Optimum Signal Constellation Design for Rotationally Symmetric Optical Channel with Coherent Detection

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Abstract: We present an optimum signal-constellation design (OSCD) method to obtain the optimum probabilities and mass points by split-step method. The OSCD increases channel capacity by 1.02bits/channel use after 2000 km of SMF.

OCIS codes: (060.4080) Modulation; (060.1660) Coherent communications; (999.9999) Optical channel capacity; (999.9999) Rotationally symmetric optical channel.

1. Introduction

The channel capacity of nonlinear optical fiber channel has been one of the major topics of interest for many researchers in recent years [1-7]. The recent approaches [4-7] use numerical method to solve the nonlinear Schrödinger equation to approximate the probability density function (pdf) for energy of pulse. It was shown in [4,5] that 128-ary based on iterative polarization quantization (IPQ) can achieve the capacity of 5.16 bits/s/Hz per single-polarization for a 2000-km transmission. The results of [4, 5] also show that IPQ outperforms than star quadrature amplitude modulation (sQAM) and square QAM. It was shown in [6,7] that the maximum achievable capacity can be improved by up to 0.5 bit/s/Hz with optimized uni-spaced ring constellations. These studies implied that the optimization on the constellation can achieve a higher channel capacity and improve overall system performance.

In this paper, we present an optimum signal constellation design (OSCD) method to achieve the channel capacity and improve system performance. We study an optical fiber system dominated by amplified spontaneous emission (ASE) noise and assume that the statistical rotational invariance can be applied to the nonlinear optical fiber channel as it has been shown in [6,7]. The first step of OSCD design is to approximate the channel transition pdfs by evaluation of histograms. Then the split-step method is employed to perform optimization over the probabilities and mass points of input distribution to maximize the channel capacity. The OSCD quantizes the optimal input distribution to obtain the required constellations by minimizing the quantization mean square error (QMSE). The numerical results show that the channel capacity of 32-sQAM can be improved by at least 1.02bits/channel use after 2000 km of single mode fiber (SMF). Moreover, the LDPC-coded 32-OSCD allows 1200 km transmission without errors, and through polarization-multiplexing a 400 Gb/s serial optical transmission can be realized.

2. Rotationally symmetric optical channel capacity

We say that the memoryless channel model is rotationally symmetric if the following condition for pdfs holds

$$p(R,\varphi \mid r,\theta) = p(R,\varphi-\alpha \mid r,\theta-\alpha) \quad \alpha \in [-\pi,\pi] \tag{1}$$

where (R,φ) and (r,θ) are the amplitude and phase of output and input signal respectively.

Assuming the input distribution $u(r)$ is rotationally symmetric, the capacity is given by

$$C = \max_u I(u) = \max_u \int u(r)dr \iint p(R,\varphi \mid r,0) \log \frac{p(R,\varphi \mid r,0)}{p(R \mid u)} dR d\varphi \tag{2}$$

where $p(R \mid u) = \frac{1}{2\pi}\int u(r)dr \int p(R,-\theta \mid r,0) dr d\theta$ is the output amplitude distribution.

The average power (AP) and peak power (PP) constraints must be considered simultaneously for fiber-optics channel due to fiber nonlinearities and physical limitations of the transmitter. If we consider the optical channel as a complex Gaussian channel under AP and PP constraints, it was shown in [8-10] that the optimal distributions are discrete amplitude, uniform independent phase (DAUIP). Similarly, we also assume that the optimal distribution for nonlinear optical fiber channel is DAUIP. Accordingly, by using equation (2) we should perform optimization not only over the probabilities but also over the mass points. Unfortunately, because the number of mass points is unknown in advance, equation (2) is difficult to solve. Moreover, the pdf obtained by evaluation of histograms is a discrete function. As a consequence, gradient based optimization methods cannot be directly applied here.

In order to overcome the first difficulty, we perform the optimization over the set of input distributions with a finite number of mass points. The admissible set of input distributions is given by

$$\Omega(u;r) = \left\{ u : \sum_{i=1}^{n} u_i r_i^2 \le P_a ,\ \sum_{i=1}^{n} u_i = 1,\ 0 \le r_i \le \sqrt{P_p} \right\} \tag{3}$$

where P_a is the AP constraint, P_p is the PP constraint, n is the number of mass points, which is selected according to the total number of constellation points, i.e. n = 3, 4 for the 32-ary constellations.

To deal with the second difficulty, the split-step method is employed to iteratively update the probabilities and mass points of input distribution. At step k we have the probabilities $u_{k\text{-}1}$ and mass points $r_{k\text{-}1}$ available, the next u_k and r_k are obtained as follows:

1) update u_k by

$$u_k = \underset{u}{arg\,max} \{ I(u) : u \in \Omega(u; r_{k-1}) \} \tag{4}$$

2) update r_k by

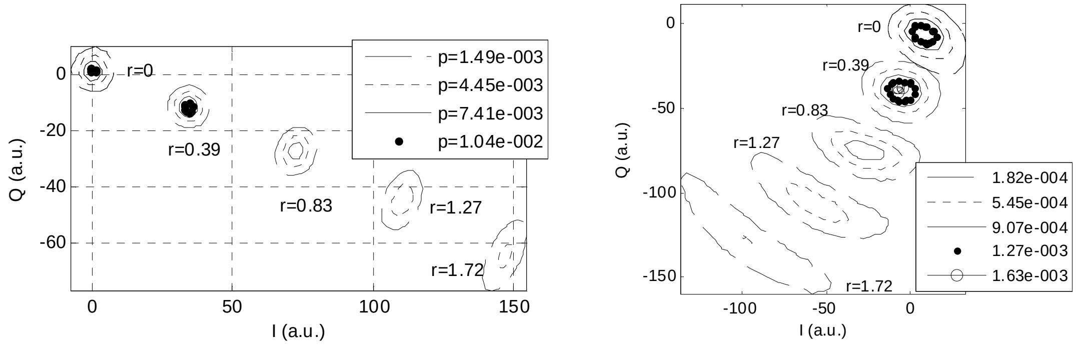
$$r_k = \underset{r}{arg\,max} \{ I(u) : u \in \Omega(u_k; r\ ) \} \tag{5}$$

Due to the concavity of the $I(u_k)$, the equation (4) can be easily solved by gradient based methods. The sequence $\{I(u_k)\}$ is non-decreasing and should converge to the channel capacity.

2. Pdfs of rotationally symmetric optical channel with coherent detection

The optical transmission system under study is similar to the LDPC-coded PM-IPM scheme of [5]. In this work, only one polarization is used. The span length is set to 120 km and each span contains 80 km SMF and 40 km dispersion compensating fiber (DCF). Pre-compensation of 20 km of DCF and corresponding post-compensation are used. The launched power is set to 0 dBm and the EDFA noise figure is 5 dB. The symbol rate is 50 Gb/s and LDPC (8547, 6922, 0.8) is used as channel code. The VPITransmissionMaker is used for channel capacity evaluation and transmission experiments.

We evaluate the pdfs $p(R, \varphi | r, 0)$ of the 32-ary amplitude-shift-keying (ASK), the peak-to-average power ratio (*Rpa*) of which is 2.95. Therefore, we can estimate the pdfs for any given input amplitude by interpolating the pdfs of 32-ary ASK. It should be noticed that the maximum *Rpa* for any constellation should be smaller than 2.95. The contour plots of the pdfs of 32-ary ASK are shown in Fig. 1. It can be seen from the Fig. 1(a) that the pdfs after three spans are similar to the Gaussian distribution. The distortion of the pdfs due to the fiber nonlinearity increases with the input amplitude. It can be seen from the Fig. 1(b) that the distortion of the pdfs becomes more serious after 18 spans than after three spans, which decreases the channel capacity compared to complex Gaussian channel.



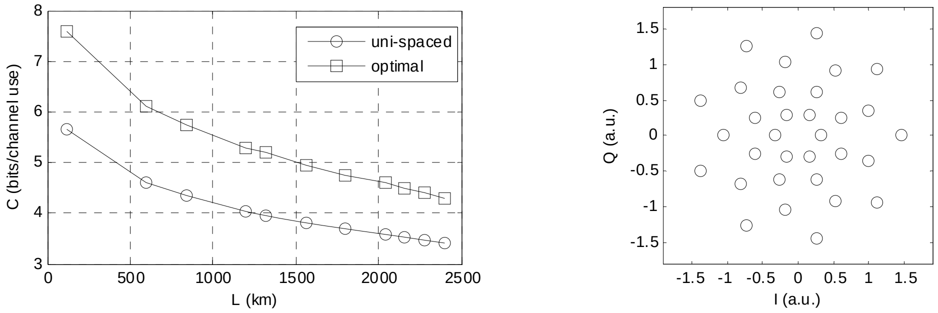
(a) (b)

Fig. 1 Contour plots of the pdfs of 32-ary ASK after: (a) three spans (b) 18 spans.

3. Optimal constellation design and performance analysis

Based on the pdfs of 32-ary ASK, we calculate the optimal distributions and channel capacities by solving equations (4) and (5). We use three-tuple (n, *Rpa*, L) to denote the pdfs after L spans with the specified PP. The optimized channel capacities for (4, 2.13, 1:20) are shown in Fig. 2(a), where 1:20 denotes the number of spans from one to 20. Compared to the achievable information rate of uni-spaced constellations, the improvement in capacity by optimizing the probabilities and mass points is 1.02 bits/channel use after 2000 km of SMF. Moreover, the improvement decreases with the transmission distance. After quantizing the optimal input distribution of (4, 2.13, 18), the optimal constellation (32-ary (4, 2.13, 18)) is shown in Fig. 2(b). It can be seen from the figure that the constellation is more compact than 32-ary sQAM and the PP is smaller than that in 32-ary sQAM. The BERs of 32-

ary sQAM and 32-ary (4, 2.13, 18) are summarized in Fig. 3. The LDPC-coded 32-ary (4, 2.13 18) allows up to 1200 km transmission without any countable errors, and outperforms its sQAM counterpart by 120 km.



(a) (b)

Fig. 2 Optimization results: (a) the optimized channel capacities for (4, 2.13, 1:20) (b) the optimal 32-ary constellation for (4, 2.13, 18).

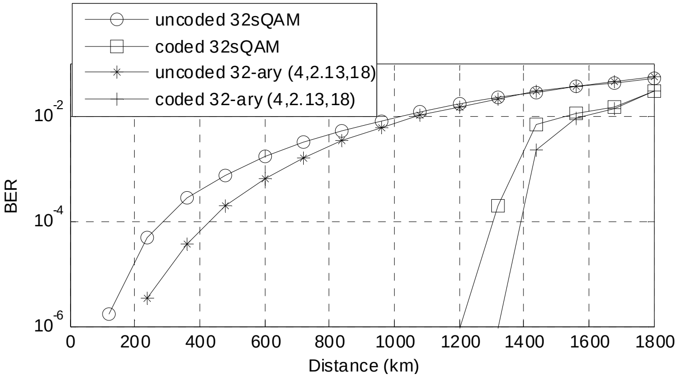


Fig. 3 BERs of optimal constellation for (4, 2.13, 18) against that of 32-sQAM.

4. Concluding remarks

In this paper, we present an optimal constellation design method to achieve the channel capacity and improve system performance. By assuming the rotational symmetry, the numerical complexity for the optimal constellation design is reduced. The numerical results show that the optimal constellation is more power efficient than sQAM and has better performance than sQAM. The proposed method is general and applicable to any rotationally symmetric optical channel. It allows determining the optimum signal constellation and channeling capacity of any particular dispersion map. The numerical results indicate that the channel capacity of 32-sQAM can be improved by at least 1.02 bits/channel use after 2000 km of SMF. The LDPC-coded 32-OSCD, in combination with polarization-multiplexing, allows 1200 km transmission without any errors enabling the 400 Gb/s serial single-carrier optical transmission.

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