# THE IMPACT OF CONFORMAL MAP PROJECTIONS ON DIRECT GEOREFERENCING

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**ABSTRACT:**

If an aerial-triangulation (AT) is performed in the National Projection System (NPS), then the *conformal map projection* (which is the base for the NPS) effects the results of the AT by raising three problems: P1) the effect of the Earth curvature, P2) the different scales in plane and height, and P3) the variation of the scale in plane across large areas of interest. Problem P1 may be solved by the so-called *Earth curvature correction*. Whereas P2 and P3 have negligible effects in plane and height (at least for non-mountainous areas) when performing a conventional AT using control- and tie-points (so-called *indirect georeferencing*), their effect in height is not negligible (even for plane surfaces) when performing *direct georeferencing* (using GPS/INS). Simple solutions for solving these problems are presented – by considering the varying planar scale with the point heights or the principal distance.

## 1. INTRODUCTION

One of the main tasks in Photogrammetry is object reconstruction with a set of (aerial) photographs. The first and most important step for this task is *image orientation*; i.e. the determination of the images' exterior orientation (XOR). The interior orientation (IOR) is generally given by means of the protocol of a laboratory calibration. Afterwards in a second step (*image restitution*) object points may be determined by means of spatial intersections. Both steps (orientation and restitution) are often combined in the term *georeferencing*.

The orientation part of georeferencing can be performed in an indirect way (aerial triangulation (AT) using ground control points (GCPs) and tie points (TPs); e.g. [Kraus 1996, 1997]) or in a more direct way (using satellite positioning systems (e.g. GPS) and Inertial Navigation Systems (INS); e.g. [Cramer 2000], [Colomina 1999], [Skaloud 1999]). Since the photogrammetric relations are based on Cartesian coordinate systems, also the georeferencing should be performed in such a system, e.g. an appropriate chosen tangential system. However, in the end the reconstructed object points commonly need to be related to the *National Projection System* (NPS), which is commonly based on a conformal projection of the *National Reference System* (NRS). Therefore, the whole georeferencing process is often carried out already in the NPS.

Since the NPS is based on a projection of the curved earth surface, during which distortions occur, the question arises, which errors are thereby introduced in the determined object points during the respective direct and indirect georeferencing.

Concerning indirect georeferencing this problem was already discussed in the literature; [Rinner 1959], [Wang 1980]. The discussion for direct georeferencing will be presented in this article. Since only the point errors due to the NPS' distortions are of interest, ellipsoidal heights will be used in the NPS.[1]

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[1] Actually in practise, instead of ellipsoidal heights *orthometric heights* are used in the NPS. Since in this case, the NPS' height reference surface (i.e. geoid) differs from the NPS' location

Further all observations (image and GPS/INS) are considered free of errors. Especially, it is assumed that the principal distance during the image flight does not differ from the value determined during the laboratory calibration.

## 2. THE CONFORMAL MAPPING OF THE NATIONAL SPATIAL REFERENCE SYSTEM

As it was already mentioned, the object points determined during georeferencing need to be related to a given coordinate system, which in general is a well defined conformal projection of the National Reference System (NRS) - the National Projection System (NPS). One part of the NRS is an appropriate ellipsoid which is used as an approximation for the Earth's shape (described by the geoid). By means of this reference ellipsoid 3D points can be described using their geodetic latitude ($\varphi$) and longitude ($\lambda$) and their ellipsoidal height (H). The mapping of an ellipsoid can never be entirely true in length, only true in angles (conformal) or true in area. Due to the importance of (terrestrial) angular measurements most national reference systems are based on conformal mappings (e.g. Gauss-Krueger (or Transverse Mercator)).

The Gauss-Krueger projection has the following properties:
- The projection is conformal. Due to a break in the series development, however, this property does not hold exactly.
- Only small stripes (1.5°) to the west and east of the central meridian (with ½° overlap) are used for a single projection.
- The central meridian is mapped in true length and serves as the ordinate axis of the strip system ($Y_{Map}$).

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reference surface (i.e. ellipsoid) by the so-called *geoid undulations*, additional errors during georeferencing may be induced. These errors, however, are of physical nature and occur independently of the (geometric) errors induced by the map projection. Therefore their treatment lies beyond the scope of this work and only ellipsoidal heights are considered throughout this paper.
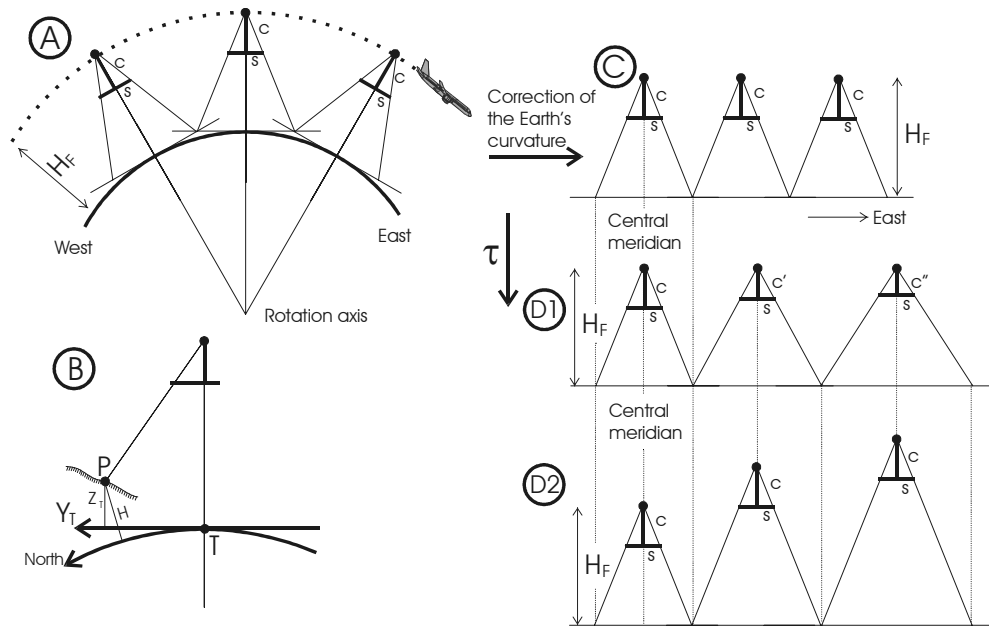
Figure 1: A flight strip in the NPS

Only the nadir point $P_N$ of a given 3D point P with ellipsoidal height H is used during the projection of the point P. This results in the planar coordinates ($X_{Map}$, $Y_{Map}$) which are affected by the length distortion $\tau$ of the map projection. $\tau$ (and hence the planar scale) increase quadratically with the distance from the central meridian. To complete the 3D coordinates in the NPS the ellipsoidal height H is used as the height coordinate of the mapped point ($Z_{Map} = H$) – as a consequence, the skew normals of the ellipsoid will be mapped to parallel lines. Thus planar scale and height scale are equal only along the central meridian – whereas with increasing distance from the central meridian the difference between these two scales gets larger. Due to these reasons the NPS does not represent a Cartesian coordinate system.

For the Gauss-Krueger projection the length distortion $\tau$ in the lateral distance $X_{Map}$ from the central meridian is computed in the following way – with R being the mean radius of curvature, depending on the reference ellipsoid (a, b) and the geodetic latitude $\varphi$ [Bretterbauer 1991]:

$$\tau = 1 + \frac{X_{Map}^2}{2R^2} + \frac{X_{Map}^4}{24R^4} \qquad (1)$$

$$with \ R = \frac{c}{1 + e'^2 \cdot \cos^2 \varphi} \ , \ c = \frac{a^2}{b} \ , \ e'^2 = \frac{a^2 - b^2}{b^2}$$

The effect of $\tau$ on 1000 m at one strip's border is ($\lambda = 1.5°$, $\varphi = 48° \rightarrow X_{Map} \sim 112$ km) $\sim +15$ cm and at the end of the overlap ($\lambda = 2.0°$, $\varphi = 48° \rightarrow X_{Map} \sim 150$ km) $\sim +28$ cm.

Another widely used projection is the Universal Transverse Mercator (UTM) which is based on Gauss-Krueger but uses a strip width of 3°. To reduce the effect of the length distortion, the planar coordinates for this map are altered by the factor 0.9996. In doing this the true length in the central meridian is lost, but is achieved in parallels to the $Y_{Map}$ axis in $\sim 180$ km distance to the west and east of the central meridian.

The effect of $\tau$ on 1000 m is in the central meridian ($\lambda = 0.0°$, $\varphi = 48° \rightarrow X_{Map} \sim 0$ km) $\sim -40$ cm, at one strip's border ($\lambda = 3.0°$, $\varphi = 48° \rightarrow X_{Map} \sim 220$ km) $\sim +20$ cm and at the end of the overlap ($\lambda = 3.5°$, $\varphi = 48° \rightarrow X_{Map} \sim 290$ km) $\sim +65$ cm.

In this distorted system of the NPS the coordinates of the object points are to be determined given the aerial images. However, the equations used in Photogrammetry (e.g. for the central projection) and the points determined with them refer to a Cartesian coordinate-system. How can this problem be solved?

The first (and theoretically best) method is to perform the AT in an Cartesian auxiliary system (e.g. a tangential system set up in the center of a given area of interest) and to transform the results to the NPS afterwards. This method, however, also has some (practical) drawbacks:

a) Problems during the so-called refraction correction may arise (esp. for large areas of interest), since this correction usually assumes the plumb line direction to coincide with the computing systems' Z-axis, which is not rigorously valid in the tangential system.
b) Due to the same reason certain leveling constraints (e.g. points along the shoreline of a lake, or the leveling of a theodolite if polar measurements are introduced into the adjustment) need some sophisticated realization in a tangential system.
c) The most severe drawback, perhaps, is that the results of the stereo restitution (roofs, streets, natural boundaries, contour lines, etc.) are finally required in the NPS. But today's analytical and digital plotters (esp. the CAD module) do not fully support (at least to the knowledge of the author) the digitisation in the tangential system and the simultaneous storage of the results in the NPS.

Therefore, in the second method, the AT is already performed in the NPS and the discrepancies between the distorted NPS and the Cartesian 'nature' of the Photogrammetric equations are minimized using suitable corrections. In the following the problems that arise with this second method are discussed in detail. Figure 1 depicts the situation schematically.

Sketch (A) in Figure 1 shows the section through three projection centers (PRCs) of a strip (flown from West to East) – for simplicity reasons the section ellipse is drawn as a circle. The plane flies in constant ellipsoidal height $H_F$. The principal distance is c and the image format is s. If the Gauss-Krueger projection is applied to the ellipsoidal area covered by these images, then this area is unwrapped in a conformal way (so to speak). The length distortions introduced this way shall be neglected at first. This projection of the ellipsoidal surface delivers the planar coordinates for the NPS. The $Z_{Map}$ coordinate is made by the ellipsoidal heights of the surface points, which are related to the curved reference ellipsoid of the NRS. This ellipsoidal curvature prevents the direct usage of the Photogrammetric Cartesian relations in the NPS.

In a first order approximation the curved surface of the ellipsoid can be replaced by a polyhedron of tangential planes, with each plane set up at the nadir point of the PRCs. Then the unwrapping of this polyhedron gives a first order approximation for the Gauss-Krueger projection. This together with the heights related to the respective tangential plane for each image create a (small) individual Cartesian system of coordinates. The correction needed for this tangential approximation therefore removes the effects of the ellipsoidal curvature.

Sketch (B) in Figure 1 depicts this correction. There a meridional section of the area around the nadir point $T_i$ of an image i together with its tangential plane and the curved surface of the ellipsoid is shown. Further a point $P_j$ on the Earth surface is depicted, having the coordinates $(X_T, Y_T, Z_T)$ with respect to the Cartesian tangential system. If the NPS coordinates of $T_i$ and of all points $P_j$, that are observed in image i, are (approximately) known, then these points can be transformed into this individual tangential system. In this system the Cartesian relations of the Photogrammetry do hold.

This transformation is commonly termed as *Earth curvature correction* and it is a standard module in today's AT packages. This correction can also be performed in the way, that the object coordinates are not altered but the image coordinates. For the Earth curvature correction the reader is directed to e.g. [Wang 1980], [Kraus 1996, 1997].

By means of this correction the flight in constant ellipsoidal height $H_F$ is flattened; i.e. now the plane flies horizontally in constant height $H_F$ above the reference plane used for the un-wrapping of the tangential polyhedron; Sketch (C) in Figure 1.

Now the previously neglected length distortion $\tau$ of the Gauss-Krueger projection together with its increase with the distance from the central meridian and the unchanged usage of the ellipsoidal heights in the NPS are taken into account. These three circumstances introduce a contradiction, which is depicted in the Sketches (D1) and (D2) in Figure 1: On the one hand, since the plane flies in constant flight height, also the PRCs in the NPS should have the same $Z_{Map}$ coordinate. This, however, induces an increase in the view angle because of $\tau$ and its increase to the East; i.e. the ratio between principal distance c and image format s must change continuously (Sketch (D1)). On the other hand, if this ratio is kept constant (since all images are taken by the same camera), then the $Z_{Map}$ coordinate of the PRCs must increase to the East (Sketch (D2)).

To sum it up, three problems occur if one wants to perform an AT in the NPS:

P1) the effect of the curvature of the Earth
P2) the difference between planar and height scale
P3) the continuous change of the planar scale throughout the considered area of interest in lateral direction

Whilst problem P1 can be solved using the afore mentioned correction of the Earth curvature, the other two problems have been neglected so far – at least to the knowledge of the author. The question now arises, which errors are induced in the determined object points during direct and indirect georeferencing, if the problems P2 and P3 are neglected.

**Note:** For terrestrial geodetic (polar) networks computed in the NPS sometimes the so-called *arc-to-chord correction* needs to be applied. This correction compensates for the angular deviation between the straight line connecting two points in the map of the NPS and the (curved) map of the straight line connecting the respective points on the earth surface. For photogrammetric networks, however, this reduction can be ignored, since the effect of this reduction referred to the image is less than 1 μm: For the Gauss-Krueger projection ($\lambda = 1.5°$, $\varphi = 48° \rightarrow X_{Map} \sim 112$ km) if the image scale is larger than 1:42.000 and for the UTM projection ($\lambda = 3.0°$, $\varphi = 48° \rightarrow X_{Map} \sim 220$ km) if the scale is larger than 1:21.000.

## 3. INDIRECT GEOREFERENCING IN CONFORMAL MAP PROJECTIONS

In this case the following quantities are given: 3D GCPs (in the NPS), the coordinates of their mappings in the aerial images, the image coordinates of TPs and the IOR according to a valid calibration protocol. The parameters to be determined are: the images' XOR and the 3D coordinates of the TPs (and of other spatial objects in the subsequent stereo restitution). Problem P1 is solved using the Earth curvature correction. What about P2 and P3?

During the *orientation step* the XOR of the images is indirectly determined using the GCPs and TPs. The free flight height adjusts to the GCP and TP situation on the ground; i.e. the local planar scale – caused by $\tau$ and realized in the GCPs – is transferred into the flight height. This situation is depicted (two images, one GCP and one unknown point N, which shall be determined during *image restitution*) in Figure 2.
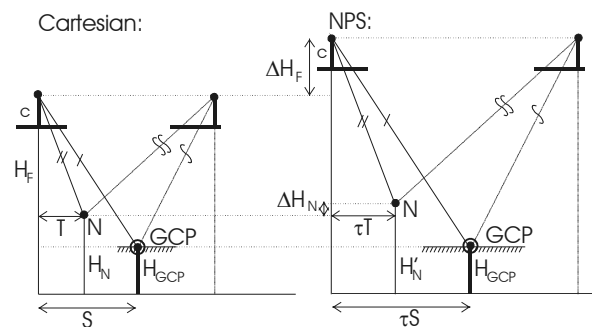


Figure 2: Indirect georeferencing

The pencil of projection rays is congruent for both systems (the Cartesian one and the distorted one, i.e. the NPS), since in each system the same value for the principal distance c is used. Therefore it holds:

$$\Delta H_F = (\tau - 1) \cdot (H_F - H_{GCP}) \qquad (2)$$

Using this relation (2) one gets the height-error of an unknown point N as follows:

$$\Delta H_N = (\tau - 1) \cdot (H_N - H_{GCP}) \qquad (3)$$

So we see, that during the *orientation step* of indirect georeferencing of aerial images with fixed IOR (according to a valid laboratory calibration) in the NPS the PRCs get a vertical shift of $\Delta H_F$ proportional to the flight height above ground (more exactly: above the level of the GCPs). Since $H_F > H_{GCP}$ and $\tau > 1$ (for Gauss-Krueger), the determined flight height will always be higher than in reality. In the restitution step the heights of unknown points in the level of the GCPs will be determined correctly, whereas points above resp. below this level $H_{GCP}$ get an error $\Delta H_N$ which is proportional to the height difference $(H_N - H_{GCP})$.

**Numerical example:** Principal distance c = 150 mm, image scale 1:10.000 $\rightarrow$ $(H_F - H_{GCP})$ = 1.5 km $\rightarrow$ $\Delta H_F$ = 40 cm (at the end of the Gauss-Krueger overlap $\lambda$ = 2.0°). With $(H_N - H_{GCP})$ = 200 m we get $\Delta H_N$ = 6 cm, which is slightly smaller than the best achievable height accuracy at this flight height of 0.06‰$(H_N - H_{GCP})$ = 9 cm [Kraus 1996].

This specified problem of indirect georeferencing in the NPS is well known in Photogrammetry for a long time; e.g. [Rinner 1959]. In the work of Wang [Wang 1980] it is addressed thoroughly. He determines the introduced error in unknown points empirically with simulation computations depending on the kind of projection (Gauss-Krueger, Lambert, stereographic – all three being conformal; true ordinate – being non conformal), the size of the block of images, the position of the block relative to the central meridian, the flight direction, the image scale, the number of planar and height control points and the number of TPs. The main outcome of his investigations is that the impact of the length distortion (i.e. P2 and P3) on the determined points in conformal map projections using Earth curvature corrected images is negligible. Although it should be mentioned that in Wang's investigations horizontal terrain was always assumed, therefore errors in the determined heights due to the height difference to the mean level of the GCPs are not documented.

**Note:** This height problem is only relevant as long as the principal distance c is not allowed to be corrected during the orientation step. If c (common for all images) is free, then problem P2 is solved in the middle of the area of interest. Problem P3, however, remains uncorrected. The equations (2) and (3) still hold but there $\tau$ needs to be replaced by $\tau/\tau_{global}$, where $\tau_{global}$ is a mean value for the area of interest (see end of section 4).

## 4. DIRECT GEOREFERENCING IN CONFORMAL MAP PROJECTIONS

In this case the following quantities are given: the elements of the images' XOR referred to the NPS, the image measurements of unknown object points and the IOR according to a valid calibration protocol. The coordinates of the unknown points are to be determined. Problem P1 is solved using the Earth curvature correction. What about P2 and P3?

In this case none of the images' XOR elements are free (they are already measured directly; i.e. the *orientation step* does not exist) and therefore the flight height can not adjust to the local

planar scale – caused by $\tau$ and realized in the known planar coordinates of the PRCs. In Figure 3 this situation is depicted (two images and one unknown point N, which shall be determined during *image restitution*).
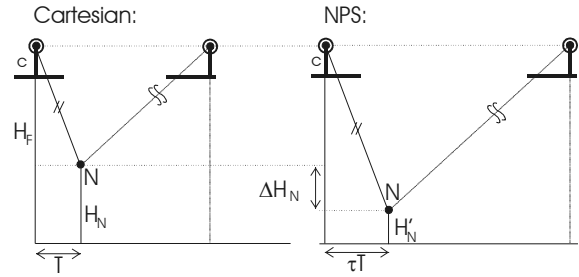


Figure 3: Direct georeferencing

The pencil of projection rays is congruent for both systems (the Cartesian one and the distorted one, i.e. the NPS), since in each the same value for the principal distance c is used. Therefore it holds:

$$\Delta H_N = (\tau - 1) \cdot (H_N - H_F) \qquad (4)$$

We see, that during the *restitution step* of direct georeferencing the heights of the unknown points will be determined with errors proportional to the imaging distance and since $H_F > H_N$ and $\tau > 1$ (for Gauss-Krueger) all determined points will always lie below their real level.

**Numerical example continued:** With the imaging distance $(H_F - H_N)$ = 1.5 km we get $\Delta H_N$ = 40 cm and this value lies clearly above the achievable height accuracy of 9 cm.

So we see, that in contrary to indirect georeferencing the height errors induced in the NPS during direct georeferencing are not negligible. If one compares equations (4) with (3) and exchanges $H_F$ by $H_{GCP}$, one sees the equivalence of these two equations; i.e. the height errors of the determined points increase for both – direct and indirect georeferencing – with the difference to the level of the height control points. During indirect georeferencing the unknown points lie approximately in the level of the (ground) control points, whereas during direct georeferencing they do not – they lie below by the amount of the flight height. This clearly shows the interpolating behavior of indirect georeferencing and the extrapolating behavior of direct georeferencing.

Now the question arises, how to remove these errors. Three possibilities can be offered:
  M1) computation in a Cartesian tangential system
  M2) correction of the heights
  M3) correction of the principal distance

The advantages and disadvantages of M1 were already discussed in section 2. The methods M2 and M3 are alternatives for solving the two problems P2 and P3 when performing direct georeferencing in the NPS and will be discussed in the following.

In M2 the principal distance remains unchanged but all heights that are used in the AT are corrected by the respective planar scale ($H_{corr} = H_{Ell} \cdot \tau$). If the area of interest is not too large, it should be sufficient to compute one representative value for $\tau$ ($\rightarrow \tau_{global}$) in the center of the area of interest and to correct all heights with $\tau_{global}$. In this case only P2 is solved and P3 is

| Projection → | Gauss-Krüger projection | | UTM projection | | |
|---|---|---|---|---|---|
| | $\lambda = 1.5°$ | $\lambda = 2.0°$ | $\lambda = 0.0°$ | $\lambda = 3.0°$ | $\lambda = 3.5°$ |
| Planar error < 10 μm·$m_b$ | c < 729 mm | c < 391 mm | c < 273 mm | c < 547 mm | c < 168 mm |
| Height error < 0.1‰·$H_F$ | c > 21 mm | c > 39 mm | c > 55 mm | c > 28 mm | c > 90 mm |

Table 5: Values for the principal distance c for which the additional planar and height errors are less than 10 μm in the image resp. 0.1‰ of the flight height, if the roll ($5^{gon}$) and pitch ($3^{gon}$) of the images are neglected and if method M3 is used.

| | $\lambda = 0.0°$ $X_0 = 0$ km | $\lambda = 1.5°$ $X_0 = 112$ km | $\lambda = 2.0°$ $X_0 = 150$ km | $\lambda = 3.0°$ $X_0 = 220$ km | $\lambda = 3.5°$ $X_0 = 290$ km |
|---|---|---|---|---|---|
| $2 \cdot \|X - X_0\|$ | 140 km | 44 km | 32 km | 22 km | 16 km |

Table 6: Maximum values for the lateral extension of the area of interest, so that the remaining height error $\Delta H_N$ (equation (6)) when neglecting P3 is below 0.06‰($H_F - H_N$) – valid for Gauss-Krüger and UTM.

neglected. If the project area is very large and P3 can no longer be neglected, then $\tau$ must be computed for each single PRC (→ $\tau_{local}$) and applied for the height correction.

This method M2, however, has the disadvantage that the artificial introduction of $\tau$ must be finally removed in all resulting heights in order to get (ellipsoidal) heights corresponding to the definition of the NPS. This is inevitable, if the heights determined during direct georeferencing need to be compared with e.g. terrestrial measured ground truth. However, it can be imagined that this kind of work is carried out by the AT package itself during an 'extended' Earth curvature correction. The user always sees ellipsoidal heights, which are corrected by $\tau_{global}$ or $\tau_{local}$ each time before an adjustment is computed. The heights after such an adjustment are immediately removed by the effect of $\tau$ and are stored in the respective memories. This 'extended' Earth curvature correction then needs to know which type of map projection needs to be applied and how the control points were reduced in advance.

In method M3 the ellipsoidal heights introduced in the adjustment remain unchanged, but the principal distance is corrected (actually falsified). Figure 4 shows how this is done.
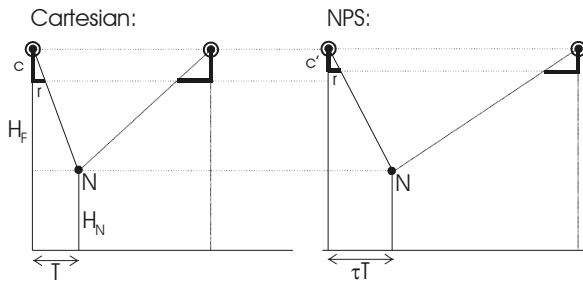

Figure 4: Change of the principal distance

Using the very simple relations in Figure 4 we get:

$$c' = \frac{1}{\tau} c \qquad (5)$$

Similar to M2 depending on the area of interest one can use for all images the same altered principal distance c'$_{global}$ (via $\tau_{global}$) or use for each image an individual value c'$_{local}$ (via $\tau_{local}$). And again it can be seen, that this work is done by an 'extended' correction of the Earth curvature.

It must be pointed out, that M2 and M3 are just approximate solutions[2] for the given problem, since the change of $\tau$ within the area covered by one image is not taken into concern.

The degree of approximation using M3 is further decreased if the images are rather oblique, since M3 holds rigorously true only for exact normal (vertical) images. If the roll and pitch angles of the images are neglected additional errors in the planar and height coordinates in the determined points on the ground are induced. Table 5 holds the values for the principal distance c for which these additional errors are less than 10 μm in the image resp. 0.1‰ of the flight height when using method M3. The deviations in roll resp. pitch from the exact vertical viewing direction were assumed to be $5^{gon}$ resp. $3^{gon}$, cf. [Kraus 1996].

This height problem is relevant only when the original GPS/INS measurements are taken as the images' XOR for the restitution in the NPS. If the GPS/INS data together with TP measurements are used to perform a so-called integrated AT [Heipke et al. 2001] where certain system parameters of GPS/INS might be corrected then this height problem remains relevant as long as no height control points on the ground are introduced into the AT. In case of given height control the operator (unaware of the real reason) would encounter large $Z_{Map}$ errors and then would either introduce a vertical shift parameter for the GPS observations (of each flight height – in case of different scales with the same camera) or allow the principal distance to be corrected. In both cases problem P2 would be solved (comparable to using $\tau_{global}$ in M2 or c'$_{global}$ in M3). Problem P3, however, would still remain uncorrected. And its effect on the height of a determined point N would be:

$$\Delta H_{N,\tau_{global}} = \left(1 - \frac{\tau}{\tau_{global}}\right) \cdot (H_F - H_N) \qquad (6)$$

$\Delta H_{N,\tau global}$ is larger than 0.06‰ ($H_F - H_N$) if (using equation (1)):

$$\|X - X_0\| > \frac{0.06‰ R^2}{X_0} \quad \text{with } \tau_{global} = \tau(X_0) \qquad (7)$$

$\|X - X_0\|$ represents half of the lateral extension of the area of interest. Table 6 gives an overview of $(X - X_0)$ depending on $X_0$ and corresponding to equation (6).

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2   If during M2 $\tau_{local}$ is applied for each individual unknown object point (iteratively using approximate values) then M2 would be a rigorous method.

## 5. SUMMARY

In this paper an old problem was recalled: The georeferencing of aerial images in the National Projection System (NPS). The planar coordinates of the NPS result commonly from a conformal map projection of the associated ellipsoid and as height coordinates the original ellipsoidal heights are used[3]. The impact of this conformal map projection on the results of the georeferencing was investigated. Due to the map projection, one encounters the following three problems:

- the effect of the Earth curvature
- the length distortion $\tau$
- the variation of $\tau$ throughout large areas of interest

Due to these three facts the NPS does not represent a Cartesian system of coordinates and they induce errors in the determined object points during restitution since Photogrammetry relies on Cartesian relations[4].

The problem of the Earth curvature can be successfully removed by the well-known Earth curvature correction; c.f. [Wang 1980], [Kraus 1997]. The problems induced by $\tau$ and its variation in the area of interest were discussed in more detail for direct and indirect georeferencing (assuming all observations are free of errors). The term georeferencing comprises *image orientation* and *image restitution*.

Concerning the *orientation step* the following can be stated:
Due to $\tau$ during indirect georeferencing a flight height larger than the real one is determined in the NPS (based on Gauss-Krueger). During direct georeferencing actually no orientation is computed, thus the correct flight height is available.

Concerning the *restitution step* the following can be stated:
For both direct and indirect georeferencing the determined heights of the object points are basically affected in the same way - by an error, that is proportional to the height difference between the object points and the (height) control points. However, since the height control points for direct georeferencing are in the height of the airplane the induced height error is much larger than for indirect georeferencing which has its height control points approximately in the height of the object points.[5]

To remove these height errors during the restitution step of georeferencing one can imagine three possible solutions:

- computation in a Cartesian tangential system
- correction of the ellipsoidal heights
- correction of the (laboratorially calibrated) principal distance

The first method is probably the best one, since a Cartesian system is used. During image restitution, however, practical problems might occur since today's stereo plotters might not fully support the required tasks (see section 2).

The second method, which can be realized as a rigorous or an approximate solution, requires a height scaling step before and after each work session, which needs to be implemented in the AT and the stereo plotter software.

The third method – although actually not a rigorous solution – can be realized quite easily already in today's AT and stereo plotter modules and works quite well as it is shown in an example in [Ressl 2001].

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[3] Actually, in practise orthometric heights (referring to the geoid) are used; see footnote 1 in the section 1.

[4] However, it is possible to change the Cartesian Photogrammetric relations in a way that they hold also in the NPS: by implicit transformation into a tangential system; c.f. [Wang 1980]

[5] As a consequence, in very high mountainous areas also for indirect georeferencing large height errors can be induced.
If for indirect georeferencing the orientation step is done in a Cartesian tangential system (where the flight height is determined correctly) and the restitution step, however, is done in the NPS, then the same large height errors as for direct georeferencing will occur.