

# Multiple randomizations

R. A. Bailey

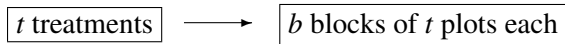


r.a.bailey@qmul.ac.uk

joint work with C. J. Brien, University of South Australia

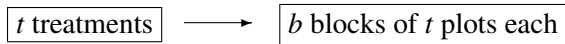
2006 International Conference on Design of Experiments and its  
Applications,  
9–13 July 2006, Tianjin, P. R. China

# Designing a simple experiment: Fisher (1935), Cox (1958), Nelder (1965)



systematic design AND randomization

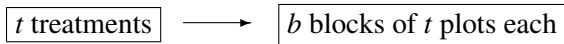
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What happens when we need to do 2 or more randomizations?

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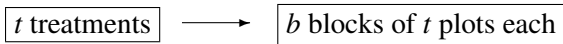


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For example,

- ▶ two-phase experiments,

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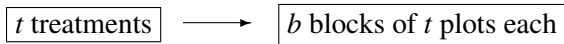


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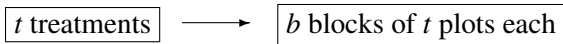


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# Plan of talk

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5. Three or more randomizations

# Concepts and terminology

A **tier** is a set of factors with the same status in randomization.

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randomized

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 $t$  Plots in B

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“ $P$  is **nested** in  $B$ ” means that  $B \wedge P$  is a meaningful factor but  $P$  is not.

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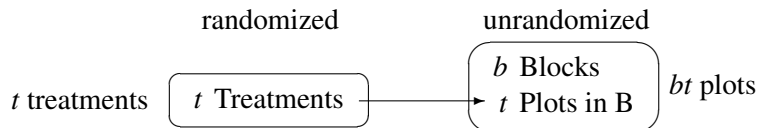
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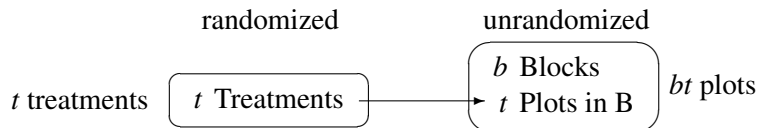


# First phase of a sensory experiment (Brien, 1983)



# First phase of a sensory experiment

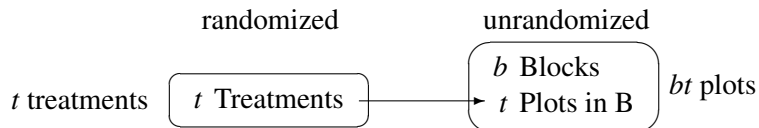
(Brien, 1983)



Systematic design: each treatment once per block

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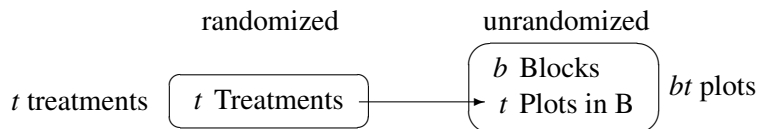


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Randomization: randomize blocks

randomize plots in each block independently

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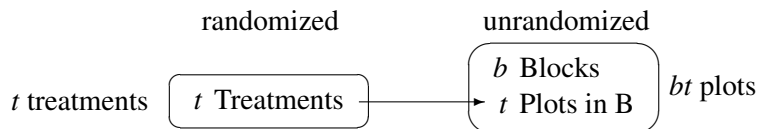
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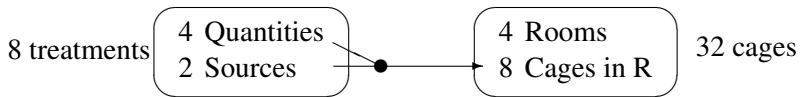
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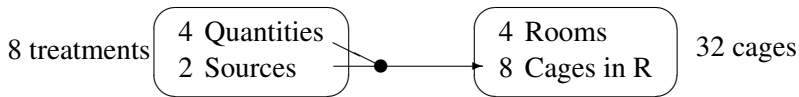
The arrow from the randomized tier to the unrandomized tier indicates both

- ▶ a systematic design (with extra explanation if necessary)
- ▶ the randomization: permute the (names of the) objects in the unrandomized set by a permutation chosen at random from among all those that preserve the relevant structure.

# A poultry-feeding experiment

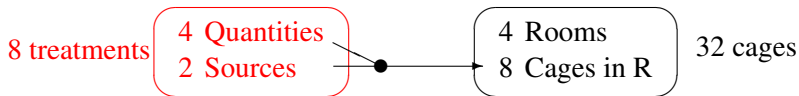


## A poultry-feeding experiment



How do we read this diagram?

# A poultry-feeding experiment

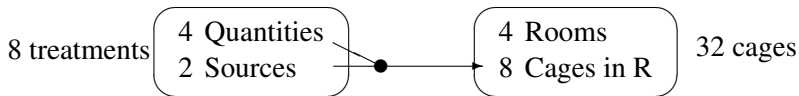


How do we read this diagram?

- ▶ There are 8 treatments:  
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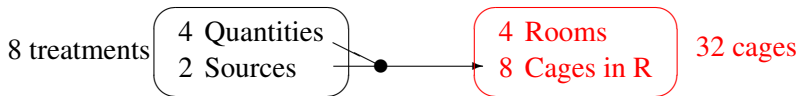
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- ▶ ● denotes the generalized factor  $Q \wedge S$  with 8 levels.

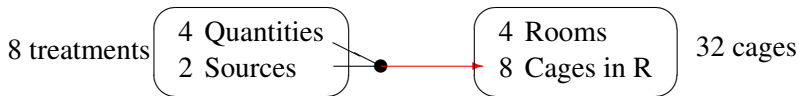
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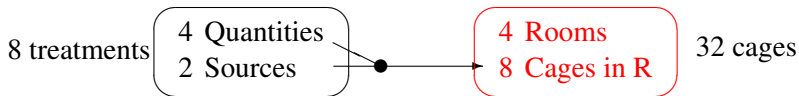
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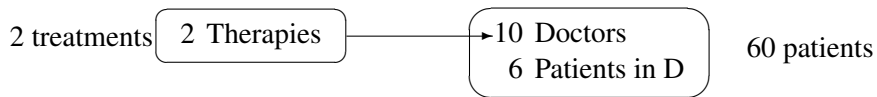
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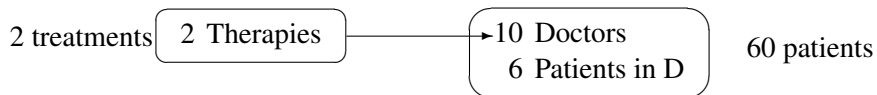
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- ▶ (Rooms are randomized);  
and cages are randomized within rooms.

# A clinical trial (R. F. White, 1975)

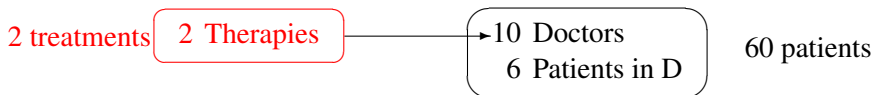


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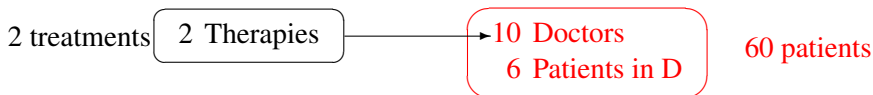
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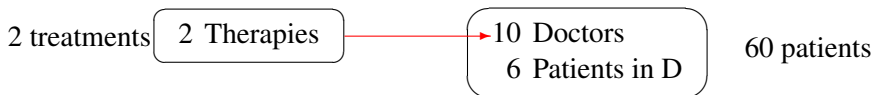


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- ▶ There are 2 therapies.
- ▶ There are 10 doctors; each doctor has 6 patients.



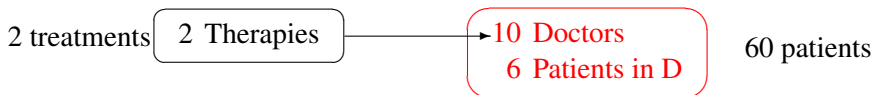
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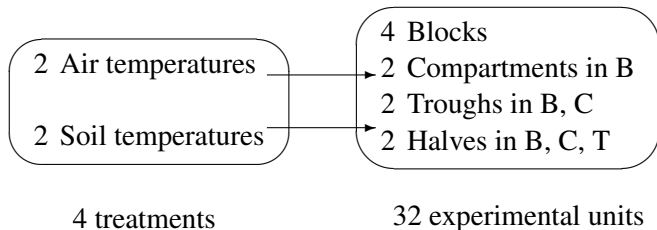
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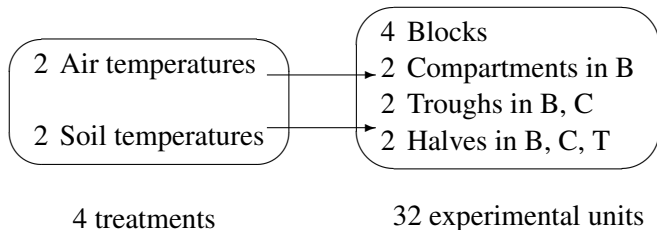
- ▶ There are 2 therapies.
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- ▶ Doctors are randomized;  
(and patients are randomized within doctors).

# A greenhouse split-split plot experiment (R. Mead, 1988)



Two arrows but a single randomization

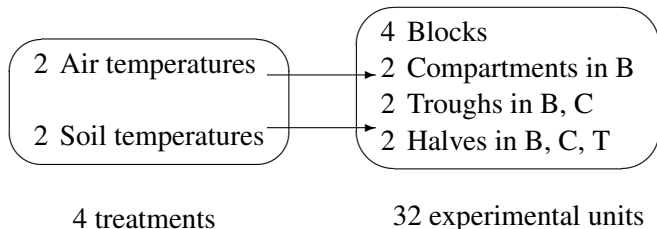
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- ▶ There are 4 blocks; each block contains 2 compartments; each compartment contains 2 troughs, each split into 2 halves.

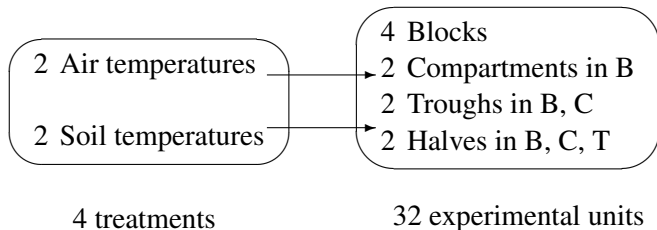
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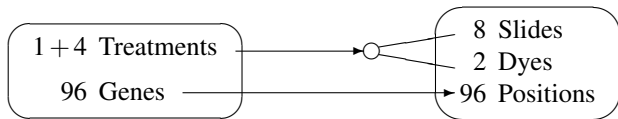
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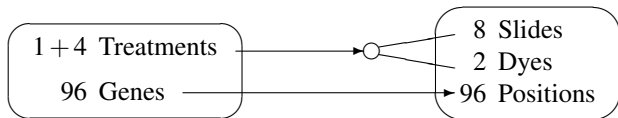
## Two arrows but a single randomization

- ▶ There are 4 blocks; each block contains 2 compartments; each compartment contains 2 troughs, each split into 2 halves.
- ▶ Each air temperature is allocated to one compartment in each block, and each soil temperature to one half of each trough.
- ▶ Blocks are randomized; compartments are randomized within blocks; troughs are randomized within compartments; and halves are randomized within troughs.

# A micorarray experiment



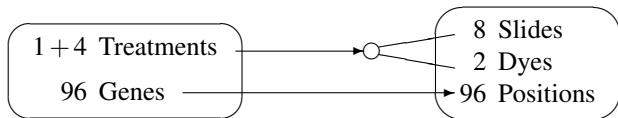
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- ▶ There is 1 'control' treatment (labelled 0) and 4 other treatments.



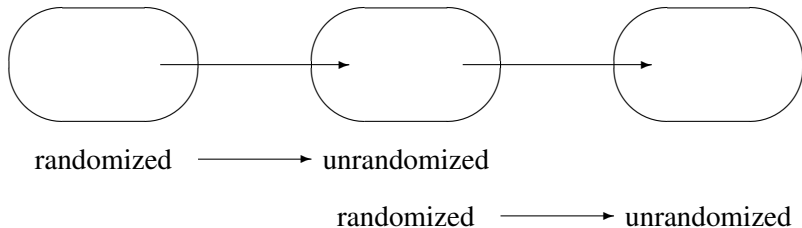
# A micorarray experiment



- ▶ There is 1 'control' treatment (labelled 0) and 4 other treatments.
- ▶ ○ shows that we need to know a specific (non-orthogonal) design for the allocation of the treatments to the dye-slide combinations, such as

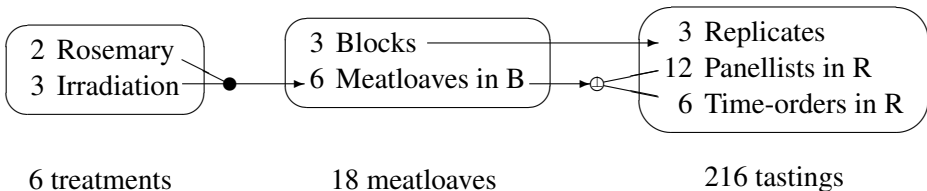
	slides							
	1	2	3	4	5	6	7	8
red	0	1	0	2	0	3	0	4
green	1	0	2	0	3	0	4	0

## Composed randomizations: Order does not matter



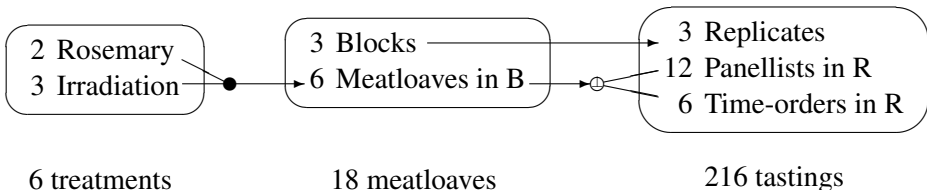
# A two-phase sensory experiment

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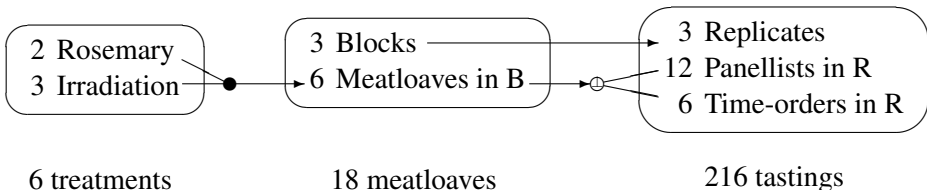
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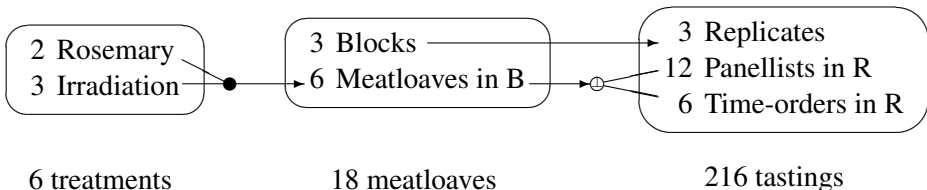


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The second phase uses an orthogonal design, indicated by  $\oplus$ : two  $6 \times 6$  Latin squares in each replicate

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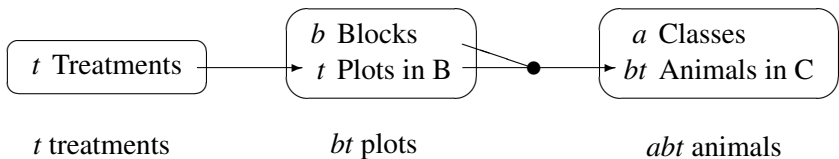


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No knowledge of the outcome of the first randomization is needed in order to perform the second.

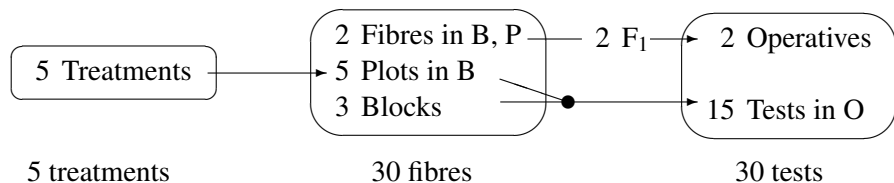
# A continuous grazing experiment (Brien and Demétrio, 1998)



A single-phase experiment with two randomizations

# Cotton fibres

(D. R. Cox, 1958)

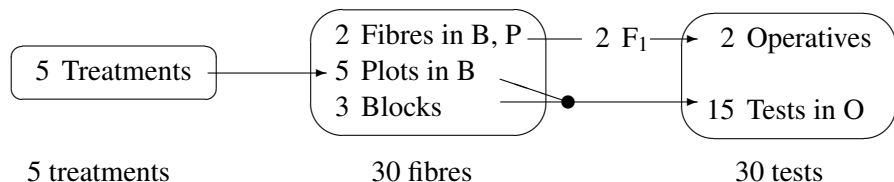


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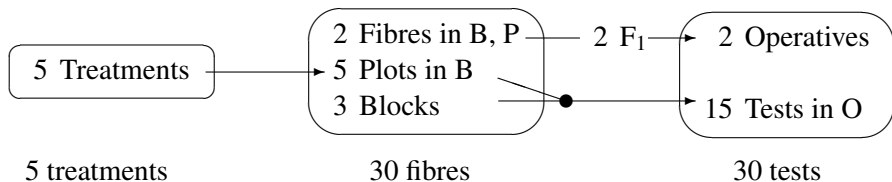


The first phase uses a complete-block design.

In the second phase, 2 fibres of cotton are sampled from each plot, and each operative tests one fibre per plot.

# Cotton fibres

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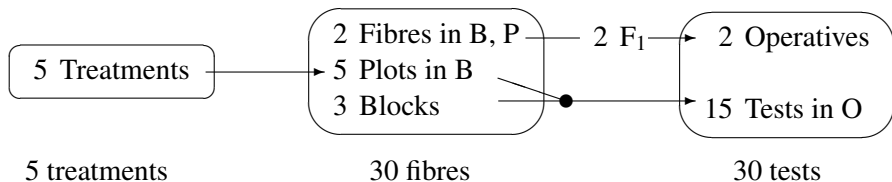
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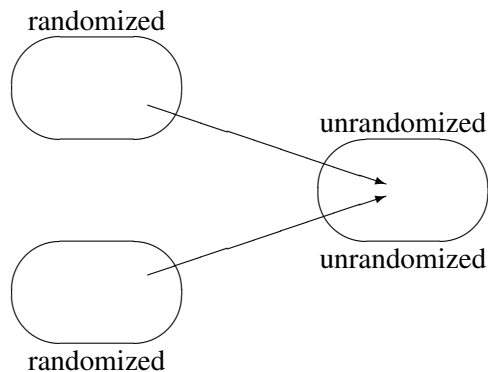
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$F_1$  is a **pseudofactor**—no inherent meaning  
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Randomization is not **consonant**: Fibres are nested in Blocks  $\wedge$  Plots  
Tests are nested in Operatives

## Coincident randomizations: Order does not matter

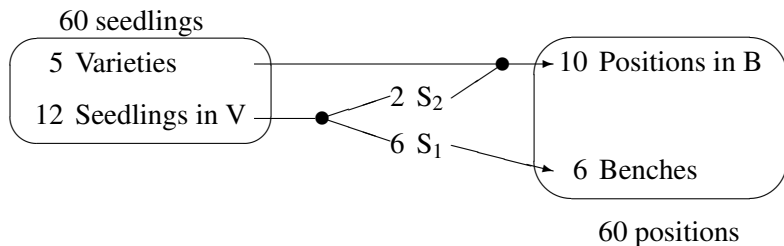


Levels of some factors from the two randomized tiers are associated by randomization.

Some effect from one randomized tier is confounded with some effect from the other randomized tier.

## A plant experiment

12 seedlings of each of 5 varieties are put into individual pots; these 60 pots are randomly assigned to 6 benches in such a way that there are 2 seedlings of each variety on each bench.

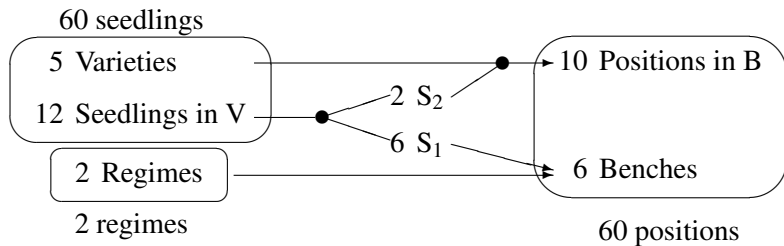


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2 spray regimes are randomly assigned to the benches so that each is applied to the pots on 3 benches.

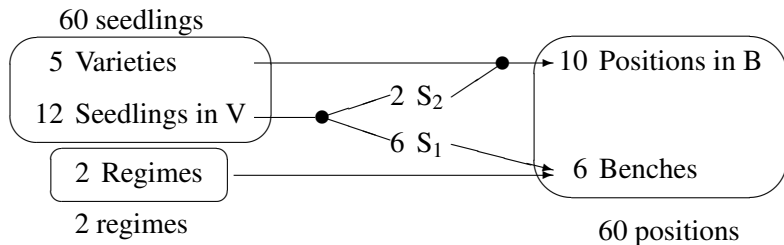


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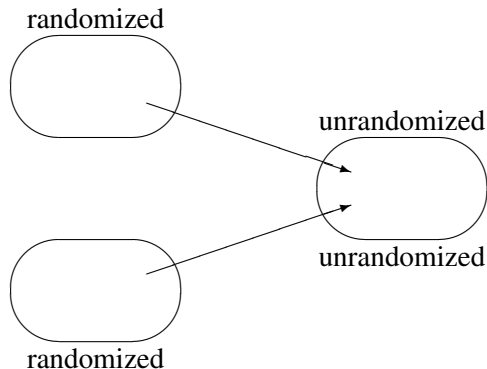
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$$S_1 \wedge S_2 = \text{Seedlings}$$

1 df for Seedlings in Varieties is confounded with Regimes.

## Independent randomizations: Order does not matter



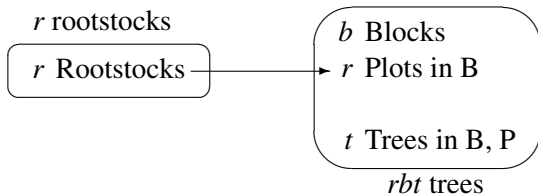
All combinations of levels of the factors from the two randomized tiers occur.

There is no confounding of effects from the two randomized tiers.



## Superimposed experiment using split plots

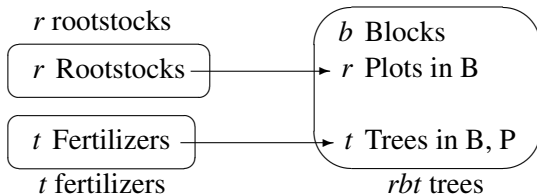
A randomized complete block experiment with  $b$  blocks is set up to investigate the yield differences between  $r$  rootstocks for orange trees, each plot containing  $t$  trees.



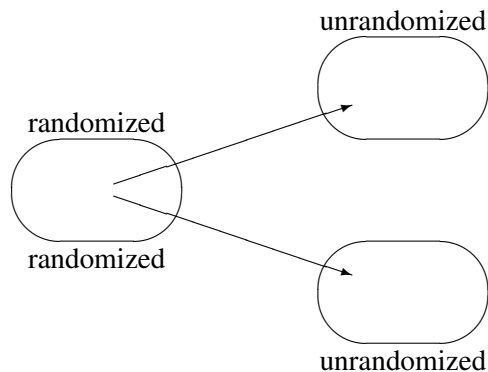
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After several years of running this initial experiment, it is decided to incorporate  $t$  fertilizer treatments by randomizing them to the  $t$  trees in each plot.



## Double randomizations: Order does not matter

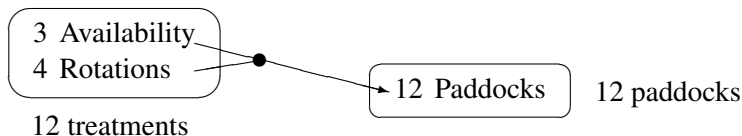


One unrandomized set has the same size as the doubly randomized set; the other contains the observational units.

Degenerate case of randomized-inclusive randomization.

# An improperly replicated rotational grazing experiment

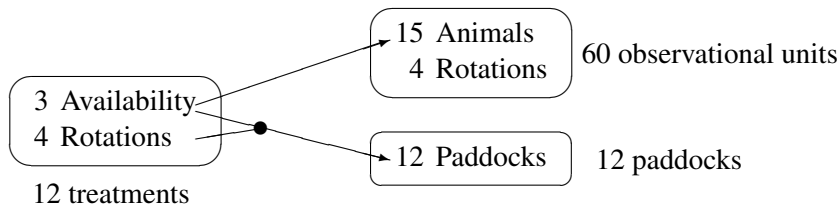
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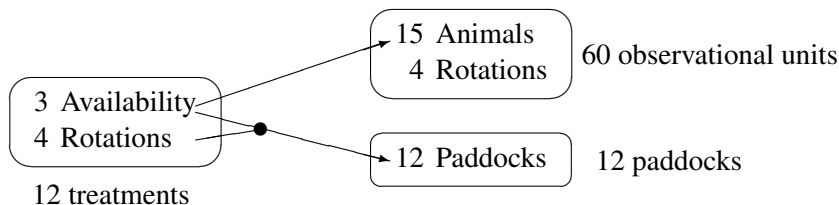
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Combinations of 3 levels of availability and 4 rotations are applied completely at random to 12 paddocks.

Also, the levels of availability are assigned completely at random to 15 animals so that each level of availability is assigned to 5 animals. The 5 animals are then grazed together in sequence on the 4 paddocks assigned to that level of availability; the sequence of 4 paddocks is determined by the rotations assigned to them.



## Two randomizations where order matters

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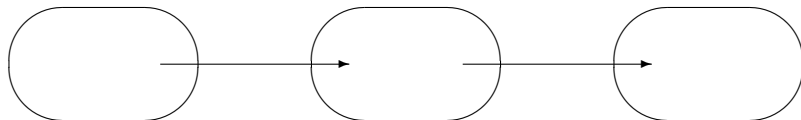
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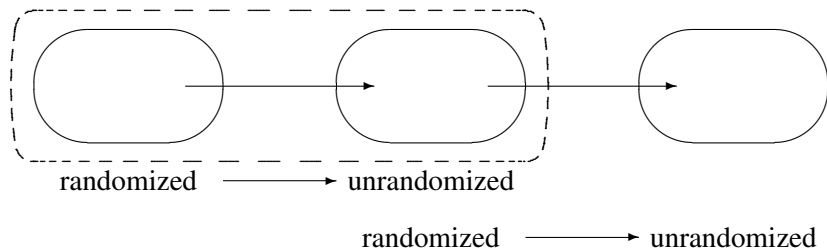
# Randomized-inclusive randomizations: Order does matter



randomized  $\longrightarrow$  unrandomized

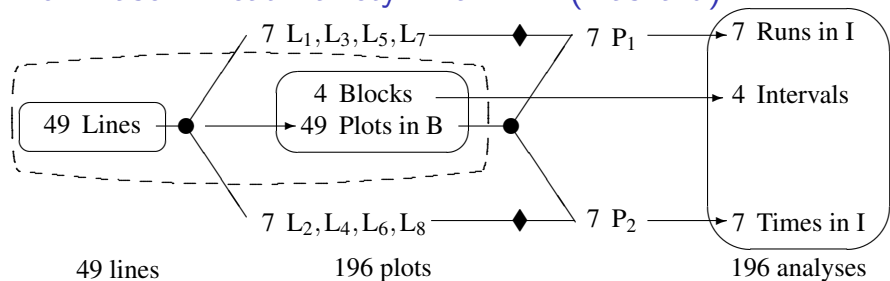
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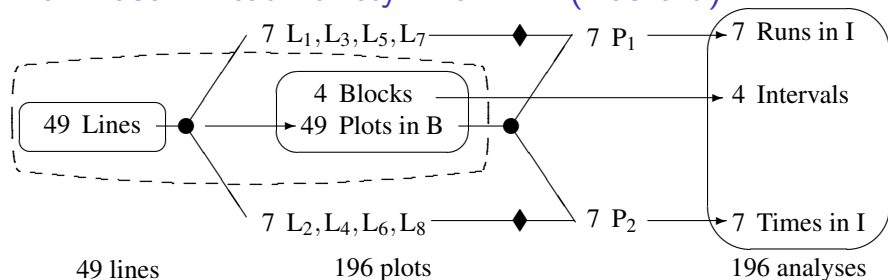


The dashed box shows the pseudotier.

# A Two-Phase Wheat Variety Trial (Haskard)



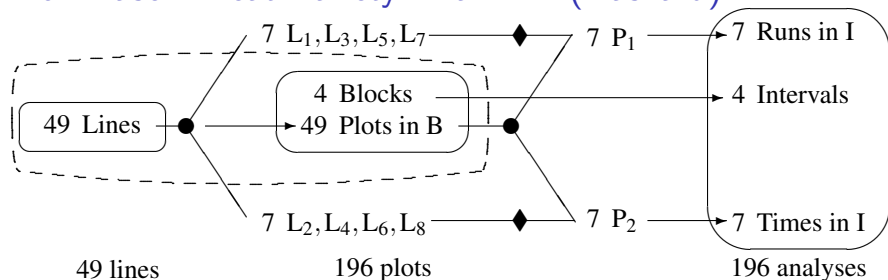
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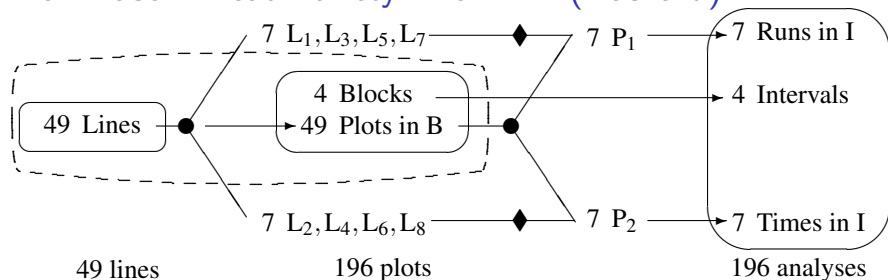


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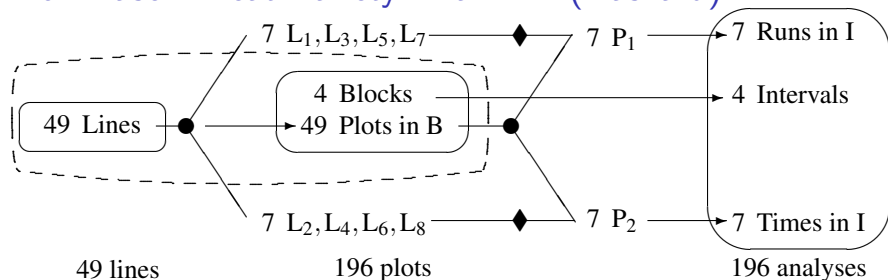
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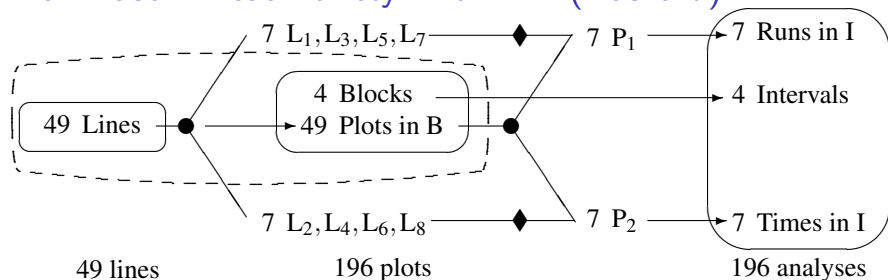
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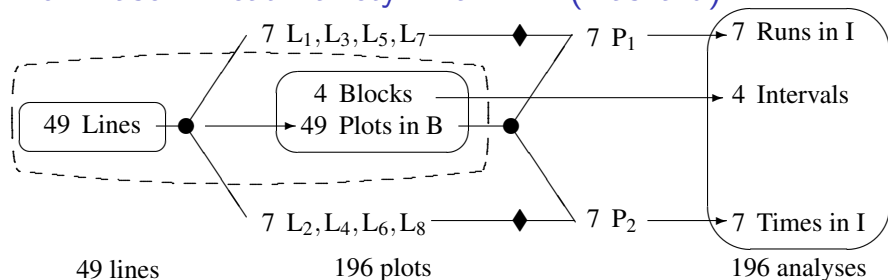
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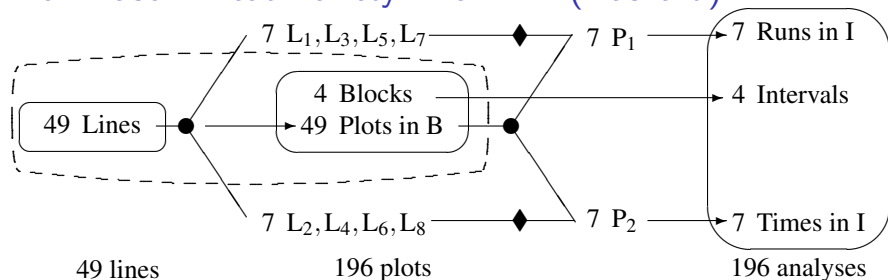
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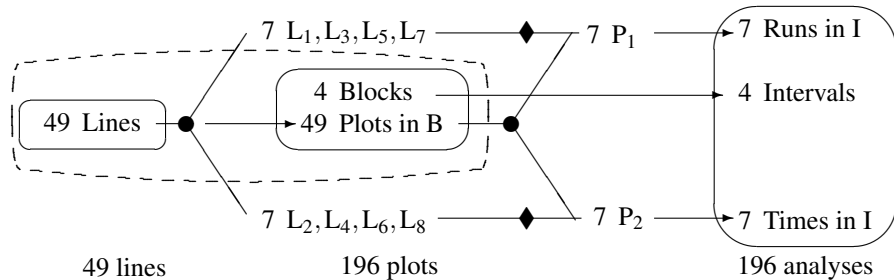
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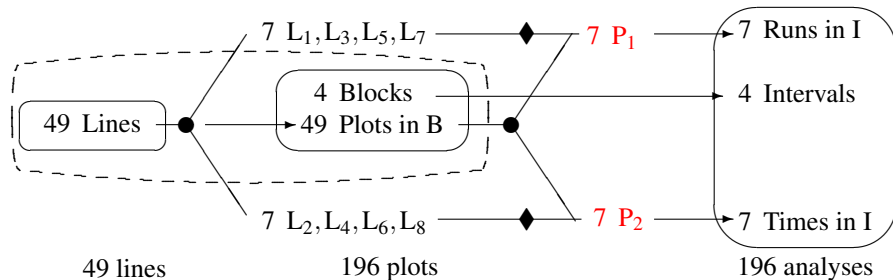


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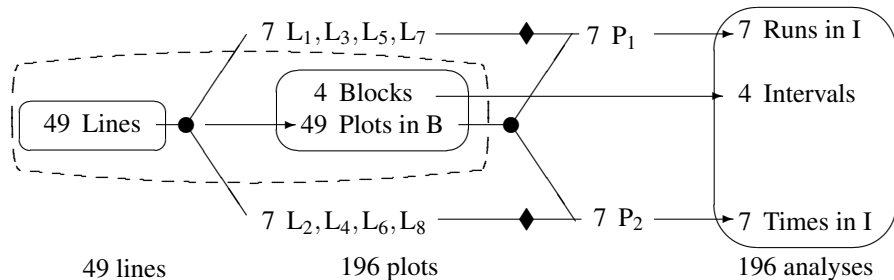
## A Two-Phase Wheat Variety Trial (Haskard)



- ▶ In each block, represent 49 Plots by 2 pseudofactors P<sub>1</sub> and P<sub>2</sub> with 7 levels; confound P<sub>1</sub> with Runs and P<sub>2</sub> with Times.

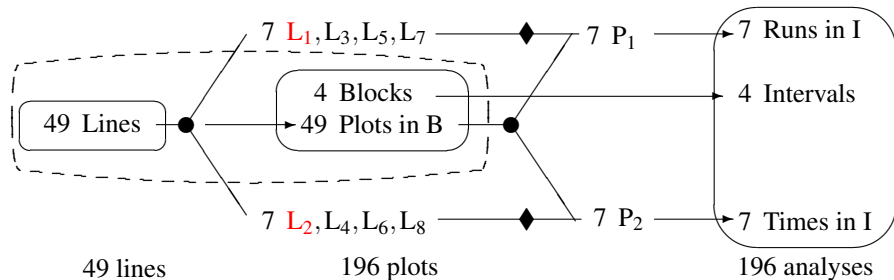


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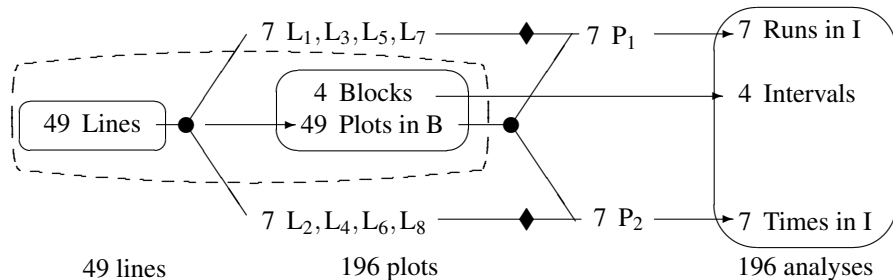
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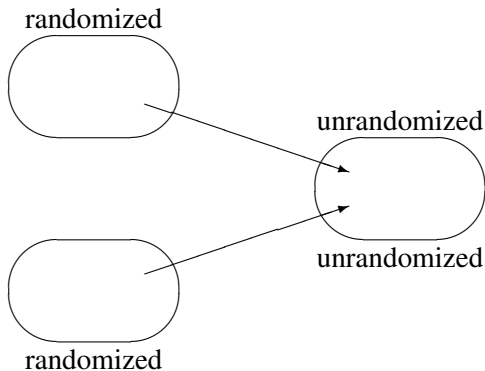
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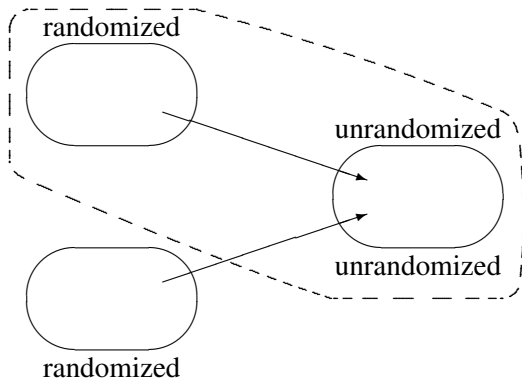


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# Unrandomized-inclusive randomizations: Order does matter

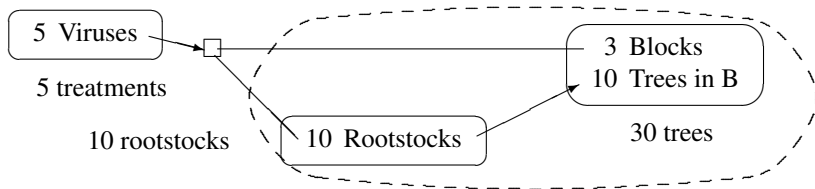


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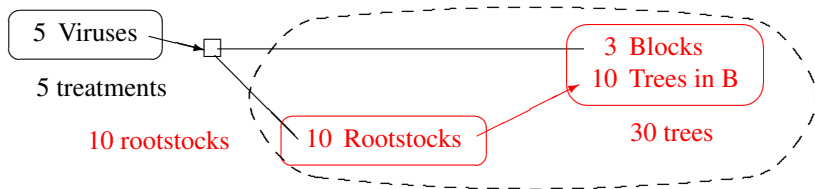


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# Superimposed Experiment in a Row-Column Design (Freeman, 1959)

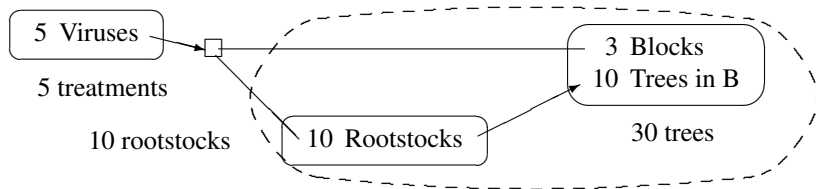


## Superimposed Experiment in a Row-Column Design (Freeman, 1959)



- ▶ Originally, 10 rootstocks were tested in 3 complete blocks, for 20 years.

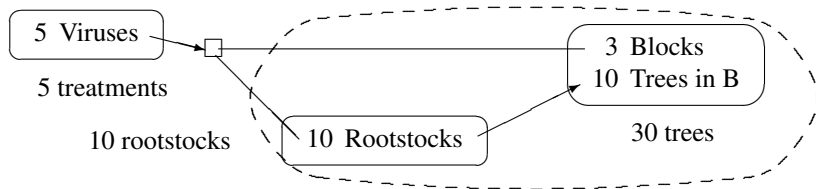
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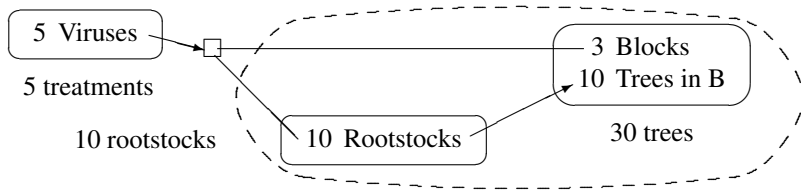


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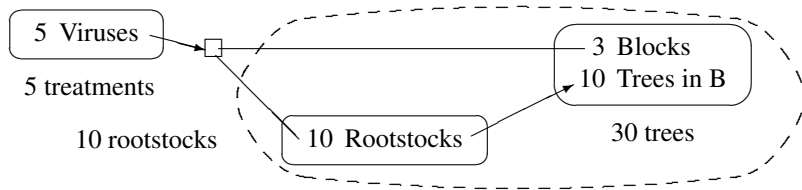
- ▶ Originally, 10 rootstocks were tested in 3 complete blocks, for 20 years.
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- ▶ In the superimposed experiment, both the systematic design and the method of randomization are constrained by the outcome of the first randomization.

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## Superimposed Experiment in a Row-Column Design (Freeman, 1959)



- ▶ □ shows that we need to know a specific (non-orthogonal) design for the allocation of the virus treatments to the block-rootstock combinations (from different tiers), such as

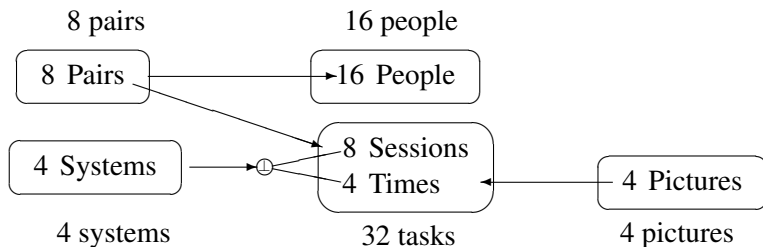
		Rootstocks									
		1	2	3	4	5	6	7	8	9	10
Blocks	I	A	B	A	C	D	C	B	E	E	D
	II	D	E	B	D	E	A	C	C	A	B
	III	E	A	C	E	B	D	D	B	C	A

- ▶ Randomize this design by randomizing blocks and randomizing rootstocks independently.

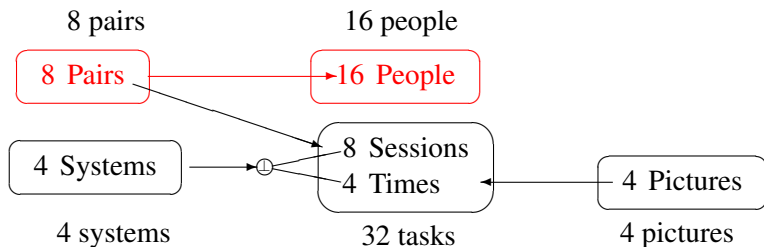
# Three or more randomizations

All these ideas extend to three or more randomizations  
(four or more tiers)  
in a straightforward way.

# Testing new telephone systems (Lewis and Russell, 1998)

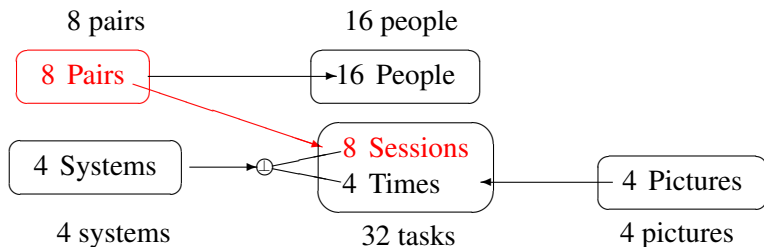


# Testing new telephone systems (Lewis and Russell, 1998)



- ▶ 16 people are divided into 8 pairs.

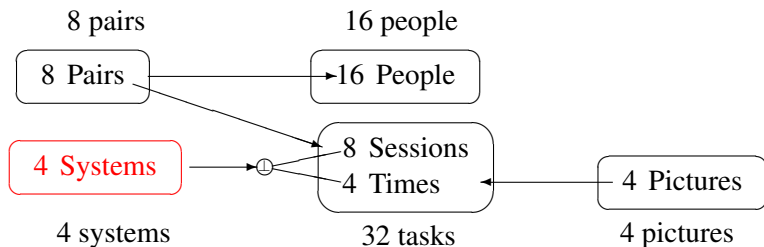
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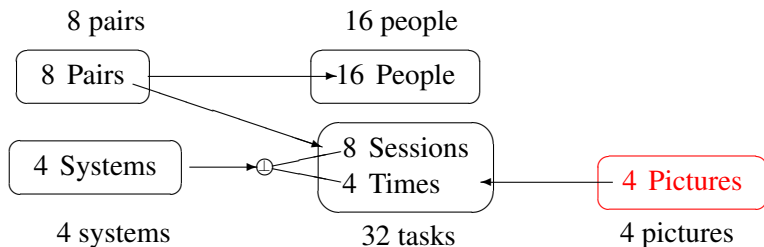


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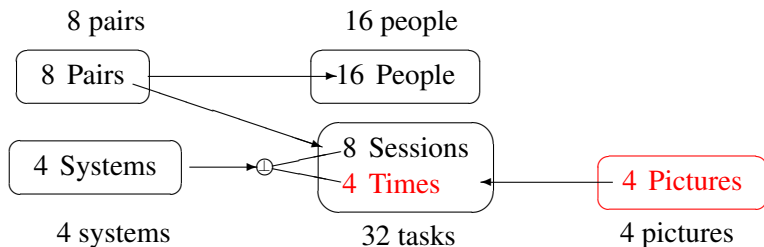
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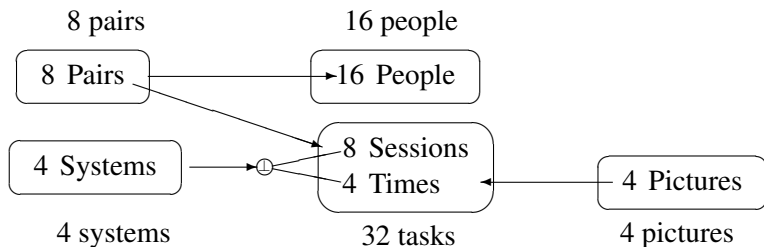
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- ▶  $\oplus$  indicates two  $4 \times 4$  Latin squares.

Read all about it!

*Multiple randomizations*

*(with discussion)*

*C. J. Brien and R. A. Bailey*

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*pages 571–609.*