Coded Multidimensional Pulse Amplitude Modulation for Ultra-high-speed Optical Transmission

Ivan B. Djordjevic[1], Lei Xu[2], and Ting Wang[2]
[1]University of Arizona, Department of Electrical and Computer Engineering, 1230 E. Speedway Blvd., Tucson, AZ 85721, USA
[2]NEC Laboratories America, Princeton, NJ 08540, USA
ivan@ece.arizona.edu

Abstract: We propose a coded *N*-dimensional pulse-amplitude modulation (ND-PAM) suitable for ultra-high-speed serial optical transport. The polarization-multiplexed-ND-PAM significantly outperforms corresponding polarization-multiplexed-QAM counterpart in terms of OSNR sensitivity (> 4 dB at symbol rate 31.25 GS/s), while enabling beyond 400 Gb/s transmission.

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1. Introduction

The optical communication systems have been rapidly evolving recently in order to meet continuously increasing demands on transmission capacity, originating mostly from the Internet and multimedia applications [1],[2]. In order to satisfy high capacity demands, according to some industry experts, the 1 TbE standard should be completed by 2012-2013 [3]. Coherent optical OFDM is one possible pathway towards achieving beyond 1 Tb/s optical transport [1]. Initial studies [1], unfortunately, show tight margin in terms of 7% overhead for RS(255,239) code, and the use of strong LDPC codes is necessary [4]. Another approach is based on multidimensional coded modulation [2],[5]. Most of the papers on multidimensional signal constellations for optical communications so far have been related to single carrier systems.

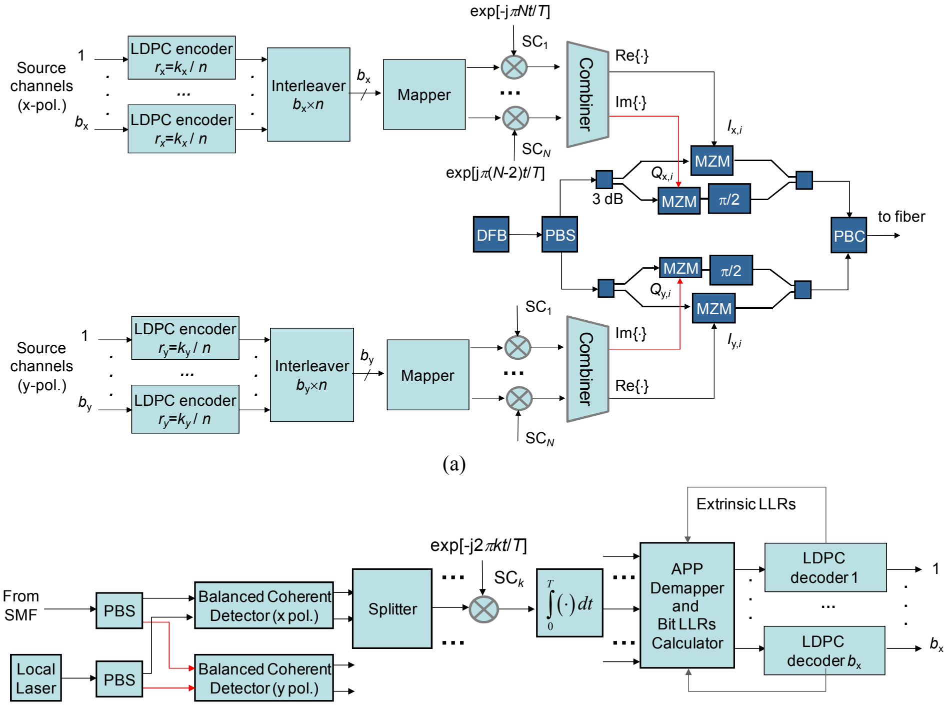
In this paper, we extend our multidimensional coded modulation approach [2],[5] to multicarrier systems as well. The proposed coded multidimensional modulation scheme is called *N*-dimensional pulse amplitude modulation (ND-PAM) in analogy to QAM that can be considered as generalization of 1D-PAM. The proposed scheme can also be considered as a generalization of OFDM. In ND-PAM scheme, the orthogonal subcarriers are used as bases functions, and the signal constellation points of corresponding ND-PAM constellation diagram are obtained as *N*-dimensional Cartesian product of one-dimensional PAM. In conventional OFDM, QAM/PSK signal constellation points are transmitted over orthogonal subcarriers and then multiplexed together in an OFDM stream. Individual subcarriers therefore carry *N* parallel QAM/PSK streams. In ND-PAM instead, the *N*-dimensional signal constellation point is transmitted over all *N* subcarriers, which serve as individual bases functions. Even if some of the subcarriers are severely affected by channel distortion, the overall signal constellation point will face only small distortion. In addition, because the channel capacity is a logarithmic function of SNR and also a linear function of number of dimensions, the spectral efficiency of optical transmission systems can dramatically be improved with proposed scheme. Notice that complexity of corresponding *a posteriori* probability (APP) demapper increases with number of dimensions, and it is clear that in practice up to 10 dimensions should be used. We further describe the frequency-interleaved scheme that properly combines subsystems with reasonable number of dimensions into a system with multi-Tb/s serial aggregate data rate.

2. Description of proposed ND-PAM

The proposed LDPC-coded ND-PAM system, which is obtained as *N*-dimensional generalization of PAM with *L* constellation points, is shown in Fig. 1. The ND-PAM signal constellation is obtained as *N*-dimensional Cartesian product of one-dimensional PAM signal constellation. The 1D-PAM is described with the following amplitude signal constellation points $X=\{(2i-1-L)d,\ i=1,2,\ldots,L\}$, where $2d$ is the Euclidean distance between two neighboring points. The ND-PAM is therefore obtained as

$$X^N = \underbrace{X\times X\times\cdots\times X}_{N\text{ times}} = \left\{(x_1,x_2,\cdots,x_N)\,|\,x_i\in X,\ \forall 1\le i\le N\right\}. \qquad (1)$$

For example, for $L=4$ and $N=3$ the corresponding constellation diagram is given by $X^3 = X\times X\times X = \left\{(x_1,x_2,x_3)\,|\,x_i\in X=\{-3,-1,1,3\},\ \forall\ 1\le i\le 3\right\}$. The number of constellation points in ND-PAM is determined by $M=L^N$, while the number of bits per symbol is $b=\log_2(L^N)$. The data streams are encoded using an LDPC (n,k) code of rate $r=k/n$, where n denotes the codeword length and k is the information word length. The codewords are written row-wise into $b\times n$ bit interleaver. The b bits are taken from bit interleaver column-wise at every symbol slot i and are used as input of ND mapper, which selects one constellation point out of L^N, depending on information content. The ND mapper is implemented as a look-up table (LUT) with b input bits serving as a memory address that selects the N-coordinates of ND-PAM signal constellation point. As an example, the LUT for L=4 and N=3 (4^3-3D-PAM) is shown in Table 1. The kth coordinate is multiplied by $\exp[j2\pi kt/T]$ (k=-N/2,…,N/2-1). Therefore, the coordinates are imposed on orthogonal subcarriers. The coordinates, upon multiplication with $\exp[j2\pi kt/T]$, are added in combiner block that provides the real and imaginary parts of such obtained signal to be used as in-phase and quadrature signals for corresponding Mach-Zehnder modulators (MZMs) as shown in Fig. 1(a). Similar architectures are used for x- and y-polarization channels. The signals at the output of I/Q modulators are combined into single stream via polarization-beam splitter (PBS), as shown in Fig. 1(a). The aggregate data rate of this scheme is $2rbR_s$, where R_s is the symbol rate. For example, by setting L=4, N=4 and R_s=31.25 Giga symbols/s (GS/s), the aggregate data rate can reach 400 Gb/s. If we increase the number of dimensions to N=10 while keeping all other parameters the same, the aggregate data rate is 1 Tb/s, which is compatible with 1 Tb/s Ethernet. The symbol rate is dictated by commercially available electronics.



(a)

(b)

Fig. 1 The LDPC-coded ND-PAM scheme: (a) Tx configuration and (b) Rx configuration (only x-polarization branch is given with all details). PBS/C: polarization beam splitter/combiner, 3 dB: 3 dB coupler, SC_k: kth subcarrier, APP: *a posteriori* probability, LLRs: log-likelihood ratios.

Table 1: Mapping rule look-up table for 4^3-3D-PAM.

Interleaver output	Signal constellation coordinates	Interleaver output	Signal constellation coordinates	Interleaver output	Signal constellation coordinates	Interleaver output	Signal constellation coordinates
0 0 0 0 0 0	{-3, -3, -3}	0 0 0 0 1 0	{-1, -3, -3}	0 0 0 0 0 1	{1, -3, -3}	0 0 0 0 1 1	{3, -3, -3}
1 0 0 0 0 0	{-3, -3, -1}	1 0 0 0 1 0	{-1, -3, -1}	1 0 0 0 0 1	{1, -3, -1}	1 0 0 0 1 1	{3, -3, -1}
0 1 0 0 0 0	{-3, -3, 1}	0 1 0 0 1 0	{-1, -3, 1}	0 1 0 0 0 1	{1, -3, 1}	0 1 0 0 1 1	{3, -3, 1}
1 1 0 0 0 0	{-3, -3, 3}	1 1 0 0 1 0	{-1, -3, 3}	1 1 0 0 0 1	{1, -3, 3}	1 1 0 0 1 1	{3, -3, 3}
0 0 1 0 0 0	{-3, -1, -3}	0 0 1 0 1 0	{-1, -1, -3}	0 0 1 0 0 1	{1, -1, -3}	0 0 1 0 1 1	{3, -1, -3}
…		…		…		…	
1 1 0 1 0 0	{-3, 1, 3}	1 1 0 1 1 0	{-1, 1, 3}	1 1 0 1 0 1	{1, 1, 3}	1 1 0 1 1 1	{3, 1, 3}
0 0 1 1 0 0	{-3, 3, -3}	0 0 1 1 1 0	{-1, 3, -3}	0 0 1 1 0 1	{1, 3, -3}	0 0 1 1 1 1	{3, 3, -3}
1 0 1 1 0 0	{-3, 3, -1}	1 0 1 1 1 0	{-1, 3, -1}	1 0 1 1 0 1	{1, 3, -1}	1 0 1 1 1 1	{3, 3, -1}
0 1 1 1 0 0	{-3, 3, 1}	0 1 1 1 1 0	{-1, 3, 1}	0 1 1 1 0 1	{1, 3, 1}	0 1 1 1 1 1	{3, 3, 1}
1 1 1 1 0 0	{-3, 3, 3}	1 1 1 1 1 0	{-1, 3, 3}	1 1 1 1 0 1	{1, 3, 3}	1 1 1 1 1 1	{3, 3, 3}

At the receiver side (see Fig. 1(b)) the optical signal is split by the polarization beam splitter (PBS) into two orthogonal polarizations that are used as input into two balanced coherent detectors. After analog-to-digital conversion, the balanced coherent detector outputs are used as real and imaginary parts of complex sequence stream, which is further split into N-branches as shown in Fig. 1(b). The kth branch determines the projection along kth coordinate. The projections are used as input of an APP demapper, in which symbol log-likelihood ratios (LLRs) are calculated in similar fashion as in [2]. The bit LLRs calculator, on the other hand, calculates the bit LLRs to be used in LDPC decoding as described in [2].

In theory, we can increase the aggregate data rate by simply increasing the number of subcarriers as long as the orhtogonality among subcarriers is preserved. However, the complexity of APP demaper (see Fig. 1(b)) increases with *N*. To keep the complexity of APP demaper reasonably low, we can employ the following approach, which can be called the *frequency-interleaving*. We first split the total number of subcarriers $N_{sc}=N^2$ into *N* subgroups of *N* subcarriers. Next, the *k*th group of subcarriers (k=1,…,N) to be used in the *N*-dimensional signal constellation is formed by taking each *k*th element of all subgroups. Finally, we perform encoding, modulation, transmission, demodulation, decoding on all groups as shown in Fig. 1. On such a way, if several subcarriers (coordinates) are affected by channel distortion they will belong to different constellation points and system will be still more immune to channel distortion compared with conventional OFDM. By using sufficiently high dimensionality of signal constellations ($N\geq 3$), the OSNR improvement advantage will be preserved.

3. Performance Analysis and Conclusion

The uncoded symbol error probability of proposed ND-PAM scheme can be calculated by

$$P_s = 1-\left[1-\left(1-1/L\right)\operatorname{erfc}\left(\sqrt{3E_{\mathrm{av}}/\left[N\left(L^2-1\right)N_0\right]}\right)\right]^N, \qquad (2)$$

where N_0 is the power spectral density and E_{av} is symbol energy, which can be expressed in terms of bit energy E_b as follows $E_{\mathrm{av}}=E_b\log_2 L^N$. In Fig. 2(left), we provide symbol-error probabilities P_s obtained by Eq. (2) in comparison with Monte Carlo simulations. An excellent agreement is found. We can see also that increase in number of dimensions results in small P_s performance degradation as long as orthogonality among subcarriers is maintained. As an illustration of the potential of the proposed scheme, we show in Fig. 2(right) the BER performance of the LDPC(16935, 13550)-coded ND-PAM schemes for symbol rate of 31.25 GS/s. Similarly to P_s performance trend, the increase in the number of dimensions for fixed *L* results in negligible BER performance degradation. The comparison of corresponding curves with L=4 and L=8 indicates that in order to increase the aggregate data rate it would be better to increase the number of subcarriers rather than the 1D-PAM signal constellation size. The PolMUX 4^3-3D-PAM outperforms corresponding PolMUX 64-QAM by even 4.281 dB at BER of 10^{-8}. PolMUX 4^4-3D-PAM performs just slightly worse than PolMux 16-QAM, but has the aggregate data rate of $2\times\log_2(4^4)\times 0.8\times 31.25$ GS/s=400 Gb/s and as such is compatible with future 400 G Ethernet. The aggregate data rate of PolMux 16-QAM is only 200 Gb/s. If we instead use the ND-PAM with L=4 and N=10 the resulting aggregate data rate is 1Tb/s. Therefore, the proposed ND-PAM is 400 Gb/s and 1 Tb/s Ethernet enabling modulation scheme.

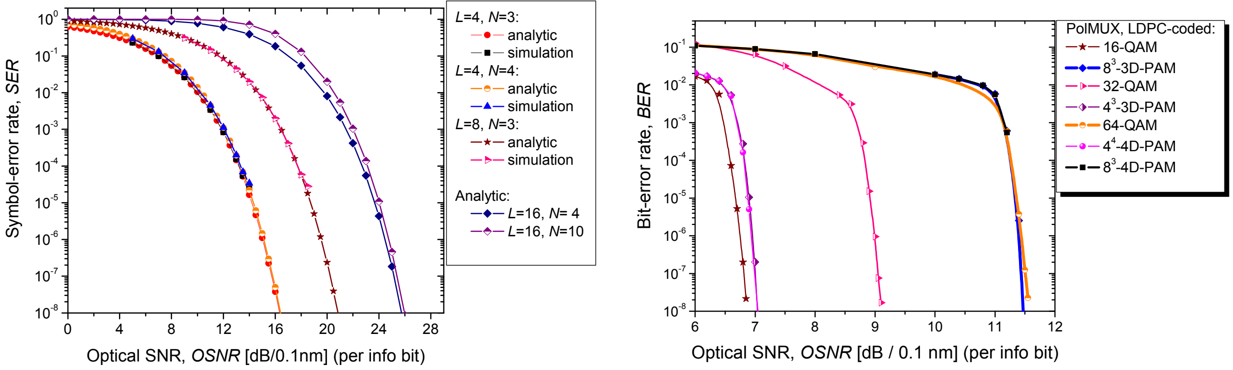


Fig. 3 ND-PAM performance: (left) uncoded symbol error-rates for symbol rate of 25 GS/s, and (right) LDPC-coded ND-PAM BER performance at symbol rate of 31.25 GS/s.

In conclusion, as response to continuously increasing demands on transmission capacity, we proposed the LDPC-coded ND-PAM. The proposed scheme can be considered as a generalization of both PAM and OFDM. In this scheme, the orthogonal subcarriers are used as bases functions, and the signal constellation points of corresponding ND-PAM constellation diagram are obtained as *N*-dimensional Cartesian product of one-dimensional PAM. This scheme is the next generations, both 400 Gb/s and 1 Tb/s, Ethernet enabling technology. The proposed scheme significantly outperforms the corresponding QAM modulations.

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