Outage calculations for spatially multiplexed fiber links

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Abstract: We calculate multiple-input-multiple-output (MIMO) outage probabilities for optical fiber links employing spatial multiplexing across linearly coupled propagation modes in the presence of mode-dependent loss and distributed optical noise.

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1. Introduction

In order to satisfy the unabated growth of data network traffic, optical communications research has been increasing transport capacities by improving spectral efficiency through techniques such as higher-order modulation or polarization-division multiplexing (PDM). However, experimentally achieved spectral efficiencies are rapidly approaching their fundamental limits for single-mode optical fiber [1], which may result in a transport capacity shortage in the near future [2]. To overcome this 'capacity crunch', techniques that potentially allow for a significant (∼100-fold) scaling in per-cable capacity are needed. With all other physical dimensions (time, frequency, quadrature, polarization) already used in today's systems, exploiting the *spatial* dimension seems to be the only option left. *Space-division multiplexing* (SDM) may be performed using parallel fiber strands, multi-core [3], or multi-mode [4] waveguides, with increasing spatial information density at the cost of (linear and non-linear) mode interactions [5]. These interactions have to be counteracted by MIMO signal processing, originally introduced for wireless systems [6], but also studied for various optical fiber applications [7–11]. These accounts have mostly focused on average SDM capacity gains in standard multi-mode fiber, neglecting the high reliability requirement of transport systems. Here, we examine the potential capacity gain from SDM in few-mode fiber that supports M propagation modes. We consider the impact of distributed optical noise and mode-dependent loss (MDL) on the outage behavior of SDM systems.

2. Space-division multiplexing system model

Neglecting fiber nonlinearities, an SDM system can be represented by the linear matrix channel $\boldsymbol{y} = \sqrt{E_0}\sqrt{L}\mathrm{H}\,x + \boldsymbol{n}$ visualized in Fig. 1(a). The SDM waveguide supports a set of M orthogonal, potentially coupled (spatial and polarization) propagation modes, $M_T \le M$ of which are assumed to be individually excitable by the transmitter, and $M_R \le M$ are assumed to be coherently extractable at the receiver. The average signal energy transmitted per symbol period and *per channel mode* is E_0, hence the total transmit energy across all modes is $M_T E_0$ per symbol. (This definition differs from the constant total power constraint used in wireless MIMO and is motivated by the fact that the optical power per mode is likely a more suitable parameter in fiber-optics [5].) Circularly symmetric complex Gaussian noise $\boldsymbol{n}$ of power N_0 per mode is added at the receiver. The SDM waveguide is described by an $M \times M$ matrix $\widetilde{\mathrm{H}}$, which we normalize

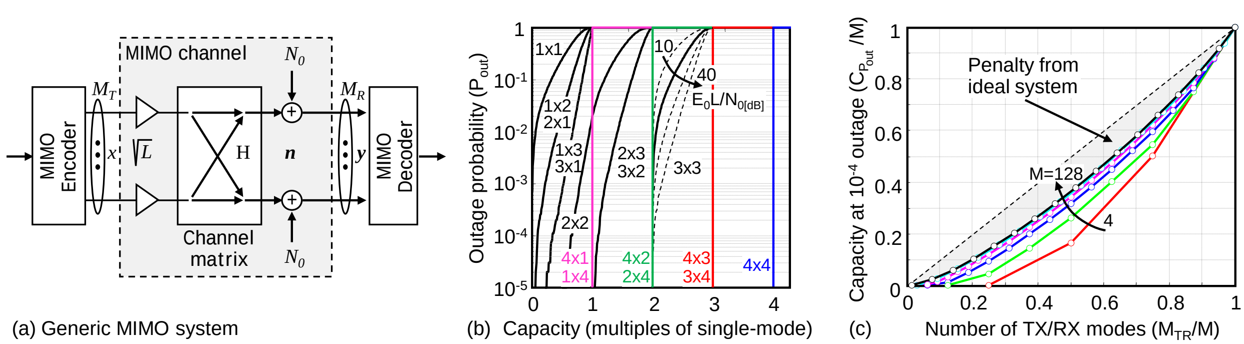

Fig. 1. (a) Basic MIMO system model; (b) MIMO capacity vs. P_{out} for $M = 4$ and different transmit × receive modes; (c) MIMO capacity (relative to its maximum value of M) at $P_{\text{out}} = 10^{-4}$ vs. number of individually addressable modes.

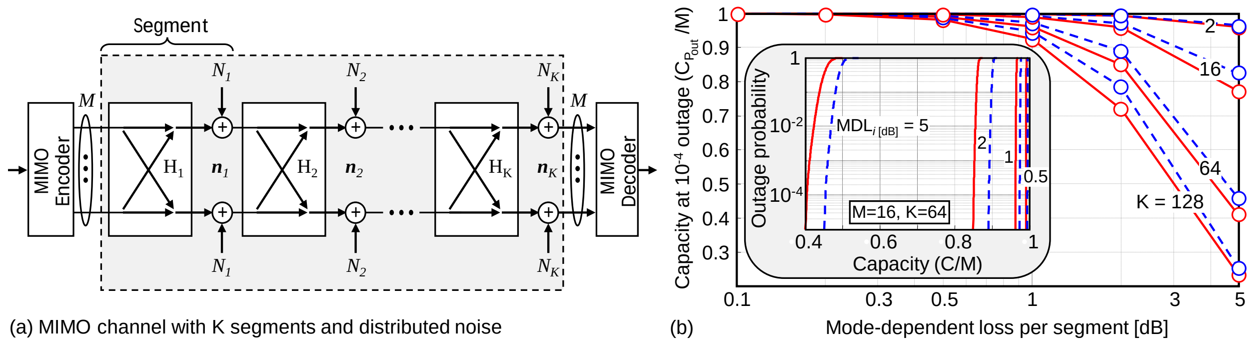

Fig. 2. (a) MIMO channel with distributed noise; (b) MIMO capacity for $M = 16$ at $P_{\text{out}} = 10^{-4}$ vs. per-segment MDL (MDL_i) for different numbers of segments (K) and noise loading at the receiver (solid) and distributed at each segment (dashed). *Inset*: MIMO capacity vs. P_{out} for $M = 16$ and $K = 64$.

to $\text{Trace}\{\widetilde{\text{H}}\widetilde{\text{H}}^\dagger\} = M$ by factoring out the mode-averaged propagation loss L; $\widetilde{\text{H}}^\dagger$ is the conjugate transpose of $\widetilde{\text{H}}$. The MIMO channel is given by the $M_R \times M_T$ matrix H spanning the subspace of $\widetilde{\text{H}}$ addressed by the transponder.

If the channel matrix H is known to the receiver (e.g., through training sequences) but unknown to the transmitter (due to long feedback delays), the MIMO channel capacity for equal launch power per transmitted mode is given by $C = \sum_{i=1}^{r} \log_2(1 + \lambda_i E_0 L/N_0)$, where λ_i are the $r \leq \min\{M_T, M_R\} \leq M$ non-zero eigenvalues of $\text{HH}^\dagger$; if transmission is done on all waveguide modes ($M_T = M$), $E_0 L/N_0$ is the mode-averaged signal-to-noise ratio (SNR) at the receiver.

3. Trade-off between system outage and individually addressable modes

Assuming that channel impairments may couple or attenuate modes, but not to an extent where modes are completely eliminated upon propagation, the channel rank r is $\min\{M_T, M_R\}$. To exploit the full MIMO capacity of an M-mode waveguide, one should hence use M transmitters and M receivers that individually excite and extract all channel modes (or mode combinations forming a complete orthogonal set). This requirement is an obvious extension of a (2×2) PDM system, where both polarizations have to be individually addressed in order to exhaust the full PDM capacity. Using less than M transmitters underutilizes the channel. Using less than M receivers can also lead to *system outage*, when the channel happens to couple transmit modes to propagation modes that are not extracted by the receiver. Figure 1(b) shows outage probability curves for $M_T \times M_R$ addressable transmit $\times$ receive modes on a waveguide supporting $M = 4$ modes. For reasons of hardware simplicity, we address only a single (arbitrary) set of M_T transmit modes, but note that blind, dynamically switched modal excitations, with equal time spent on all $\binom{M}{M_T}$ options, can give better outage results. Our numerical simulations are based on 100,000 random channel realizations assuming uniformly distributed unitary mode coupling among all propagation modes [12], an extension of uniform Poincare sphere coverage in a PDM system. We set $E_0 L/N_0$ to 20 dB, except for the 3×3 case, where it is parameterized as 10, 20, 30, and 40 dB. In contrast to wireless MIMO systems, the assumption of constant per-mode transmit power lets the SDM capacity of this channel be symmetric in M_T and M_R. Deterministic capacities (i.e., step-like outage curves) are only observed for M transmitters or M receivers, with $M \times M$ exhausting the available channel capacity for an M-fold increase over a single-mode system. Capacity loss and system outage have to be accepted when choosing M_T or M_R less then M. Figure 1(c) focuses on the low outage regime applicable to optical transport systems, assuming a 10^{-4} outage probability (or 99.99% availability). The figure shows the achievable SDM capacity as a fraction of the maximum possible value of M times the single-mode capacity for uniformly coupled waveguides supporting $M = 4, 8, 16, 32, 64$, and 128 propagation modes, when transmitter and receiver are able to individually address the $M_T = M_R = M_{TR}$ modes given on the x-axis. The curves converge to a common asymptote for large M. If SDM is implemented on an M-mode fiber but transmitter and receiver are only able to address $M_{TR} < M$ modes, Fig. 1(c) also indicates the (asymptotic) capacity penalty compared to an SDM system that uses the same amount of terminal hardware on an M_{TR}-mode fiber.

4. Distributed noise loading

With reference to Fig. 2(a), we next assume an $M \times M$ system operating on an M-mode waveguide that is composed of K concatenated segments (segment matrices H_i) with optical noise power N_i added at the end of each segment. We assume that optical amplification fully compensates for the mode-averaged propagation loss of each segment (i.e., $L_i = 1$). If all noise sources are statistically independent, we find for the noise correlation matrix at the receiver:

$$\mathrm{R}_{\boldsymbol{n}} = \langle \boldsymbol{n}\boldsymbol{n}^\dagger \rangle = \left(N_1 \mathrm{H}_K \cdots \mathrm{H}_3 \mathrm{H}_2 \mathrm{H}_2^\dagger \mathrm{H}_3^\dagger \cdots \mathrm{H}_K^\dagger + N_2 \mathrm{H}_K \cdots \mathrm{H}_3 \mathrm{H}_3^\dagger \cdots \mathrm{H}_K^\dagger + \cdots + N_{K-1} \mathrm{H}_K \mathrm{H}_K^\dagger + N_K \mathrm{I}_M \right) ;$$

I_M is the $M \times M$ identity matrix. If only modal crosstalk occurs within each fiber segment (*unitary* segments H_i), the correlation equals that of spatio-temporally white noise, $\mathrm{R}_{\boldsymbol{n}} = \sum_{i=1}^{K} N_i \mathrm{I}_M = N_0 \mathrm{I}_M$. Hence, a concatenation of unitary segments with distributed noise has the same capacity as a unitary channel for the same amount of total noise loading.

If the segment matrices are *not unitary*, the channel can be written as $\boldsymbol{y} = \sqrt{E_0}\ \mathrm{H}_0\ x + \mathrm{G}\ \boldsymbol{n}_0$, where $\mathrm{H}_0 = \mathrm{H}_K\ \mathrm{H}_{K-1} \ldots \mathrm{H}_2\ \mathrm{H}_1$ is the overall channel matrix for the signal, and white noise $\boldsymbol{n}_0$ (power N_0) is colored by G such that $\mathrm{R}_{\boldsymbol{n}} = \mathrm{G}\,\mathrm{G}^\dagger$. Placing G^{-1} as a whitening filter inside the receiver leads to the standard MIMO channel of Fig. 1(a) with white noise and channel matrix $\mathrm{H} = \mathrm{G}^{-1}\mathrm{H}_0$. (The matrix G is found from the measured noise correlation $\mathrm{R}_{\boldsymbol{n}} = \mathrm{U}\,\Theta\,\mathrm{U}^\dagger$, where U is a unitary and Θ is a diagonal matrix containing the eigenvalues θ_i of $\mathrm{R}_{\boldsymbol{n}}$. The matrix G is given as $\mathrm{U}\,\Theta^{\frac{1}{2}}\,\mathrm{U}^\dagger$, where $\Theta^{\frac{1}{2}}$ is diagonal with elements $\sqrt{\theta_i}$, and $\mathrm{G}^{-1} = \mathrm{U}\,\Theta^{-\frac{1}{2}}\,\mathrm{U}^\dagger$, where $\Theta^{-\frac{1}{2}}$ is diagonal with elements $1/\sqrt{\theta_i}$.)

5. Capacity statistics for channels with mode-dependent loss (MDL)

We next investigate MIMO capacity statistics for a channel with K segments and M modes, all of which can be addressed by transmitter and receiver. Each segment represents uniformly random, lossless mode coupling in the few-mode transmission fiber, followed by some mode-dependent loss (MDL) induced by discrete components at the end of the segment (e.g., splices or optical amplifiers). Hence, each segment matrix is composed of a random unitary matrix U_i, followed by a random diagonal matrix V_i whose M real-valued, positive elements $\sqrt{v_{kk}}$ satisfy $\sum_{k=1}^{M} v_{kk} = M$. The v_{kk} are drawn from a uniform distribution such that the ratio of maximum to minimum element equals a pre-specified value for the MDL of each segment, which is a parameter in our analyses, $\mathrm{MDL}_i = \max\{v_{kk}\}/\min\{v_{kk}\}$.

Figure 2(b) shows the results of a statistical analysis of a 16-mode SDM channel based on 100,000 realizations for the K segments. The inset gives the achievable MIMO capacity at a certain outage probability, both for noise loading at the receiver (solid) and for distributed noise loading at each segment (dashed) for 64 segments and an SNR of 20 dB. The MDL of each segment is varied from 0.5 to 5 dB. As expected, the achievable MIMO capacity drops with MDL, and distributed noise loading gives slightly better capacities than noise loading at the receiver. Evaluating the MIMO capacity at an outage probability of 10^{-4} as a function of the per-segment MDL for 2, 16, 64, and 128 segments results in the curves shown in the main part of Fig. 2(b). The SDM channel capacity drops with the per-segment MDL as well as with the number of segments, since the MDL of the entire system increases approximately as $\mathrm{MDL}_i^{\sqrt{K}}$. For 128 segments, a per-segment MDL of 1 dB is tolerable to still achieve over 90% of the ideal channel capacity.

6. Conclusions

To exploit the outage-free capacity of a randomly mode coupling SDM system, transmitter and receiver should match the number of individually addressable waveguide modes. Even though mode-dependent loss reduces the SDM capacity, low-outage transmission for per-segment MDLs below 1 dB seems possible.

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