# Optical fiber as a conical microresonator

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**Abstract:** High Q-factor localized conical modes are discovered theoretically and demonstrated experimentally in an optical fiber. The theory of these modes provides a means for exceptionally accurate local characterization of the optical fiber nonuniformity.

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## 1. Introduction

This paper describes a new type of high Q-factor optical mode, a conical mode, which is found to be commonly met in conventional optical fibers. Usually, a high Q-factor of a whispering gallery mode (WGM) in dielectric microresonators (e.g., in microspheres and microtoroids [1]) is a result of strong WGM localization due to the total internal reflection from the resonator surface and/or classical conservation laws, which confine the mode between two caustics separating the classically allowed and classically forbidden regions. It is commonly accepted that the absence of caustics or strongly reflecting material interfaces, which confine the classical rays, or periodic structures, which perform multiple wave reflection, makes the corresponding wave motion unbounded and leads to the disappearance of the high Q-factor resonances. For this reason, it could be expected that all the WGMs in a lossless dielectric cone illustrated in Fig. 1 are delocalized. In fact, the classical motion at the conical surface is bounded on the narrower side of the cone and is unbounded on its wider side, so that any geodesic (classical ray) propagating at the conical surface eventually moves off to infinity (Fig. 1(a)). In contrast, it is shown below that, for a cone with a small half-angle $\gamma$, a wave beam launched normally to the cone axis (using, e.g., an optical microfiber [2]) can be completely localized (Fig. 1(b)).

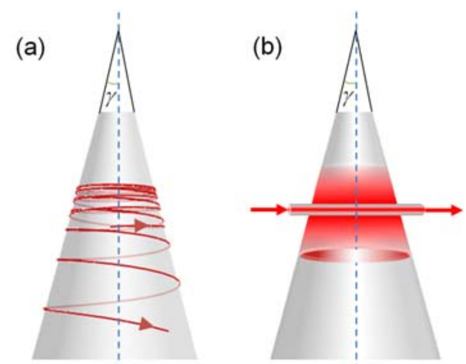


Fig. 1 (a) – Illustration of a geodesic propagating along the conical surface. (b) – Illustration of a localized WGM launched by a microfiber.

The discovered localized conical modes are common for conventional optical fibers which usually have $\gamma \sim 10^{-5}$ or less. It is found that the transmission resonance shape of a conical WGM exhibits asymmetric Airy-type oscillations, which allow one to determine the local slope $\gamma$ of a slightly nonuniform microcylinder (e.g., of an optical fiber) from a single measurement. As cone half-angle $\gamma$ decreases, the size of the localized mode grows very slowly, as $\gamma^{-1/3}$. The developed theory, applied to the investigation of the local slope of an optical fiber, is in excellent agreement with the experimental data.

## 2. Theory

Following the experimental situation considered below, it is assumed that a light beam propagating close to the conical surface is launched by a microfiber waveguide which touches the cone surface normally to its axis (Fig. 1(b)). Similar to [3], in the developed theory, the value of a circulating and self-interfering WGM at a point $(\varphi, z)$ of the cone surface (here $\varphi$ is the azimuthal angle and $z$ is the cone axial coordinate) is shown to be determined by the following superposition of fundamental Gaussian beams which are launched at point $\varphi = z = 0$ and make $m$ turns before approaching point $(\varphi, z)$:

$$\Psi(\varphi, z) \sim \sum_m S_m(0,0)^{-1/2} \exp\left[i(\beta + i\alpha) S_m(\varphi, z)\right] \quad (1)$$

Here $S_m(\varphi, z)$ is the distance between the launch point $(0,0)$ and point $(\varphi, z)$ calculated along the geodesic which connects these points after completing $m$ turns. For $2\pi m\gamma << 1$, we have $S_m(\varphi, z) \approx S_m^0 + \varphi r - \pi m\gamma z + z^2 / (2S_m^0)$

where $r$ is the local cone radius of the circumference $(\varphi, 0)$ and $S_m^0$ is the length of the geodesic crossing itself at the original point after $m$ turns:

$$S_m^0 \approx 2\pi rm - \frac{\pi^3}{3}\gamma^2 rm^3 \tag{2}$$

The resonance propagation constant $\beta_q$ is defined by the quantization condition along the circumference $(\varphi, 0)$: $\beta_q = q/r$, where $q$ is a large integer. If the deviation of the propagation constant, $\Delta\beta = \beta - \beta_q$, attenuation, $\alpha$, and the cone slope, $\gamma$, are small, i.e., if $\Delta\beta, \alpha << (2\pi r)^{-1}$, and $\gamma << \pi^{-3/2}(\beta r)^{-1/2}$ then the sum in Eq. (1) can be replaced by an integral. Using Eq. (2) we get

$$\Psi(\varphi, z) \sim \exp(iq\varphi)\int_0^\infty \frac{dm}{m^{1/2}} \exp\left[2\pi(i\Delta\beta - \alpha)rm - \frac{i\pi^3\beta_q}{3}\gamma^2 rm^3 - \pi m\gamma z + \frac{i\beta_q z^2}{4\pi rm}\right] \tag{3}$$

The resonant transmission power is found from Eq. (3) as $P = |1 - D - C\Psi(0,0)|^2$ where parameters $C$ and $D$ are constants in a vicinity of the resonance. For weak coupling, $|D|, |C\Psi(0,0)| << 1$:

$$P \approx 1 - 2\operatorname{Re}(D) - 2\operatorname{Re}\left\{C\int_0^\infty \frac{dm}{m^{1/2}} \exp\left[2\pi r(i\Delta\beta - \alpha)m - \frac{i\pi^3\beta_q}{3}\gamma^2 rm^3\right]\right\}. \tag{4}$$

The integral in Eq. (4) determines the generalized Airy function of $\Delta\beta + i\alpha$. For $\alpha = 0$, similar to the ordinary Airy function, this function has asymmetric oscillations vanishing away from the principal peak along the real axis $\alpha = 0$.

Numerical and asymptotic analysis of the integrals in Eqs. (3) and (4) lead to the following results. At the narrower side of the cone, for large positive *z*, vanishing of the WGMs is explained by reflection from a turning point, similar to the conventional ray picture (Fig. 1(a)). Crucially, at the wider side of the cone, for large negative *z*, the asymptotic calculation of the integral in Eq. (3) shows that for the discrete sequence of propagation constant determined by the formula $\beta_{qn} = q/r + (\beta\gamma^2 r^{-2})^{1/3}(9\pi/4 + 3\pi n)^{2/3}/2$, ( $n = 0, 1, 2, \ldots$, $q$>>1, integer), the circulating WGMs experience the complete destructive wave self-interference which leads to the full mode localization. The characteristic size of the discovered localized conical mode is $z_{res} = \beta_q^{-2/3} r^{1/3}\gamma^{-1/3}$. The $\gamma^{-1/3}$ dependence of $z_{res}$ is very slow so that a conical resonator with an extremely small slope $\gamma$ can support strongly localized states. In fact, at wavelength $\lambda \sim 1.5\ \mu\text{m}$, for a silica optical fiber with radius $r \sim 50\ \mu\text{m}$ and slope $\gamma \sim 10^{-5}$, we have $z_{res} \approx 50\ \mu\text{m}$.

### 3. Experiment

Experimentally, a 50 mm segment of an uncoated silica optical fiber with radius $r \approx 76\ \mu\text{m}$ was investigated. First, the radius variation of this fiber was accurately measured (Fig. 2). To this end, a biconical adiabatic fiber taper having the microfiber waist of a 1.3 $\mu$m diameter and 3 mm length was fabricated. The ends of the taper were connected to the JDS Uniphase tunable laser source and detector. The wavelength resolution of this system was 3 pm. The microfiber was attached normally to the tested fiber where the WGMs were excited as illustrated in Fig. 1(b). The resonant transmission spectra were measured at points spaced by 2 mm along the tested fiber in the wavelength interval between 1535 nm and 1545nm. The coupling between the microfiber and the tested fiber was tuned to small values by shifting the microfiber/tested fiber contact point to a thicker part of the microfiber (see, e.g., samples of the measured spectra in Fig. 3(a) and (b)). To arrive at the plot of Fig. 2, the radius variation, $\Delta r$, was calculated from the shift of resonance positions, $\Delta\lambda$, as $\Delta r = \lambda\Delta\lambda / r$ [5].

In order to experimentally verify the described theory, the transmission spectra at two positions of the tested fiber segment were examined. At the first position (point $a$ in Fig. 2) the fiber slope was negligible and the corresponding transmission resonant spectrum shown in Fig. 3(a) was primarily determined by the WGM attenuation $\alpha$. At this position, the characteristic resonance width was smaller than the measurement resolution (see e.g., the magnified resonance sample in Fig. 5(c)) that allowed to estimate the attenuation constant as $\alpha < 10^{-6}$. Alternatively, at point $b$ and the next 2 mm spaced measurement point in Fig. 2, the fiber slope was $\gamma \approx 1.07 \cdot 10^{-5}$ and $\gamma \approx 1.63 \cdot 10^{-5}$, respectively. These values, together with the estimate $\alpha < 10^{-6}$, suggests that the attenuation constant in Eqs. (3) and (4) can be ignored. The magnified structure of a separate resonance, which is outlined by a rectangle in Fig. 3(b), is shown in Fig. 3(d). The experimental data in this figure was fitted with the generalized Airy function defined by Eq. (4) assuming the fiber radius $r \approx 76\ \mu\text{m}$, refractive index $n_r = 1.46$, and radiation wavelength $\lambda = 1.54\ \mu\text{m}$, which resulted in $\gamma \approx 1.413 \cdot 10^{-5}$ with better than 0.1% accuracy. This value is in reasonable agreement with the directly-measured slope values. Thus, the excellent agreement between the experimentally-obtained and theoretically-predicted shapes of resonances shown in Fig. 3(d) strongly support the developed theory of localized conical modes. In addition, variation of the position of the principal spectral peak in Fig. 3(d) found from Eq. (4) allows to find corrections to the previously developed method of measurement of the fiber radius variation [5].

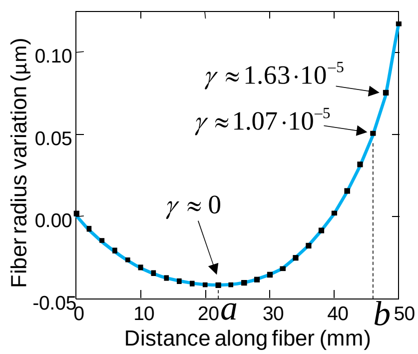


Fig. 2. Radius variation of a 50 mm optical fiber segment obtained experimentally with 2 mm measurement steps.

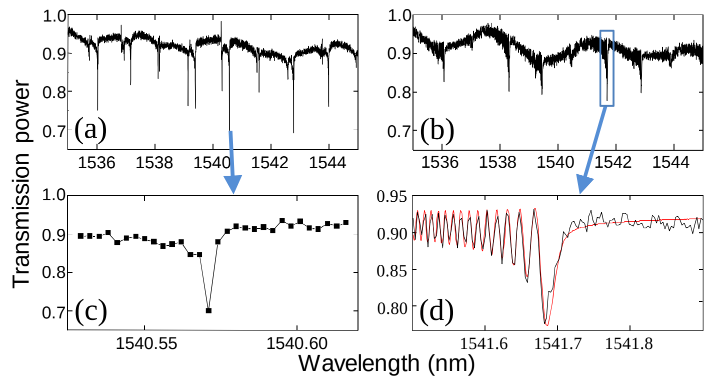


Fig. 3. (a) and (b) – resonant transmission spectra of the optical fiber at positions *a* and *b*, respectively, shown in Fig. 4; (c) – plot (a) magnified near a single resonance; (d) – plot (b) magnified near a single resonance and its theoretical fit.

## 4. Summary and discussion

Thus, a linear variation of an optical fiber radius leads to the appearance of strongly localized WGMs. The developed theory of these modes predicts asymmetric oscillatory behavior of transmission spectrum which allows to determine the local change of fiber radius with a single measurement. Generally, an optical fiber with radius variation $\Delta r(z)$ can host different types of localized WGMs. The considered case $\Delta r(z) = \gamma z$ is related to the situation when the classical motion at the fiber surface is unbounded and, in particular, the circumference $(\varphi, 0)$ is not a geodesic. Alternatively, for characteristic quadratic dependencies of $\Delta r(z) = -(z/z_0)^2$, this circumference is a stable geodesic which supports WGMs that are bounded along the axial direction by two caustics. These WGMs correspond to the conventional Lorentzian spectral resonances [1]. For other fiber profiles, more complex localized structures (e.g., WGM bottles [4]) can be present. It should be noted that the type of a resonance mode described in this paper is not limited to the special case of a constant conical slope. Deformation of the conical shape will result in appearance of other oscillatory spectral dependencies (then, the term $\sim m^3$ in Eqs. (2), (3), and (4) should be replaced by a more general function of $m$ determined by the equations of classical motion on the deformed surface). These more complex cases will be considered elsewhere.

## References

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