$  the amount of errors that can be corrected by the code.

Note that in order for  $k_F$  to be positive, we need to have  $\rho + R > 1$ ; since  $\rho$  approaches  $\alpha - \gamma$  from below in the asymptotic limit, an upper bound for  $\delta$  is obtained by setting  $R > 1 - \alpha + \gamma$  and  $\theta = 0$  in Equation (4).

There is a construction based on random linear codes which achieves a better bound, namely, the Gilbert-Varshamov bound: for a given relative distance  $\eta$  and  $\theta > 0$ , the code has rate  $R \geq 1 - h(\eta) - \theta$ . Applying again the constraint that  $\rho + R > 1$  and that  $\rho \rightarrow \alpha - \gamma$  in the asymptotic limit, and using the fact that a code that can correct  $\delta n$  errors has relative distance  $\eta = 2\delta + 1/n \rightarrow 2\delta$ , this gives an upper bound for  $\delta$ : we must have  $h(2\delta) < \alpha - \gamma$ . However, as noted in Remark 2.16, the random linear code construction does not have efficient decoding. It is an open question whether an efficient construction can achieve better parameters than the one from [30].

### Transmission phase:

- Alice chooses uniformly  $k$  positions from  $X$ . Similarly, Bob samples  $k$  positions from  $\tilde{X}$ . We call their sets of positions  $A$  and  $B$ , respectively.

### Setup phase:

- Alice sends  $A$  to Bob. Bob computes  $D = A \cap B$ . If  $|D| < \ell$ , Bob aborts. Otherwise, Bob picks a random subset  $C$  of  $D$  of size  $\ell$ .
- Bob computes the encoding  $W$  of  $C$  (as a subset of  $A$ ). Alice and Bob interactively hash  $W$ , producing two strings  $W_0, W_1$ . They compute the subsets  $C_0, C_1 \subset A$  that are respectively encoded in  $W_0, W_1$ . If either encoding is invalid, they abort.

### Transfer phase:

- Bob sends  $e = c \oplus d$ , where  $W_d = W$ .
- For  $i \in \{0, 1\}$ , Alice picks  $R_i \xleftarrow{\$} \{0, 1\}^r$ , computes  $(Y_i, P_i) \leftarrow \text{Ext}(X^{C_i}, R_i)$  and  $Z_i = s_{i \oplus e} \oplus Y_i$ , and sends  $(Z_i, R_i, P_i)$  to Bob.
- Bob computes  $Y' \leftarrow \text{Rec}(\tilde{X}^C, R_d, P_d)$  and outputs  $s = Y' \oplus Z_d$ .

## 4.3 Proof of security for the oblivious transfer protocol

In this section it is proved that the protocol presented above is  $(\lambda_C, 0, \lambda_A)$ -secure for  $\lambda_C$  and  $\lambda_A$  negligible in  $\ell$ .

**Lemma 4.1** *The protocol is  $\lambda_C$ -correct for  $\lambda_C$  negligible in  $\ell$ .*

**Proof:** The probability of an abort is analyzed first. The protocol will abort if either  $|D| < \ell$ , or if one string obtained in the interactive hashing protocol is an invalid encoding of subsets of  $A$ . By Lemma 2.24,  $\Pr[|D| < \ell] < e^{-\ell/4}$ . Out of the two outputs of the interactive hashing protocol, one of them is always a valid encoding (since  $W_d = W$ , which is the encoding of  $C$ ). The other output  $W_{1-d}$  is  $2^{-m}$ -close to distributed uniformly over the  $2^m - 1$  strings different from  $W_d$ . Since it is a dense encoding, Lemma 2.20 implies that the probability that it is not a valid encoding is thus less than or equal to  $2^{-m} + \frac{\binom{k}{\ell}}{2^m - 1} \leq 2^{-m} + 2^{\ell \log k - m + 1} \leq 4k^{-\ell}$  for  $m \geq 2\ell \log k$ .



If both parties are honest and there is no abort, then  $s = s_c$  if and only if  $\text{Rec}(\tilde{X}^C, R_d, P_d) = Y_d$ . By the properties of the employed fuzzy extractor, this last event happens if  $\text{HD}(X^C, \tilde{X}^C) \leq (\delta + \xi)\ell$ . By Lemma 2.23,  $\text{HD}(X^C, \tilde{X}^C) > (\delta + \xi)\ell$  with probability at most  $e^{-\xi^2\ell/2}$ . Putting everything together this concludes the proof.  $\blacksquare$

**Lemma 4.2** *The protocol is 0-secure for Bob.*

**Proof:** There are two possibilities: either the protocol aborts or not. If the protocol aborts in the setup phase, Bob still has not sent  $e = c \oplus d$ , so Alice's view is independent from  $c$ . On the other hand, if the protocol does not abort, then  $W_{1-d}$  is a valid encoding of some set  $C'$ . Due to the properties of the interactive hashing protocol, Alice's view is then consistent with both

1. Bob choosing  $c$  and  $C$ , and
2. Bob choosing  $1 - c$  and  $C'$

Hence Alice's view is independent of  $c$ . Thus the protocol is 0-secure for Bob.  $\blacksquare$

**Lemma 4.3** *The protocol is  $\lambda_A$ -secure for Alice for  $\lambda_A$  negligible in  $\ell$ .*

**Proof:** There should be an index  $i$  (determined at the setup stage) such that for any two pairs  $(s_0, s_1), (s'_0, s'_1)$  with  $s_i = s'_i$ , Bob's view of the protocol executed with  $(s_0, s_1)$  is close to his view of the protocol executed with  $(s'_0, s'_1)$ . The view of Bob is given by the function computed from the public random source  $f(\tilde{X})$  along with all the messages exchanged and his local randomness.

The proof's strategy is to show that for  $i$ ,  $X^{C_{1-i}}$  has high enough min-entropy, given Bob's view of the protocol, in such a way that  $Y_{1-i}$  is indistinguishable from a uniform distribution. Indistinguishability of Bob's views will then follow.

By the bounded storage assumption,  $|f(\tilde{X})| \leq \gamma n$  with  $\gamma < \alpha$ . Then, by Lemma 2.7,

$$R_\infty^{\varepsilon'}(X | f(\tilde{X})) \geq \alpha - \gamma - \frac{1 + \log(1/\varepsilon')}{n} = \rho.$$

Since Alice is honest,  $A$  is randomly chosen. Lets consider a random subset  $\tilde{C}$  of  $A$  such that  $|\tilde{C}| = \ell$ . This is an  $(\mu, \nu, e^{-\ell\nu^2/2})$ -averaging sampler for any  $\mu, \nu > 0$  according to Lemma 2.10. By setting  $\mu = \frac{\rho - 2\tau}{\log(1/\tau)}$ ,  $\nu = \frac{\tau}{\log(1/\tau)}$ , we have by Lemma 2.9 that

$$R_\infty^{\varepsilon' + \varepsilon''}(X^{\tilde{C}} | A, \tilde{C}, f(\tilde{X})) \geq \rho - 3\tau$$

for  $\varepsilon'' = e^{-\ell\nu^2/2} - 2^{-\Omega(\tau n)}$ . For  $\tilde{\varepsilon} = (\varepsilon' + \varepsilon'')^{1-\zeta}$ , let **Bad** be the set of  $\tilde{C}$ 's such that  $R_\infty(X^{\tilde{C}} | A, \tilde{C}, f(\tilde{X}))$  is not  $\tilde{\varepsilon}$ -close to  $(\rho - 3\tau)$ -min entropy rate. Due to the above equation the density of **Bad** is at most  $(\varepsilon' + \varepsilon'')^\zeta$ . Then the size of the set  $T \subset \{0, 1\}^m$  of strings that maps (in the encoding scheme) to subsets in **Bad** is at most  $(\varepsilon' + \varepsilon'')^\zeta 2^m \leq 2^t$ . Hence the properties of the interactive hashing protocol guarantee that with probability greater than or equal to  $1 - \theta$  there will be an  $i$  such that

$$R_\infty^{\tilde{\varepsilon}}(X^{C_{1-i}} | A, C_{1-i}, f(\tilde{X}), M_{IH}) \geq \rho - 3\tau$$

where  $M_{IH}$  are the messages exchanged during the interactive hashing protocol. We now show that  $X^{C_{1-i}}$  has high min-entropy even when given  $Z_i, Y_i, P_i$ . We can see  $(Z_i, Y_i, P_i)$  as a random variable over  $\{0, 1\}^{(2m_F+1-R)\ell}$ . Then, by Lemma 2.7,

$$R_{\infty}^{\hat{\varepsilon}+\sqrt{8\hat{\varepsilon}}}(X^{C_{1-i}} \mid A, C_{1-i}, f(\tilde{X}), M_{IH}, Z_i, Y_i, P_i) \geq \rho + R - 3\tau - 2m_F - 1 - \frac{1 + \log(1/\hat{\varepsilon})}{\ell} = k_F.$$

Thus setting  $\varepsilon'$  and  $\hat{\varepsilon}$  to be negligible in  $\ell$ , the use of the  $(k_F\ell, \varepsilon_F, \delta + \xi, 0)$ -fuzzy extractor to obtain  $Y_i$  that is used as an one-time pad guarantees that only negligible information about  $s_{i\oplus e}$  can be leaked and that the protocol is  $\lambda_A$ -secure for Alice for negligible  $\lambda_A$ . ■

## 5 Conclusion

In this work we presented the first protocols for commitment and oblivious transfer in the bounded storage model with errors, thus extending the previous results existing in the literature for key agreement [18]. The proposed protocols are based on different techniques: while the commitment protocol is based on universal hashing and a typicality test, the oblivious transfer protocol is based on error-correcting information. As expected, our protocols work for a limited range of values of the noise parameter  $\delta$ . The allowed range for our commitment scheme is different than the one for the oblivious transfer protocol. A natural question is whether this is an intrinsic property of the functionalities. We do not believe that our protocols are optimal regarding the level of noise tolerated by them. Particularly, interactive hashing maybe could be used for obtaining commitment protocols that work for a larger range of noise. However, we do conjecture that there exists an intrinsic difference between oblivious transfer and commitment schemes in the sense that there exists levels of noise so that one of them is possible but not the other. If this conjecture is proven, this would sharply contrast with the noise-free bounded memory model, where there is an all-or-nothing situation: either one has oblivious transfer and bit commitment or one has nothing. These are all interesting sequels to the present work.

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