

## HANDLING TEMPORAL UNCERTAINTY IN GIS DOMAIN: A FUZZY APPROACH

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Commission IV, WG IV/1

**KEY WORDS:** Temporal, Uncertainty, Fuzzy Set, Model, Reasoning, GIS

### ABSTRACT:

This paper is an attempt to introduce a new framework to handle both uncertainty and time in spatial domain. The application of the fuzzy temporal constraint network (FTCN) method is proposed for representation and reasoning of uncertain temporal data. A brief introduction of the fuzzy sets theory is followed by description of the FTCN method with its main algorithms. The paper then discusses the issues of incorporating fuzzy approach into current spatio-temporal processing framework. The general temporal data model is extended to accommodate uncertainties with temporal data and relationships among events. A theoretical FTCN process of fuzzy transition for the imprecise information is introduced with an example. A summary of the paper is given together with outlining some contributions of the paper and future research directions.

### 1. INTRODUCTION

Geographic information systems (GIS) have been popularly applied in modeling environmental and ecological systems. However, real-world system analysis is typically challenged by difficulties in handling temporal change and uncertainty. Current GIS provide little or no support for modeling dynamic and uncertain phenomena. The GIS, more broadly, lack the ability to effectively model change and vagueness. There has been much research activity over the past decade focused on dynamic applications with geographic reference and handling uncertain features of geographic applications (Leung and Leung, 1993; Fischer, 1994; Stefanakis 1997). Fuzzy set theory, as a promising logical foundation for an intelligent GIS (Stefanakis, 1997), has drawn a lot of attention in its application to handling uncertainty and temporal reasoning in spatial domain. The fuzzy approach has been applied in geography on problems related to soil classification and definitions (Burrough, 1989; Kollias and Voliotis, 1991; Banai, 1993; Davidson et al., 1994; Cheng et al., 2001). The fuzzy aspect of boundaries in geographic space is now fully recognized (Mark and Csillag, 1989; Wang and Hall, 1996). A general spatial data model was extended to deal with the uncertainty with geographic entities in a database repository (Stefanakis, 1999). The fuzzy set theory was also applied to perform temporal interpolation in a raster GIS database (Dragicevic and Marceau, 2000).

There is little research trying to handle uncertainty in temporal presentation in spatial domain. This seems to be due to the fact that the handling of time and the management of uncertainty are two distinct important issues. Each of both possesses its own specific problems, and their solutions require a lot of research effort separately. Some modeling issues in spatial domain require a representation of time and of the temporal

relationships between events. Most systems rely on a mechanism in which time is associated with each piece of knowledge. In spatial data models currently used, temporal information is stored through a series of snapshots associated to particular instants in time. Relationships are then the time ordering. In the complex environmental and ecological systems modeling with GIS, the current time mechanism is not sufficient. One must be able to represent situations with relative knowledge like “precedes” or “during”. Real-world modeling requires a unified model of time and uncertainty. Here is an example of the scenario in wildlife migration modeling analysis: All animals of species  $S_1$  moved away from area A before October 1st. Animals of species  $S_2$  entered the area A after June 1st. But between mid-July and mid-August, we seldom saw both species of animal appeared at same time in the area. This gives a sample description of temporal definitions and relationships necessary for modeling wildlife migration properly. One obvious characteristic with the description is that those definition and relationships are vague. A major task of the analysis is to translate it and capture the knowledge in a form which we can correctly process and reason over.

The purpose of this paper is to introduce a novel fuzzy approach, fuzzy temporal constraint network (FTCN) for reasoning uncertain temporal data in spatio-temporal information processing. Section 2 presents a brief introduction of the fuzzy sets theory as the basis of the model. Section 3 describes the FTCN method with its main algorithms. Section 4 addresses the issues of incorporating the fuzzy approach into current spatio-temporal processing model, and of FTCN inferences in a GIS domain. Section 5 concludes the discussion by summarizing the contribution of the paper and giving some directions for future research.

## 2. FUZZY SET THEORY

Fuzzy theory is a method that facilitates uncertainty analysis of systems where uncertainty arises due to vagueness or “fuzziness” rather than due to randomness alone (Evans et al., 1986). This is based on a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth - truth values between “completely true” and “completely false”. It was introduced by Zadeh as a means to model the uncertainty of natural language (Zadeh, 1965). Fuzzy theory uses the process of “fuzzification” as a methodology to generalize any specific theory from a crisp (discrete) to a continuous (fuzzy) form.

Classical set theory has a “crisp” definition as to whether an element is a member of a set or not. However, certain attributes of systems cannot be ascribed to one set or another. For example, an attribute of a system can be specified as either “low” or “high”. In such a case, uncertainty arises out of vagueness involved in the definition of that attribute. Classical set theory allows for either one or the other value. On the other hand, fuzzy theory provides allows for a gradual degree of membership. This can be illustrated as follows:

In the classical set theory, the truth value of a statement can be given by the membership function  $\mu_A(x)$ , as

$$\mu_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \end{cases} \quad (1)$$

On the other hand, fuzzy theory allows for a continuous value of  $\mu_A$ , between 0 and 1, as

$$\mu_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \\ p; 0 < p < 1 & \text{if } x \text{ partially belongs to } A \end{cases} \quad (2)$$

In fuzzy theory, statements are described in terms of membership functions, that are continuous and have a range [0,1]. For example, given the measured value of a parameter, the membership function gives the “degree of truth” that the parameter is “high” or “low”.

Further, fuzzy logic is defined by the following the set relationships:

$$\begin{aligned} \mu_{A'}(x) &= 1 - \mu_A(x) \\ \mu_{A \cap B}(x) &= \min(\mu_A(x), \mu_B(x)) \\ \mu_{A \cup B}(x) &= \max(\mu_A(x), \mu_B(x)) \end{aligned} \quad (3)$$

Using fuzzy arithmetic, based on the grade of membership of a parameter of interest in a set, the grade of membership of a model output in another set can be calculated. Fuzzy theory can be considered to be a generalization of the classical set theory. It must be noted that if the membership grades are restricted to only 0 and 1, the fuzzy theory simplifies to classical set theory.

## 3. FUZZY TEMPORAL CONSTRAINT NETWORK

Temporal constraint network algorithm was proposed to handle time reasoning in artificial intelligence systems with explicit temporal representation (Dechter et al., 1991). However, it is not enough for domains where knowledge about time is highly

pervaded with vagueness and uncertainty. Several pieces of work have considered the representation of approximate temporal knowledge. Vila and Godo (1994) described an extended method by applying fuzzy set theory, the Fuzzy Temporal Constraint Network (FTCN).

Timeline is assumed to be a discrete and totally ordered set as the basis of temporal entities,  $\tau = \{t_0, t_1, \dots, t_b, \dots\}$ , where each  $t_i$  represents a precise point of time and  $t_0$  represents the time origin. Time points are equally spaced and the unit of time is the distance between two consecutive time points. An imprecise time point  $a$  is represented by means of a possibility distribution  $\pi_a$  over  $\tau$ . Given any precise time point  $t \in \tau$ ,  $\pi_a(t) \in [0,1]$  represents the possibility (membership) of  $a$  being precisely  $t$ . The extreme values 1 and 0 represent, respectively, the absolute and null possibilities of  $a$  being  $t$ . There is always a fuzzy set  $A$  associated with a possibility distribution  $\pi_a$ , whose membership function coincides with  $\pi_a$ . The concept of imprecise time extent was introduced to represent fuzzy quantities of time. An imprecise time extent  $m$  is represented by a possibility distribution  $\pi_m$  over the set of integer numbers  $\mathbf{I}$ , where the elements of  $\mathbf{I}$  represent units of time. Thus, given an  $n \in \mathbf{I}$ ,  $\pi_m(n) \in [0,1]$  represents the possibility of the quantity of time  $m$  being precisely  $n$  units of time. The duration of the time interval between two fuzzy time points is an imprecise time extent. Given an ordered pair of imprecise time points  $(a, b)$ , the temporal distance from  $a$  to  $b$ ,  $D(a, b)$ , is represented by means of the time extent possibility distribution,  $\pi_{d(a,b)}$ :

$$\forall n \in \mathbf{I}, \pi_{d(a,b)}(n) = \max_{n=t-s; t, s \in \tau} \min \{ \pi_a(s), \pi_b(t) \}, \quad (4)$$

corresponding to the fuzzy difference  $D(a, b) = B \ominus A$

An imprecise time interval  $i$  may be defined as a triplet  $(a, b, d)$ , in which  $a$  and  $b$  are fuzzy time points representing, respectively, the initial and final time points of the intervals, and  $d$  is a fuzzy time extent representing its duration. Assume we have a set of instantaneous events whose time points of occurrence are completely unknown. Each of them is represented by means of a temporal variable  $X_i$  that, in absence of any other information, can take as its value any precise time point  $t_i$ . When any imprecise information is added about the position of event  $X_i$ , the information can be represented as the possibility distribution of an imprecise time point  $\pi_i$  and establishes a constraint over the possible values for  $X_i$ . It is called unary constraint  $L_i$  over the variable  $X_i$ . Then assume some imprecise information is added about the relative position of two variables  $X_i$  and  $X_j$ , that is, over the time elapsed between the two events they represent. This information can be represented as the possibility distribution of an imprecise time extent and  $\pi_{ij}$  establishes a constraint over the possible values of other constraints, the assignments  $X_i = t_i$  and  $X_j = t_j$  are possible if  $\pi_{ij}(t_j - t_i) > 0$ . This is called binary constraint  $L$  over variables  $X_i$  and  $X_j$ .

Based on the above arbitrary set of information, a fuzzy temporal constraint network (FTCN) can be defined as follows: A FTCN  $L$  is a finite set of  $n + 1$  temporal variables  $X_0, X_1, \dots, X_n$ , and a finite set of binary constraints  $L_{ij}$ , over those variables, where  $X_0$  is a precise origin of times. Minimized imprecision in the position  $X_i$  can be obtained when all the constraints over  $X_i$  are combined. The formalization of concept of minimizing imprecision is also supported by a few more definitions on  $L$ . Here we just introduce how to obtain the minimal network  $M$

corresponding to a network  $L$ . Between each pair of  $L, X_i, X_j$ , there is a direct constraint  $L_{ij}$ , but there will also be induced constraints. Each induced constraint will correspond to a possible path connecting to them time points associated with  $X_i$  and  $X_j$ . In order to obtain the minimal constraint between two nodes  $X_i$  and  $X_j$ , it is necessary to obtain the intersection of the direct constraint  $L_{ij}$  and the induced constraints between  $X_i$  and  $X_j$ .

Given a path  $k$  from a network node of index  $i$  to another one of index  $j$ :

$$i_0 = i, i_1, i_2, \dots, i_k = j, \quad (5)$$

The induced constraint is represented as  $C_{i_0, i_1, \dots, i_k}^k$ . It is given by the composition of the direct constraints between each pair of consecutive nodes belonging to the path.

$$C_{i_0, i_1, \dots, i_k}^k = L_{i_0 i_1} \oplus L_{i_1 i_2} \oplus \dots \oplus L_{i_{k-1} i_k} = \sum_{p=1}^k L_{i_p - i_{p-1}} \quad (6)$$

$L_{ij}^k$  is used as the intersection of the induced constraints corresponding to all the paths of length  $k$  from the node with index  $i$  to the one with index  $j$ :

$$L_{ij}^k = \bigcap_{i_0=i, i_1=0, \dots, i_k=i, i_{k-1}=0, \dots, i_1=j} C_{i_0, i_1, \dots, i_k}^k \quad (7)$$

Finally  $M_{ij}$  is used to present the intersection of all the induced constraints corresponding to paths of any length between 1 and  $n$  going from  $X_i$  to  $X_j$ :  $M_{ij} = \bigcap_{k=1}^n L_{ij}^k$

It can be proved that a network  $L$  is consistent if and only if the constraints  $M_{ij}$  obtained by means of this expression are normalized. Also in this case, the constraints  $M_{ij}$  correspond to the minimal network  $M$  associated with  $L$ . These results have practical interest from the computational viewpoint. It is easy to operate with normalized distributions which are mostly common. Figure 1 shows a trapezoidal distribution represented by means of the four parameters ( $\alpha, \beta, \gamma, \delta$ ). The  $M_{ij}$  can be obtained by solving constraint satisfaction problems with path-consistency algorithms (Rit, 1986; Van Beek, 1990; Dechter et al., 1991).

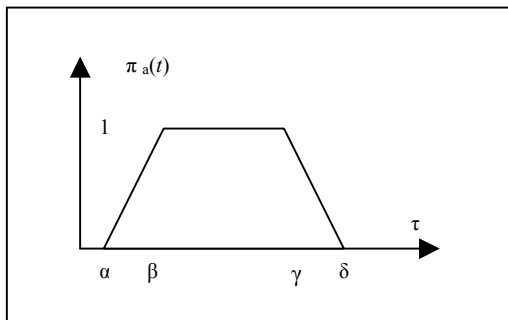


Figure 1. Trapezoidal possibility distribution for an imprecise time point

#### 4. FUZZY TIME LAYER: MODEL AND REASONING

A critical issue for this research is how to incorporate fuzziness into a general spatio-temporal data model in GIS with the fuzzy temporal constraint network method. The most usual way of handling the time factor in current GIS is to look on time as an attribute to the objects in the same way as for other attributes. This view corresponds with the usual way of presenting spatial data, and can thus be realized for both vector and raster data (Bernhardsen, 1999). Temporal information is stored through a series of time layers (snapshots) associated to particular instants in time. To accommodate uncertain time information, the current data model must be extended to represent the partial memberships (possibilities) of each time point. Thus the fuzzy time layer concept is introduced here. Corresponding to the definition in the FTCN method, a fuzzy time layer is an imprecise time interval with initial and final time points and possibility distribution function (Figure 2). The fuzzy information can be specified in various ways based on practical observation and individual type of possibility distribution.

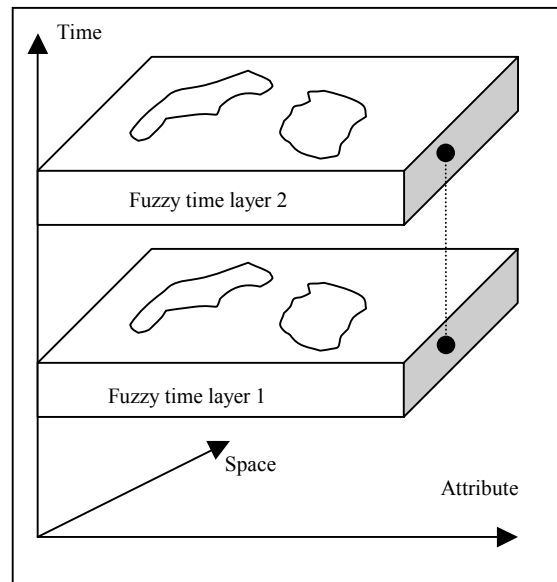


Figure 2. Uncertain times are realized in fuzzy layers (Dashed line between two layers indicates fuzzy relationship)

New components and related definitions have to be brought into data model for incorporation of the temporal fuzziness. Also the model structure has to be rebuilt to fit the change. Obviously, the number of layers increases for characterizing a temporal uncertainty. More complicated structure will be seen due to the requirements for storing and retrieving fuzzy membership functions, and if necessary, reasoning results with the FTCN method as to be discussed later. The specific concept of fuzzy temporal layer should not be unique. It may vary depending on specific uncertainty pattern on individual time layer and query requirements.

The temporal reasoning regarding fuzzy relationships among time layers has to be tackled following reconstruction of data model. This involves the incorporation of FTCN algorithms into basic data interpretation operations in GIS modeling packages. Let us still use the wildlife migration modeling example mentioned in Section 1. As we can see now, it is an ideal trapezoidal distribution problem. As shown in Figure 1,  $\alpha =$

June 1st,  $\beta$  = July 15th,  $\gamma$  = August 15th, and  $\delta$  = October 1st. A logical translation of the fuzzy temporal problem starts with construction of separate possibility distributions (Figure 3). A simplified inference procedure in mathematical form can be as follows:

The fuzzy temporal possibility distributions  $\pi^*$ , objects of fuzzy subsets:

$$\begin{aligned}\pi_1^* &= \tau_2 \cap \tau_4 \cap \tau_5 \\ \pi_2^* &= \tau_2 \cap \tau_3 \\ \pi_3^* &= \tau_1 \cap \tau_4 \\ \pi_4^* &= \tau_1 \cap \tau_3 \cap \tau_5\end{aligned}\quad (8)$$

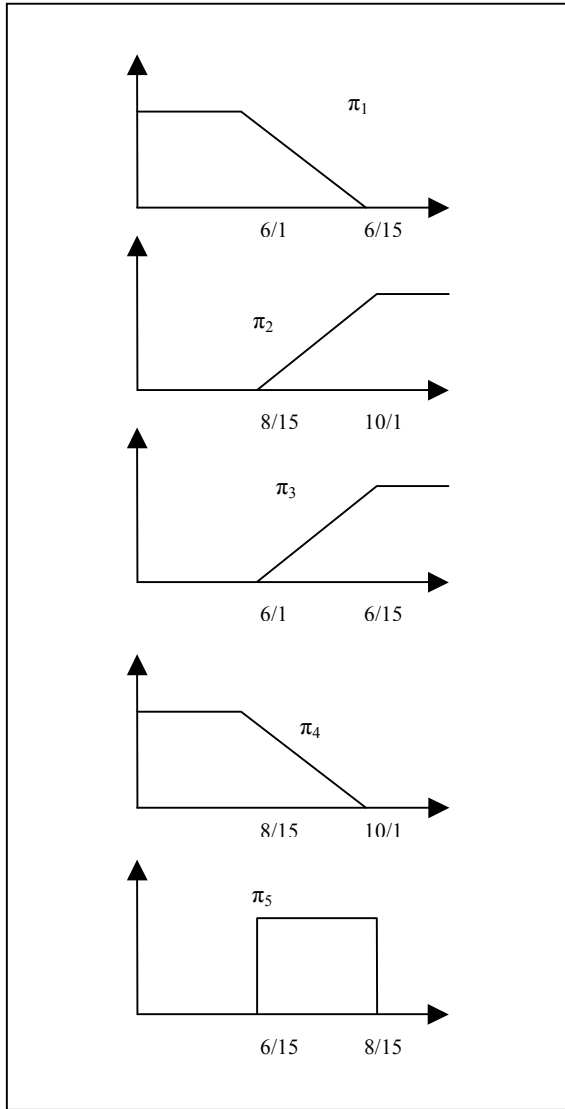


Figure 3. Separated possibility distributions

Then we can have the fuzzy complementary of the union of these four fuzzy subsets:

$$\begin{aligned}\tilde{U} &= \pi_1^* \cup \pi_2^* \cup \pi_3^* \cup \pi_4^* \\ &= (\tau_2 \cup \tau_4 \cup \tau_5) \cap (\tau_2 \cup \tau_3) \cap (\tau_1 \cup \tau_4) \cap (\tau_1 \cup \tau_3 \cup \tau_5) \\ &= (\tau_2 \cup \tau_4 \cup \tau_5) \cap \tau_3 \cap \tau_1 \cap (\tau_1 \cup \tau_3 \cup \tau_5) \\ &= \tau_1 \cap \tau_3 \cap (\tau_2 \cup \tau_4 \cup \tau_5) \\ &= (\tau_1 \cap \tau_2 \cap \tau_3) \cup (\tau_1 \cap \tau_3 \cap \tau_4) \cup (\tau_1 \cap \tau_3 \cap \tau_5)\end{aligned}\quad (9)$$

$$= \tau_1 \cap \tau_3 \cap \tau_5 = \tau_1 \cap \tau_5$$

Now we can compute temporal necessity functions  $N^*(1)$  and  $N^*(2)$  induced by  $\pi^*$  for the following two expressions in the example: (1) Species  $S_1$  has certainly left the area, and (2) Species  $S_2$  has certainly left the area.

$$\begin{aligned}N^*(1) &= (\tau_1 \cap \tau_4) \cup (\tau_1 \cap \tau_3 \cap \tau_5) \\ &= (\tau_1 \cup \tau_4) \cap (\tau_1 \cup \tau_3 \cup \tau_5) = (\tau_1 \cup \tau_4) \cap T = \tau_1\end{aligned}$$

$$N^*(2) = (\tau_2 \cup \tau_3) \cap (\tau_1 \cup \tau_3 \cup \tau_5) = \tau_3 \quad (10)$$

Generally, this approach makes possible to compute the fuzzy set of instants when we are more or less certain that a proposition is true, and on the other hand, to compute the fuzzy set of instants when we are more or less certain that proposition is false. The above translation procedure and algorithm can be built into GIS as one of spatio-temporal operation functions. This would be possible if we have a well defined fuzzy data model with much knowledge on uncertainties for specifying possibility distributions and fuzzy relationships. It is necessary for large-scale dynamic modeling analysis which may have a lot of uncertainty in time dimension and interactions among events. Expected queries to related information in a GIS database should also be considered before constructing a built-in system. A set of computer programs are being developed to implement the FTCN approach to model the uncertain temporal reasoning for a case study we are working on. The major steps and tasks in the program include:

- Generation of fuzzy time layers;
- Quantification and construction of fuzzy relationships between timed events;
- Verification of generalized fuzzy parameters;
- Implementation of the FTCN process to parse the fuzzy data, and storage of results in a GIS database.

## 5. CONCLUSIONS

Fuzzy set theory has been proved to be effective in supporting GIS-based decision-making analysis. The fuzzy temporal constraint network method is examined in this paper with regards to its application to handling uncertain temporal information in spatio-temporal modeling. It shows the FTCN concept can be adopted as a useful tool for representation and reasoning of temporal uncertainty in real-world applications. The paper discusses how the general temporal data model is extended to accommodate uncertainty with temporal data and relationships among events. A theoretical FTCN process of fuzzy translation for the imprecise information is introduced with an example. The research is going on with an experimental study of the application of the method to environmental systems modeling. The focus will be on streamlining the FTCN reasoning procedure for various circumstances of temporal uncertainty. Then effort will be made to develop a versatile computer program built into a GIS database.

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