TWO NEW ALGORITHMS TO RETRIEVE THE CALIBRATION MATRIX FROM THE 3-D PROJECTIVE CAMERA MODEL

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ABSTRACT:

By relating the projective camera model to the perspective one, using homogenous coordinates representation, the interior orientation parameters constitute what is called the calibration matrix. This paper presents two new algorithms to retrieve the calibration matrix from the projective camera model. In both algorithms, a collective approach was adopted, using matrix factorization. The calibration matrix was retrieved from a quadratic matrix term. The two algorithms were framed around a correct utilization of Cholesky factorization to decompose the quadratic matrix term. The first algorithm used an iterative Cholesky factorization to retrieve the calibration matrix from the quadratic matrix term. The second algorithm used Cholesky factorization to factor the quadratic matrix term but after its inversion. The basic argument behind the two algorithms is that: the direct use of Cholesky factorization does not reveal the correct decomposition due to the missing matrix structure in terms of lower-upper order. In both algorithms, a successful retrieval of the calibration matrix was achieved. This paper explains the key ideas behind the two algorithms, accommodated with a simulated example to demonstrate their validity.

1. INTRODUCTION

Calibration of cameras, analog and digital-alike, is a prerequisite task for the precise extraction of metric information from imagery in photogrammetry, computer vision, and other vision applications in which precise quantitative measurements are needed.

Most current vision applications, employed off-the-shelf digital cameras that exhibit a considerable amount of distortions due to various reasons. The camera assembly is often misaligned, the CCD chip may not be orthogonal to the optical axis, the effective focal length may not be known, and the camera lens may exhibits a high radial and decentric distortions. The removal of these distortions constitutes the objectives of geometric camera calibration; see (Seedahmed et al., 1998). Generally, the camera calibration problem is formulated under the perspective or the projective camera model. Under the perspective camera model an extended calibration can be achieved and retains the geometric fidelity of the extracted features, but at the cost of solving a non-linear system of equations. On the other hand, a partial calibration can be achieved using the projective camera model but with the main advantage of having a closed form solution.

In the context of the 3-D projective transformation, the camera interior orientation parameters are implicitly confined to five parameters, namely, the principal point coordinates location, two camera constants, and a non-orthogonality factor. By establishing the relationship between the projective or the Direct Linear Transformation (DLT) and the perspective camera model, the interior orientation parameters can be retrieved either sequentially; see (Abdel-Aziz and Karara, 1971), or simultaneously. The sequential retrieval leads to the original relationship between the Direct Linear transformation and the collinearity model. The simultaneous or collective retrieval leads to what is called the calibration matrix. This study provides two new algorithms to the simultaneous retrieval of the calibration matrix. In addition, this study showed that one of the classical simultaneous retrieval algorithm does not provide the general solution.

Cholesky factorization in its original format is suggested as a decomposition method to retrieve the calibration matrix from the projective camera model, see (Forstner, 2000; Urbanek et al., 2001). We showed in sequel of this paper that this is not a valid factorization when the principal point is displaced from its true location.

This paper introduces two algorithms based on the correct use of Cholesky factorization and can accommodate any amount of principal point displacement without affecting the quality of the solution. The first algorithm used an iterative Cholesky factorization to retrieve the calibration matrix from a quadratic matrix term. The second algorithm used Cholesky factorization to factor the quadratic matrix term but after its inversion. The key idea behind the two algorithms is that: the direct use of Cholesky factorization will not reveal the correct decomposition for the calibration matrix housed in the quadratic matrix term, despite the fact that we have a symmetric positive definite matrix, and this is due to the missing matrix structure in terms of lower-upper order. The quadratic matrix term, which housed the calibration matrix, has an upper-lower ordering. The two algorithms rebuild the missing matrix structure for Cholesky factorization and enable the correct retrieval of the calibration matrix.

This paper is organized as follows. Section 2 briefly reviews the 3-D projective camera model and emphasizes its linearity. Section 3 presents the principle of matrix

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factorization and explains the relationship between the projective and the perspective camera model using matrix factorization. Section 4 and 5 present the new algorithms for the retrieval of the calibration matrix and are explaining the key ideas behind them. Section 6 presents the experimental results. Finally, section 7 concludes the paper.

2. THE 3-D PROJECTIVE CAMERA MODEL

In the projective model, the camera is considered as a system that performs a linear projective transformation from the projective space P^3 into the projective plane P^2. Mathematically this mapping can be written as:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} L_1 & L_2 & L_3 & L_4 \\ L_5 & L_6 & L_7 & L_8 \\ L_9 & L_{10} & L_{11} & L_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (1)$$

where x, y image coordinates.
X, Y, Z Object space coordinates.
$L_1..L_{12}$ camera parameters.

In addition, this model can be written as :

$$x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + L_{12}} + e_x \quad (2)$$

$$y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + L_{12}} + e_y \quad (3)$$

Equation (2) and (3) are the non-linear version of the 3-D projective model. By setting $L_{12}=1$ as normalization criterion, see (Seedahmed and Schenk, 2001). A linear version of equations 2 and 3 can be written as:

$$x = XL_1 + YL_2 + ZL_3 + L_4 - (x-e_x)XL_9 - (x-e_x)YL_{10} - (x-e_x)ZL_{11} + e_x \quad (4)$$

$$y = XL_5 + YL_6 + ZL_7 + L_8 - (y-e_y)XL_9 - (y-e_y)YL_{10} - (y-e_y)ZL_{11} + e_y \quad (5)$$

where e_x *and* e_y are the true unknown errors associated with the image coordinate measurements.
X, Y, and Z are treated as error free coordinates in equations 4 and 5.

3. PRINCIPLE OF MATRIX FACTORIZATION

This section reviews the principles of matrix factorization. The matrix factorization technique provides a compact link between the projective and the perspective camera model. By using matrix factorization, the perspective camera model can be written as a product of the following matrices (Hartley and Zisserman, 2000):

$$x = KR[I_3 \quad | \quad -X_o]X \quad (6)$$

$$X_o = [X_o \quad Y_o \quad Z_o]^T \quad (7)$$

where x : homogenous image coordinates vector.
K : calibration matrix.
R : rotation matrix.
X_o : the position of the camera in the object space.
X : homogenous coordinates vector of a point in the object space.
I_3 : the identity matrix.

The general form of the calibration matrix is:

$$K = \begin{bmatrix} C_x & \alpha & x_p \\ 0 & C_y & y_p \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

where x_p, y_p are the coordinates of the principal point.
C_x, C_y focal length along the x and y axes.
α : skewness factor.

From equations (1) and (6), the following equivalency between the projective and perspective cameras can be inferred:

$$\begin{bmatrix} L_1 & L_2 & L_3 & L_4 \\ L_5 & L_6 & L_7 & L_8 \\ L_9 & L_{10} & L_{11} & L_{12} \end{bmatrix} = KR\,[I \quad | \quad -X_o] \quad (9)$$

From equation (9), we can write:

$$KR = D \quad (10)$$

where:

$$D = \begin{bmatrix} L_1 & L_2 & L_3 \\ L_5 & L_6 & L_7 \\ L_9 & L_{10} & L_{11} \end{bmatrix} \quad (11)$$

Also from equation (9), we can write:

$$-KRX_o = d \quad (12)$$

where

$$d = [L_4 \quad L_8 \quad 1]^T \quad (13)$$

From equation (10), we can write a quadratic term for the calibration matrix as follows:

$$(KR)(KR)^T = DD^T \quad (14)$$

Then:

$$KK^T = DD^T \quad (15)$$

since:

$$RR^T = I_3 \quad (16)$$

At this stage, we should denote that equation (15) is rotation and translation invariant.

where I_3 : is the identity matrix.

The normalized calibration matrix can represented by:

$$K = \frac{1}{K_{33}} \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ 0 & K_{22} & K_{23} \\ 0 & 0 & K_{33} \end{bmatrix} \qquad (17)$$

Equation (15) is the starting point for the two algorithms presented in this paper. The basic argument behind the two algorithms is that: the direct use of Cholesky factorization will not reveal the correct decomposition to the matrix (DD^T), despite the fact that we have a symmetric positive definite matrix, and this is due to the missing matrix structure in terms of lower-upper ordering. The missing matrix structure can be confirmed by checking the structure of the calibration matrix *(K)* in connection with the matrix (DD^T).

4. THE FIRST ALGORITHM

The first algorithm utilized Cholesky factorization coupled with an iterative update of the principal point assuming that the skewness factor is very small. At every iteration the principal point is updated and the observed image coordinates were corrected due to the principal point displacement. We observed that after a very few iterations, the solution converged to the correct calibration parameters.

The key idea behind this algorithm can be captured by the following two arguments. First, Cholesky factorization alone will not reveal the correct decomposition of the matrix (DD^T) since we had upper-lower matrix structure instead of lower-upper matrix structure. Second, the iterative solution reduced the factored matrix to a diagonal structure, which made Cholesky factorization a valid decomposition. Step-wise this algorithm can be stated as follows:

1. Compute the camera parameter using equation (4) and (5).
2. Form the quadratic matrix term, (DD^T), using equation (15).
3. Apply Cholesky factorization to the quadratic matrix. This step leads to un-normalized calibration matrix *(K)*.
4. Normalize the calibration matrix by dividing its elements by *K(3,3)* using equation (17).
5. Extract the principal point from the normalized calibration matrix.
6. Update the principal point.
7. Displace the observed image coordinates using the updated principal point.
8. Repeat steps 2-8 until the convergence of the solution to a stable principal point.

The net result of this algorithm is a reduced calibration matrix in the sense that the elements correspond to x_p and y_p are equal to zero. The principal point solution is recovered in two separate terms.

5. THE SECOND ALGORITHM

The second algorithm is based on a very simple idea. This idea states that: by inverting the matrix (DD^T) we will end-up with the correct order in terms of lower-upper matrix structure, which will lend itself to a direct Cholesky factorization. Step-wise this algorithm can be stated as follows:

1. Compute the camera parameters using equations (4) and (5).
2. Form the quadratic matrix term, (DD^T), using equation (15).
3. Invert the matrix (DD^T).
4. Find the Cholesky factorization of the matrix $(DD^T)^{-1}$.
5. Invert the factored matrix and this is represents the un-normalized calibration matrix *(K)*.
6. Normalize the calibration matrix, by dividing its elements by *K(3,3)*, to end-up with the calibration matrix *(K)* using equation (17).

The net result of this algorithm is the full calibration matrix as depicted by equation (17).

6. EXPERIMENTAL RESULTS

This section presents the experimental results of a simulated example using a single image. We set-up 8 control points at the object space, as shown in table 1, and project them to the image space using the perspective camera model with specified exterior orientation parameters presented in table 3. Table 4 shows the interior orientation parameters, used in connection with the exterior orientation parameters to project the control point into the image space.

POINT ID	X	Y	Z
P_1	-200.0	-200.0	100.0
P_2	-200.0	2200.0	100.0
P_3	2200.0	2200.0	100.0
P_4	2200.0	-200.0	100.0
P_5	2200.0	1000.0	100.0
P_6	200.0	1000.0	100.0
P_7	900.0	2000.0	50.0
P_8	1100.0	100.0	150.0

Table 1: Object space Points in meters.

POINT ID	X	Y
P_1	-96.9105	-90.3249
P_2	-81.8805	95.4951
P_3	105.4855	85.5611
P_4	94.1925	-106.8049
P_5	99.9025	-9.5429
P_6	-58.6375	1.4631
P_7	1.4425	73.6151
P_8	6.7605	-76.9579

Table 2: Image space points in mm.

Xo (m)	Yo (m)	Zo (m)
1000.1	999.81	2000.1
ω^o	φ^o	κ^o
1.0002	1.5	3.9999

Table 3: Exterior camera parameters.

where *ω, φ, and κ:* are the elements of the rotation matrix *R*.

In this study four experiments are presented to show the validity of the two algorithms. The only difference from experiment to experiment is that the image coordinates are shifted from their true locations using four different sets of principal point as depicted in table 4. In all experiments an identical skewness factor was used that is equal to 0.13615.

Experiment#	C_x mm	C_y mm	x_p mm	y_p mm
1	150.01	149.91	0.0	0.0
2	150.01	149.91	0.130	5.4
3	150.01	149.91	9.01	11.97
4	150.01	149.91	19.01	21.97

Table 4: Interior camera parameters.

In the sequel of this section, we used experiment #4 to demonstrate the validity of the two algorithms. Using the Cholesky factorization in its original format we end-up with the following calibration matrix:

$$K = \begin{bmatrix} 154.0212 & 2.9509 & 19.7230 \\ 0. & 154.2948 & 22.4161 \\ 0. & 0. & 1. \end{bmatrix}$$

It is evident that the direct use of Cholesky factorization alone will not reveal the correct calibration matrix.

By using the first algorithm, we are able to retrieve the correct calibration parameters. By examining the graph depicted in figure 1, we can deduce that we need 2 to 3 iterations to obtain the correct solution for the reduced calibration matrix and the principal point.

The reduced calibration matrix is:

$$K = \begin{bmatrix} 150.01 & 0.13615 & 0.0 \\ 0. & 149.91 & 0.0 \\ 0. & 0. & 1. \end{bmatrix}$$

The principal point solution is:

$$x_P = 19.01$$
$$y_P = 21.97$$

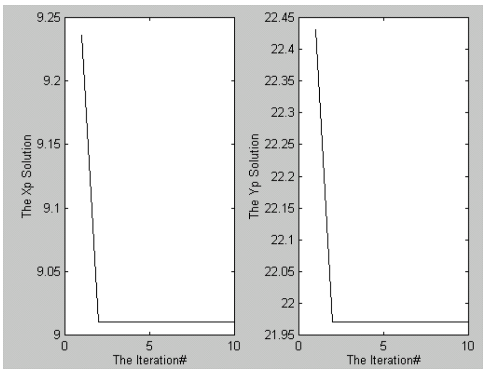

Figure 1: Principal Point Solution vs. Iteration#

By applying the second algorithm the quadratic matrix term, we end-up with the calibration matrix *K*.

$$K = \begin{bmatrix} 150.01 & 0.13615 & 19.01 \\ 0. & 149.91 & 21.97 \\ 0. & 0. & 1. \end{bmatrix}$$

7. CONCLUSIONS

The basic idea behind the two algorithms is that: the direct use of Cholesky factorization will not reveal the correct decomposition, despite the fact that we have a symmetric positive definite matrix, and this is due to the missing lower-upper matrix structure. The two algorithms rebuild the missing matrix structure and enable the correct retrieval of the calibration matrix. The first algorithm adopts an iterative strategy that leads to the correct retrieval of the calibration matrix. The second algorithm avoid the iterative strategy by applying Cholesky factorization in the inverse domain and this establishes the correct matrix structure in term of lower-upper order, which in turn achieved the correct retrieval of the calibration matrix. Direct Cholesky factorization, applied to DD^T, is a valid algorithm if the principal point displacement is very small. The two algorithms support the proposed argument of the missing matrix structure and provide the correct solution, but for practical applications the second algorithms is a suitable choice since it obviates the need of the iterative solution.

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