Study on the Performance of Decision-Aided Maximum Likelihood Phase Estimation with a Forgetting Factor

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Abstract: A structure of non-adaptive decision-aided maximum likelihood is demonstrated by introducing a forgetting factor via both analysis and experiments, which achieves the same performance as the adaptive phase estimation while reducing the algorithm complexity.
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1. Introduction

Coherent detection with advanced modulation formats has been well established as the technology for the next-generation optical networks to support 100 G Ethernet or even higher bit rates [1]. One of the major impairments in coherent receiver is the carrier phase perturbation originating from laser phase noise and/or fiber nonlinear phase noise. Various data-aided and non-data-aided phase estimators have been proposed for coherent detection [2–5]. Among these algorithms, decision-aided (DA) maximum likelihood (ML) is well investigated in both wireless and coherent optical communications due to its excellent characteristics [6, 7]. However, no experimental study has been carried out to investigate its performance in the presence of fiber nonlinearity. In this paper, we will first introduce a forgetting factor into the conventional DA ML estimator and examine its performance via both analysis and experiments, focusing on self-phase modulation (SPM).

2. Principles of DA ML with a forgetting factor

The recovered signal after digital equalization can be represented by $r(k) = m(k)\exp[j\theta(k)] + n(k)$ [4], where $m(k)$ is the kth data symbol, and M, a power of 2, is the total number of signal points in the constellation. $\theta(k)$ is the Wiener phase noise given by $\theta(k)=\theta(k\text{-}1)+\nu(k)$ and the increment $\nu(k)$ is a Gaussian variable with mean zero and variance σ_p^2, and $n(k)$ is assumed to be the dominant additive white Gaussian noise with mean zero and variance σ_0^2. The signal-to-noise ratio (SNR) per symbol is defined as $\gamma_s = |m(k)|^2/\sigma_0^2$. The conditional probability density function $p(r(k)|\theta(k), m(k))$ is Gaussian[7]. By taking the derivative of the likelihood function given L received signals with respect to $\theta(k)$, the DA ML phase estimate is represented by [6] $V(k) \equiv U^{-1}(k)\sum_{l=k-L}^{k-1} r(l)\hat{m}^*(l)$, where the superscript * denotes complex conjugate, $\hat{m}(l)$ is the receiver's decision on the lth received symbol, and $U(k) = \sum_{l=k-L}^{k-1} |\hat{m}(l)|^2$ is the normalization factor. Here, L is called the memory length. The phase error $\Delta\theta = \theta(k) - \hat{\theta}(k)$ of the DA ML estimator has a variance derived in [4] as

$$E\left[\Delta\theta^2\right] = \frac{(L+1)(2L+1)}{6L}\sigma_p^2 + \frac{1}{2L}\sigma_{n'}^2, \tag{1}$$

where $\sigma_{n'}^2 = 1/\gamma_s$. The phase error derived in Eq. (1) is still valid even in M-quadtraure amplitude modulation (M-QAM) format due to the average over L symbols [8].

The conventional DA ML estimator shows the individual phase estimates $\hat{\theta}(l)$ for symbols k–L to k–1 are all weighted equally. Due to the fact that phase noise is a Wiener process, which de-correlates over time, the DA ML phase estimator would be suboptimal. It is straightforward to conjecture that the phase estimate $\theta(k-1)$ at symbol k–1 ought to be a better estimate of $\hat{\theta}(k)$ than the phase estimate $\theta(k-L)$. In view of the aforementioned characteristics of the phase noise, a forgetting factor α ($0 < \alpha \leq 1$) is introduced into the conventional DA ML estimator, yielding

$$V(k) \equiv U^{-1}(k) \sum_{l=k-L}^{k-1} \alpha^{k-l} r(l) \hat{m}^*(l) \, . \qquad (2)$$

The variance of its phase error $\Delta\theta$ is easily derived as ($\alpha \neq 1$)

$$E\left[\Delta\theta^2\right] = \frac{1}{(1-\alpha^L)^2}\left[\frac{1-\alpha^{2L}}{1-\alpha^2} + L\alpha^{2L} - 2\alpha^L \frac{1-\alpha^L}{1-\alpha}\right]\sigma_p^2 + \frac{1}{2}\frac{(1-\alpha)(1+\alpha^L)}{(1-\alpha^L)(1+\alpha)}\sigma_{n'}^2 \qquad (3)$$

Note that the conventional DA ML corresponds to the form of Eq. (2) with α=1.

Fig. 1 demonstrates that the analytical standard deviation (STD) in Eq. (3) for the phase error variance is accurate whichever value for the forgetting factor is selected. As can be observed, a small forgetting factor is optimal at high SNR region while a large forgetting factor is preferred at low SNRs. Moreover, we plot the optimal forgetting factor as a function of phase noise in Fig. 2. The dotted line indicates the transition of the optimal forgetting factor: the faster the carrier phase varies, the smaller the forgetting factor α is. The explanation for the observations in both Figs. 1 and Fig. 2 is that a smaller forgetting factor is able to make the first phase estimate $\theta(k-1)$ dominant over the previous phase estimates, which are less correlated with the current phase estimate $\hat{\theta}(k)$. The STD of the adaptive phase estimator is also plotted in Fig. 1, where the adaptive DA algorithm is no longer optimum in 16-QAM due to the constellation penalty [4]. By carefully adjusting either the memory length L [4] or forgetting factor α, the DA ML can outperform the adaptive phase estimation.

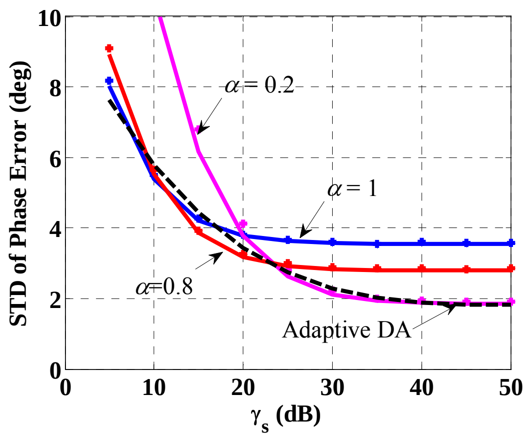

Fig. 1. The comparison of STD between DA ML with different forgetting factor α in 16-QAM format obtained from analysis (line) and MC simulations (marker '+'). σ_p^2=10^{-3} and L=10.

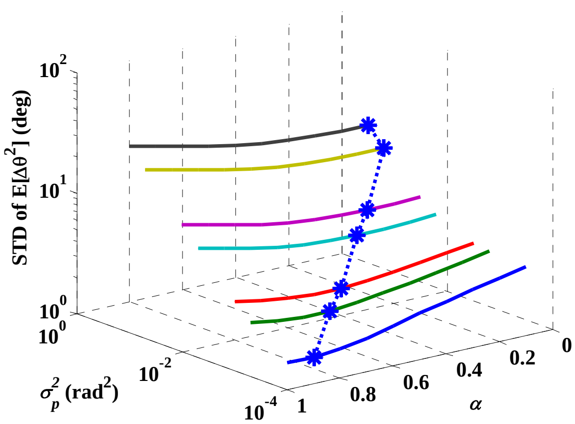

Fig. 2. The STD of the DA ML with different forgetting factor α and phase noise at γ_s=20 dB in 16-QAM format. The dot line highlights the optimal forgetting factor at different phase noise levels. L=10.

4. Experiments and Results

A long-haul transmission was carried out to investigate the proposed DA ML with the forgetting factor. As displayed in Fig. 3, a typical polarization-multiplexing (PolMux) RZ-QPSK transmitter [9], centering at wavelength 1550.0 nm modulated by 44 Gbits/s bit rate, was coupled into the transmission loop. The other channels were occupied by continuous-wave (CW) lasers for noise suppression in erbium-doped fiber amplifiers (EDFA). The fiber loop consisted of 6-span ~80km dispersion managed fiber (DMF), resulting in a total ~800ps/nm residual chromatic dispersion after each loop. The measured signal was transmitted over 5 loops, resulting in a total transmission length of 2, 400 km. The spectra after 5-loop transmission are depicted in Fig. 4. The received signal was filtered through a 1nm optical tunable filter, and was then fed into a coherent receiver. The received signal power was kept at –5 dBm throughout all the measurements.

A series of typical digital signal processing (DSP) algorithms were conducted for channel equalization and correlation was then computed to search for the beginning of the training data that are designed for low-density parity check codes. To address the phase ambiguity in the system, the training data was differentially encoded first and then uploaded into two pulse pattern generators (PPGs), which were synchronized with the aid of a colleague from the University of Arizona. In the offline processing, the cross correlation between the recovered bits and training data was calculated, as illustrated in Fig. 5, where the peak indicates the beginning of the training data. It is worthy to point out that the correlation for two tributaries (in-phase and quadrature) is helpful to monitor the synchronization of two PPGs. The use of training data can resolve the phase ambiguity and facilitate various data-aided DSP algorithms. In this paper, we will utilize those training data for the adaptive DA phase estimation.

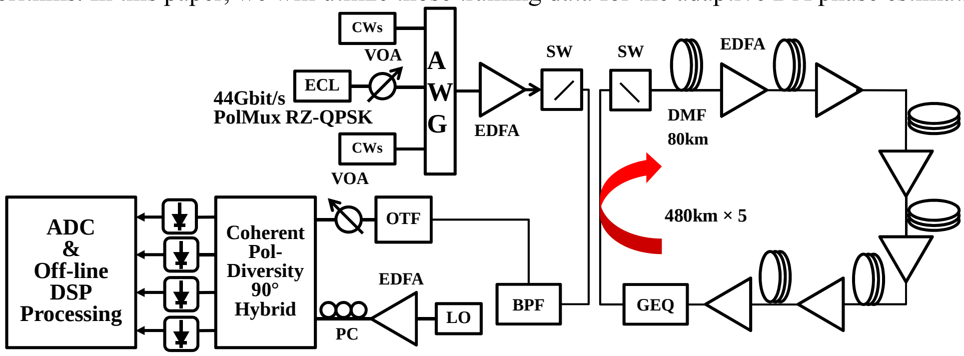

Fig. 3. Experimental setup for the 2,400 km transmission of 44 Gbits/s PolMux RZ-QPSK. ADC: analog-to-digital converters; AWG: arrayed waveguide grating; BPF: band-pass filter; GEQ: gain equalizer; OTF: optical tunable filter; PC: polarization controller; SW: switch; WSS: wavelength selected switch; VOA: variable optical attenuator;

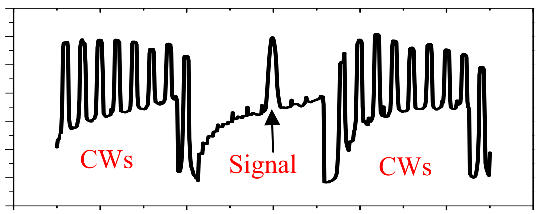

Fig. 4. Received optical spectra after 5 loops. Res. 0.2 nm.

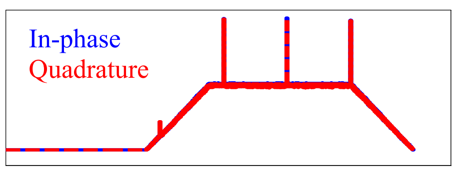

Fig. 5. The cross correlation between the recovered bits and training bits.

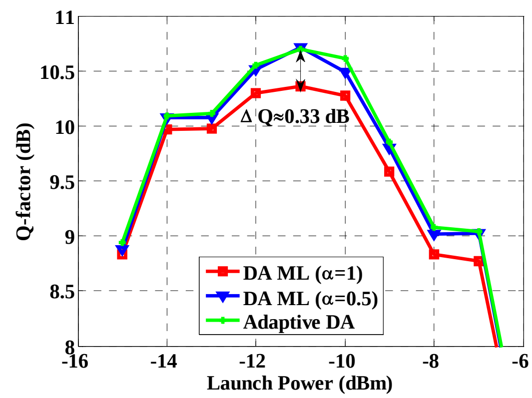

Fig. 6. The measured *Q*-factor (X-pol) as a function of input power.

Fig. 6 shows the measured *Q*-factor for the DA ML with different forgetting factors (α=1 and 0.5) as a function of the launched signal power. It demonstrates that the DA ML with a proper forgetting factor (α=0.5) can actually outperform the conventional DA ML (α=1) by 0.33 dB at the optimal launch power. The adaptive DA algorithm is also plotted in Fig. 6 for reference. It is seen that the forgetting factor drives the DA ML to approach the adaptive DA algorithm, which is proven to have the optimal performance in constant-amplitude *M*-PSK systems [4]. Thus, the DA ML with the forgetting factor can achieve optimum performance without the need for training as in the adaptive DA, or dynamically changing the memory length, thus reducing the algorithm complexity.

5. Conclusions

We propose a novel scheme for implementing DA ML phase estimation by introducing a forgetting factor. The phase error variance of the proposed DA ML structure is firstly demonstrated and analytically verified. In addition, a long-haul transmission experiment is carried out to study the proposed DA ML structure. It is experimentally shown that the proposed DA ML with a forgetting factor can gain a 0.33 dB *Q*-factor improvement over the conventional DA ML. It achieves the same performance as the adaptive phase estimator while reducing the algorithm complexity.

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