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Research article

Kinetic Constrained Optimization of the Golf Swing Hub Path

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Abstract

This study details an optimization of the golf swing, where the hand path and club angular trajectories are manipulated. The optimization goal was to maximize club head velocity at impact within the interaction kinetic limitations (force, torque, work, and power) of the golfer as determined through the analysis of a typical swing using a two-dimensional dynamic model. The study was applied to four subjects with diverse swing capabilities and styles. It was determined that it is possible for all subjects to increase their club head velocity at impact within their respective kinetic limitations through combined modifications to their respective hand path and club angular trajectories. The manner of the modifications, the degree of velocity improvement, the amount of kinetic reduction, and the associated kinetic limitation quantities were subject dependent. By artificially minimizing selected kinetic inputs within the optimization algorithm, it was possible to identify swing trajectory characteristics that indicated relative kinetic weaknesses of a subject. Practical implications are offered based upon the findings of the study.

Key words: Golf swing biomechanics, hand path, optimization, kinetics, kinematics.

Introduction

The golf swing is one of the most difficult and complex sport motions (Dillman & Lange, 1994). Much research has been applied to the biomechanical analysis of the golf swing in an effort to understand the complex mechanics of the motion to provide a basis for improving performance (Hume et al, 2005).

A natural extension to the basic biomechanical analysis of golf swing mechanics are efforts to identify modifications that could potentially improve the swing beyond its current capabilities. Previous studies have shown that only a small percentage (20.2–26.8%) of the energy developed by the body during the downswing is transferred to the club (Nesbit and Serrano, 2005). This finding suggests that there may be an opportunity to increase this energy transfer and the resulting club head velocity, through some modification of the swing. Lampsa (1975) and later Sharp (2009) applied optimal control theory to double-pendulum models of the swing to identify joint torque profiles that maximized club head velocity for the drive shot. Both found that it was theoretically possible to increase club head velocity through modification of the arm and wrist torque profiles without exceeding the maximum torque capabilities of the subjects. In the case of Lampsa, it was revealed that the required power output of the subject was exceeded suggesting that power limiting should be considered during the search for optimal swing torque profiles (Kaneko and Sato, 1994). Sharp (2009) also presented the development of a triple-pendulum model, where shoulder, elbow, and wrist torque profiles were identified which minimized the difference between model-predicted wrist and club head positions and those obtained from subject data. The torques were then manipulated to maximize club head speed at impact. White (2006) utilized a torque driven double-pendulum model to determine means to improve the energy transfer efficiency from the arms to the club through modifications of the wrist-cock angle, release delay, and wrist torque magnitude. Ultimately, these approaches are limited by the accuracy of the model employed, and logically, cannot account for un-modeled affects. Moreover, it is difficult to provide an assessment of joint torques outside of a motion capture laboratory, thus limiting the ability to translate these results directly to the golfer and the coach. An optimization methodology that focuses primarily on the manipulation of swing trajectories (as opposed to joint torques) may provide a more visual and thus practical means towards helping the golfer improve their swing velocity.

When discussing the kinematics of the golf swing, it is natural to focus on the club head, as club head speed, direction and orientation at impact ultimate dictate the success of a shot (Jorgensen, 1999). However, the only control the golfer is able to influence over the club head is derived from the linear and angular trajectories imposed at the grip. Specifically, the path of the hands, also referred to as the hub path, is the point where the summation of the forces, torques, energy, and momentum developed by the golfer through the various joint and body movements are ultimately transferred to the club. The subtle non-circular nature of the hub path (Figure 1) has been recognized since the early days of golf biomechanical study (Cochran and Stobbs, 1969; Williams, 1966) however, its specific role in the golf swing has been ignored due to the popularity of the double-pendulum models for analyzing swing mechanics which removed this swing characteristic. Recent studies have determined that the non-circular nature of the hub path is a fundamental, yet subject dependent characteristic of the golf swing (Miura 2001; Nesbit and McGinnis, 2009), and that a reduction in radius of curvature nearing impact is indicative of skill (Nesbit 2005, Miura 2001). In addition to the trajectory of the hub path, the golfer is also responsible for controlling the three distinct angular motions of the club throughout the swing (Nesbit, 2005). Specifically, the manner in which the golfer controls the swing plane

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component (alpha component) of the angular motions has an important effect on the golfer/club energy transfers, and ultimately the club head velocity at impact (Jorgensen, 1970; Nesbit, 2005; Pickering and Vickers, 1999; Sprigings and Mackenzie, 2002; White, 2006).

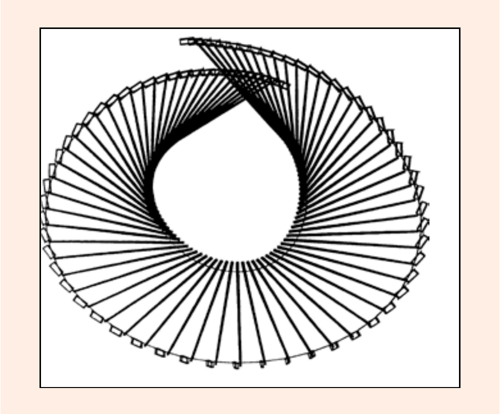

Figure 1. Superimposed golf swing illustrating non-circular hub path. (Inner Black Line Path. Frame Spacing is 0.01 sec)

As evidenced from the above discussions, the hub path and swing angular trajectories are important contributors to generating and maximizing club head velocity at impact. Therefore, the primary objective of this optimization study is to identify golfer-specific hub path trajectories (linear position and derivatives) and swing angular trajectories (angular position and derivatives) which maximize the club head velocity at impact while constraining the golfer kinetic inputs (force, torque, work, and power) within the empirical limits for each golfer. An important aspect of this study is to avoid the same model-based simplifications that limited previous optimization studies. A secondary objective of the optimization is to determine the most efficient hub paths and swing trajectories that minimize a specific kinetic input while maintaining the original club head velocity at impact. A possible outcome of the secondary objective is to identify particular kinematic actions that suggest specific kinetic weaknesses. Such information may prove useful for visually identifying limiting factors in a golfer's ability to generate club head velocity, and could provide insight into possible methods of improvement.

Methods

Subjects and testing protocol

Four amateur golfers, three males and the one female had their golf swings analyzed for this study. All subjects are right-handed and their relevant data are given in Table 1. A diversity of skill levels and swing styles was the criteria for selecting these subjects in an attempt to yield a range of results (Nesbit and Serrano, 2005; Nesbit, 2005). Stylistically, the male scratch and male 5H subjects had aggressive, powerful, and quick swings, whereas the male 13H and female 18H subjects had smoother, longer, and slower swings. All subjects used the same club (driver of length = 1.092 m; mass = 0.382 kg; cg location from top of club = 0.661 m; I_{CG} = 0.07104 kg.m^2). All subjects were informed of the purposes of the study, and gave written consent for the following testing procedures, and the use of their data for research purposes, in accordance with local IRB requirements. A rigid triad of passive reflective markers was attached to the club near the bottom of the grip. The three-dimensional paths of these markers were tracked at 200 Hz using an 8-camera motion capture system (Motion Analysis Corporation, Santa Rosa, CA, USA). The system was calibrated until the combined 3D residual for all cameras was under 1.00 mm (Test/retest of static marker locations varied by less than 0.20 mm for a given calibration) prior to testing. Subjects were asked to execute a series of swings that included hitting a ball into a net after being advised to swing the club in a manner similar to hitting a driver in a competitive situation where distance and accuracy were both important. The subjects were instructed to practice swinging the club as many times as necessary until they became comfortable with the testing situation and felt they could swing "normally" and consistently. Subsequently, a minimum of eight swings from each subject were recorded and tracked then presented to the subjects for their review. It was found that the club head velocities were consistent among the acceptable trials within a maximum range of 5% for all subjects. The subjects each selected what they considered to be representative swings in terms of club head velocity, impact feel, partial flight of the ball, and overall visual assessment of the motion capture data. One of the self-selected swings from each subject was then analyzed for this study. This manner of conducting trials and selecting swings for subsequent analyses is consistent with previous studies (Nesbit and Serrano, 2005; Nesbit, 2005; Nesbit and McGinnis, 2009).

Forward kinematics model

The free-body-diagram of the golf club model is shown in Figure 2. The club model with representative mass and inertia properties, constrained the swing to one non-moving vertical plane (Coleman and Anderson, 2007), ignored rotations about the club shaft, and treated the shaft as rigid. These simplifications are consistent with many biomechanical models of the golf swing (Budney and Bellow, 1979; 1982; Cochran and Stobbs, 1969; Jorgensen, 1970; Lampsa, 1975; Neal and Wilson, 1985; Vaughn, 1981; Williams, 1966) with the exception that this model did not constrain the hub path to follow a constant radius circular arc. This model included the primary

Table 1. Subject data.

Subject	Age	Height (m)	Weight (kg)	Handicap	Experience (yrs)	Round per Year
1 (Male)	42	1.83	86.3	0 (scratch)	24	150
2 (Male)	35	1.79	93.1	5	20	100
3 (Male)	21	1.88	74.9	13	7	120
4 (Fem)	31	1.70	59.0	18	11	50

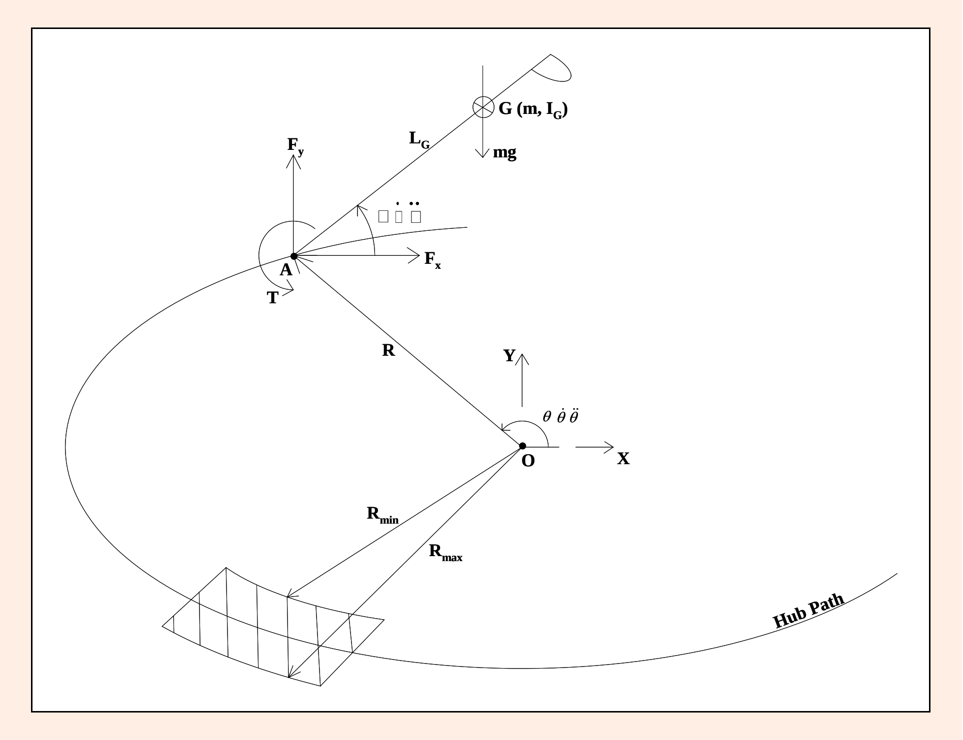

Figure 2. **Planar free-body-diagram of club model.**

kinematic and kinetic parameters responsible for affecting club head velocity, and the golfer/club interacting forces, torques, and energy transfers (Nesbit, 2005).

The *X*-*Y* coordinate system illustrated in Figure 2 is in the plane of the swing and fixed to the ground (global coordinate system). The position of the grip along the hub path, relative to the global coordinate system, is described by the radial coordinate *R* and the transverse coordinate *θ*.

The following scalar equations of motion were developed from Figure 2:

$$F_X = MA_{GX} \quad (1)$$

$$F_Y - Mg = MA_{GY} \quad (2)$$

$$T + F_X L_G \sin\gamma - F_Y L_G \cos\gamma = I_G \ddot{\gamma} \quad (3)$$

where F_X and F_Y are the X and Y components of the applied linear force, M is the mass of the club, A_{GX} and A_{GY} are the X and Y components of the acceleration of the club mass center (G), g is the acceleration of gravity, T is the applied swing torque, L_G is the distance from the grip point to the club mass center, I_G is moment of inertia of the club about the mass center, and γ, $\dot{\gamma}$, and $\ddot{\gamma}$ are the angular position, velocity, and acceleration of the club respectively.

The acceleration of the club mass center is given by the following relative acceleration vector expression:

$$\vec{A}_G = \vec{A}_A + \vec{A}_{G/A} \quad (4)$$

This vector equation yielded the following scalar equations in the *X* and *Y* directions:

$$A_{GX} = A_{AX} - \dot{\gamma}^2 L_G \cos\gamma - \ddot{\gamma} L_G \sin\gamma \quad (5)$$

$$A_{GY} = A_{AY} - \dot{\gamma}^2 L_G \sin\gamma + \ddot{\gamma} L_G \cos\gamma \quad (6)$$

Data to kinematically drive the model were obtained from the subject-selected golf swings as described previously. Two of the reflective markers attached to the club were aligned with the long axis of the shaft, and the third was offset perpendicular to the shaft and parallel to the club face. The three-dimensional marker triad paths were recorded then smoothed via a Butterworth low-pass filter (6 hz), and processed to yield global body *1-2-3* angular motions of the club and the three-dimensional global positions of the hub path using methods described in Nesbit (2005). The orientation of the swing plane (*X*-*Y* plane) during the downswing was established from the angular motion data (Coleman and Rankin, 2005). The global position data of the hub path and global orientation of the club were mapped onto this plane using methods described in Kane et al (1983) to yield the *X* and *Y* position data of the hub path and the swing angle of the club (γ) within the swing plane.

Numerical differentiation of the swing plane linear and angular position data yielded the linear acceleration of the hub path (A_{AX} and A_{AY}), and angular velocity and acceleration of the club $(\dot{\gamma} \text{ and } \ddot{\gamma})$ (Dean and Nesbit, 1988). From this information, A_{GX} and A_{GY} were completely specified from Equations (5) and (6) for the duration of the swing. With the accelerations of the club specified, Equations (1) through (3) were solved to determine the time histories of the applied linear force (F_X and F_Y) and torque (*T*).

From the linear force components and swing torque at the club handle, the total work done by the golfer on the club was determined from the following:

$$Work = \sum_{i=\gamma_o}^{\gamma_f} F_{Xi}(\Delta X_i) + F_{Yi}(\Delta Y_i) + T_i(\Delta\gamma_i) \quad (7)$$

Where the linear and angular portions of the total work are given by:

$$Work_{Linear} = \sum_{i=\gamma_o}^{\gamma_f} F_{Xi}(\Delta X_i) + F_{Yi}(\Delta Y_i) \tag{8}$$

$$Work_{Angular} = \sum_{i=\gamma_o}^{\gamma_f} T_i(\Delta\gamma_i) \tag{9}$$

Where i indicates the value of the quantity at point i in the hub path, and the Δ *function indicates a change in the associated quantity from hub point i to i + 1. The total, linear, and angular power were determined by numerically differentiating the work expressions of Equations (7) through (9).*

Solving the dynamic model yielded the kinematic and kinetic profiles during the downswing of each subject. From the kinetic profiles of force, torque, linear and angular work, and linear and angular power, the maximum values (and when they occurred) were identified and assumed to represent the capacity of the subject (with the exception of work which is a cumulative quantity).

Parameterized hub path and swing angle kinematic models

Referring to Figure 2, the kinematic components of the swing manipulated by the optimization algorithm were the hub path radial and transverse coordinates (*R* and θ), and club swing angle (γ) during the downswing. These variables were required to be modeled in a form that yielded accurate representations of the original subject swings, while containing sufficient variability to allow for wide ranging, yet reasonable modifications for optimization. To this end, the transverse coordinate of the hub path (θ) was represented with the following 4° polynomial

$$\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 \tag{10}$$

The first and second time derivatives yielded the transverse angular velocity and acceleration of the hub path per

$$\dot{\theta}(t) = C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 \tag{11}$$

$$\ddot{\theta}(t) = 2C_2 + 6C_3 t + 12C_4 t^2 \tag{12}$$

The angular velocity (11) and acceleration (12) of the hands relative to the hub path were derived via differentiation of (10), and therefore were also functions of the unknown coefficients and thus may be greater or less than presented by the subject. Equations (10)-(12) included five unknown constants C_0 through C_4. To define these constants, we considered (kinematic) boundary conditions at the initiation of the downswing and impact. Specifically, the start of the downswing was defined as $t = 0$ seconds, when the hands and club were assumed to have zero velocity, and the transverse position and acceleration of the grip are θ_0 and $\ddot{\theta}_0$ respectively. Impact occurred at $t = t_f$ seconds, when the transverse position and acceleration of the hands were $\theta_f = 270^\circ$ and $\ddot{\theta}_f$ respectively. Substitution of these boundary conditions into (10)-(12) yielded expressions for the constants C_0 through C_4 per

$$C_0 = \theta_0 \tag{13}$$

$$C_1 = 0 \tag{14}$$

$$C_2 = \frac{\ddot{\theta}_0}{2} \tag{15}$$

$$C_3 = \frac{18\pi - 12\theta_0 - 5t_f^2\,\ddot{\theta}_0 - t_f^2\,\ddot{\theta}_f}{11t_f^3} \tag{16}$$

$$C_4 = \frac{-3\pi + 2\theta_0 - t_f^2\,\ddot{\theta}_0 + 2t_f^2\,\ddot{\theta}_f}{22t_f^3} \tag{17}$$

With the substitution of (13)-(17) into (10)-(12), the transverse position of the grip and its derivatives were parameterized completely by the boundary conditions t_f, θ_0, $\ddot{\theta}_0$, and $\ddot{\theta}_f$. (See Appendix Figures A1 through A3 for example curves generated by this algorithm.)

A parallel development was applied to the club swing angle (γ), where a 4° polynomial was again used to specify the angular position of the club as a function of time per

$$\gamma(t) = C_5 + C_6 t + C_7 t^2 + C_8 t^3 + C_9 t^4 \tag{18}$$

where the unknown coefficients $C_5 - C_9$ *were determined by considering the kinematic behavior of the club at the initiation of the downswing and impact. Specifically, at the start of the downswing the club has zero velocity and the angular position and acceleration are* γ_0 *and* $\ddot{\gamma}_0$ *respectively, while at impact the club has angular position and acceleration of* $\gamma_f = 270^\circ$ *and* $\ddot{\gamma}_f$ *respectively. Following a similar approach to that taken for* $\theta(t)$ *,we parameterized* $\gamma(t)$ *with the four unknown boundary conditions* t_f, γ_0, $\ddot{\gamma}_0$, and $\ddot{\gamma}_f$.

Finally, the radial position of the grip point along the hub path (see Figure 2) was specified as a function of the transverse angle *θ,* by a 6° polynomial curve fit to radial coordinates (*R values*) specified from the hub path at seven equally spaced angular positions (θ) from θ_0 to $\theta_f = 270^\circ$ during the downswing by

$$R(\theta) = k_0 + k_1\theta + k_2\theta^2 + k_3\theta^3 + k_4\theta^4 + k_5\theta^5 + k_6\theta^6 \tag{19}$$

where $k_0 - k_6$ are unknown curve-fit constants determined using standard methods once $R_1 - R_7$ was specified. Here the path of the grip point was parameterized completely by the radial boundary conditions $R_1 - R_7$.

Optimization algorithm

These parameterized models of the hub path radial and transverse coordinates (*R* and θ: Eqns. 19 and 10 respectively), and club angle (γ: Eqn. 18) fully described the trajectory and resulting kinematics of the club during the downswing, and yielded the following fourteen independent parameters for manipulation by the optimization algorithm:

- R_1- R_7 radial positions of grip point within the grip point hub path.
- t_f duration of downswing.
- θ_0 angular position of hub path at initiation of downswing.
- γ_0 angular position of the wrist at start of downswing.
- $\ddot{\theta}_0$ *and* $\ddot{\theta}_f$ initial and final angular acceleration of hub path.
- $\ddot{\gamma}_0$ *and* $\ddot{\gamma}_f$ initial and final angular acceleration of swing wrist angle.

These optimization parameters were initially specified based upon values obtained from a subject's recorded swing (nominal values) which recreated the original swing, and yielded the same associated kinematic and kinetic values. They were subsequently independently manipulated within reasonable limits adjacent to their nominal values to optimize the swing trajectory with the following primary and secondary goals.

The primary optimization goal was to maximize the club head velocity at impact within the kinetic constraints of maximum force, torque, and linear and angular power as determined for the subjects via the analyses of their original swings. This (maximize velocity) optimization was applied to all four subjects. The secondary optimization goal was to minimize required maximum force, torque, work, and power while maintaining the original maximum club head velocity. This (kinetic minimization) optimization was applied to subject 1 only.

The club model and optimization algorithms were implemented in MATLAB (The MathWorks, Inc.). For both optimization goals, the fourteen optimization parameters were varied exhaustively via nested loops using relatively fine increments. This method, while computationally inefficient, avoided the possibility of converging to local optima. For each swing iteration (for a given set of the optimization parameters), the geometry of the hub path (Eqns. (10) and (18)) and swing trajectory (Eqn. (19)) were specified, and the resulting maximum force, torque, linear and angular work, linear and angular power, and club head velocity were determined from the equations of motion. If any of the kinetic quantities for a given optimization trial exceeded the subject maximum (limiting) values, then that iteration was discarded. If an optimization trial did not exceed any of the subject kinetic limitations, and resulted in an improvement in the optimization goal quantity, then the parameters of that iteration were captured and it became the new standard for comparison. A sample iteration trial is shown in Figure 3 which illustrates the downswing through impact in increments of 0.003 seconds. Outputs include the count (or trial number), the maximum club head velocity, force, torque, total work, and total power, and the ratios of the max force, torque, linear and angular work, and linear and angular power to the maximum values for the subject.

Results

For each subject, the maximum kinematic and kinetic quantities occurring during the original (down)swing are provided in Table 2. The time of occurrence relative to impact ($t = 0$ seconds) is given in parentheses. The original swing hubs for each of the subjects are shown in Figure 4, while the original angular position profiles of

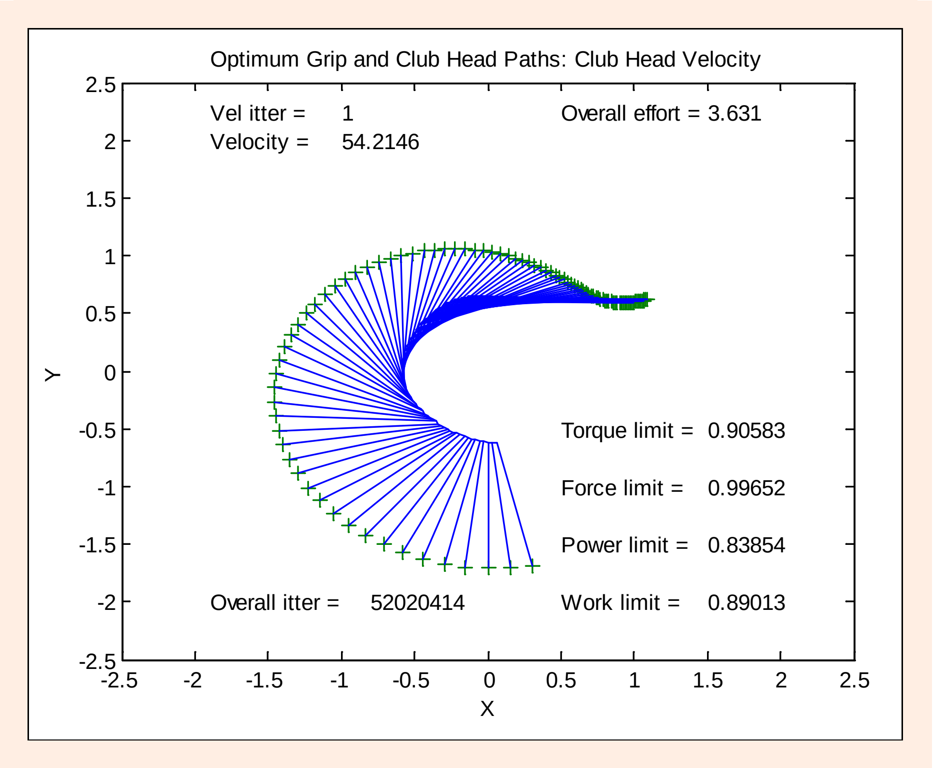

Figure 3. **Sample swing trajectory with associated kinetic maxima and ratios.**

Table 2. Selected subject data during downswing.

Data Type	Units	Sub #1	Sub #2	Sub #3	Sub #4
Club Head Vel	m/s	51.5	48.7	46.8	42.3
(time of max)	(s)	(0)	(0)	(0)	(0)
Swing Torque	N.m	45.1	38.8	26.2	23.5
(time of max)	(s)	(-0.08)	(-0.09)	(-0.11)	(-0.19)
Mag of Force	N	512	453	390	304
(time of max)	(s)	(0)	(0)	(0)	(0)
Total Work	N.m	355	309	288	235
(time of max)	(s)	(-0.01)	(-0.02)	(0)	(0)
Linear Work	N.m	206	185	140	124
(time of max)	(s)	(0)	(-0.01)	(0)	(0)
Angular Work	N.m	146	134	158	121
(time of max)	(s)	(-0.02)	(-0.02)	(-0.03)	(-0.05)
Total Power	N.m/s	3875	3005	2310	1720
(time of max)	(s)	(-0.05)	(-0.06)	(-0.06)	(-0.07)
Linear Power	N.m/s	2775	2316	1402	1188
(time of max)	(s)	(-0.04)	(-0.03)	(-0.03)	(-0.02)
Angular Power	N.m/s	1150	890	1078	698
(time of max)	(s)	(-0.05)	(-0.04)	(-0.06)	(-0.05)

the club (γ) for each subject are given in Figure 5. Note that some of the data of Table 2 have been previously presented for these subjects (Nesbit and McGinnis, 2009; Nesbit and Serrano, 2005). Some quantities are slightly different from that previously reported due to refinements in the dynamic model, adjustments for subject specific grip point location, and interpretation of the precise time of impact.

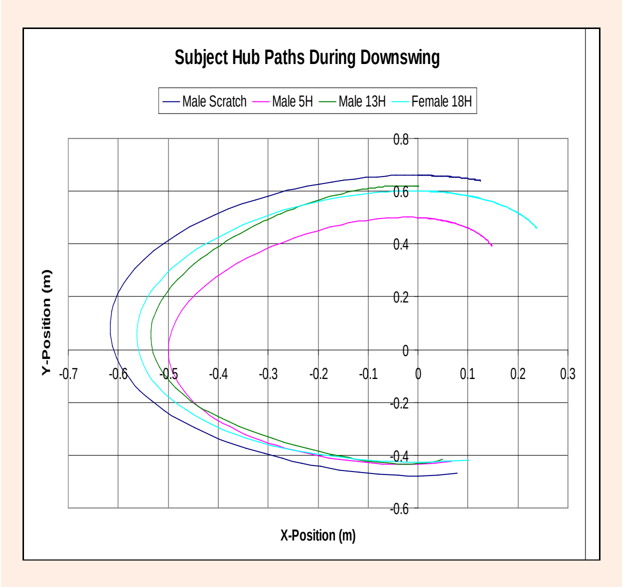

Figure 4. Superimposed subject hub paths during downswing.

The results of the primary (maximize velocity) optimization for all subjects are given in Table 3. The results for all quantities are the maximum values occurring during the downswing, and are given as a percentage of the original swing maximum value (Table 2). The time of occurrence for each quantity is also given. The time ratio quantity is the duration of the optimized downswing relative to the duration of the original downswing. The resulting hub paths and swing angular trajectories during the downswing for this optimization, for the four subjects, are given in Figures 6 and 7.

Figure 5. Superimposed subject angular position of club during downswing.

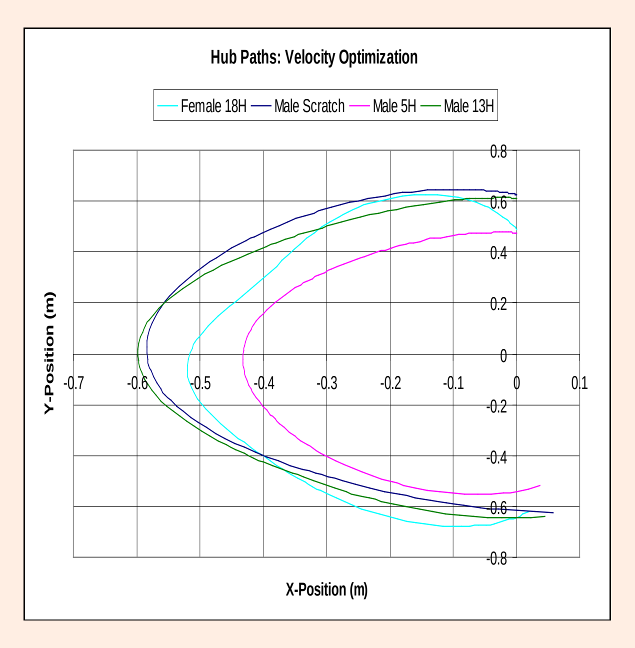

Figure 6. Hub paths for the club head velocity optimization for the subjects.

Table 3. Velocity optimization data for the subjects.

Velocity Optimization	Subject #1		Subject #2		Subject #3		Subject #4	
Data Type	**Time**	**Percent**	**Time**	**Percent**	**Time**	**Percent**	**Time**	**Percent**
Club Head Vel	.00	104.40	.00	106.68	.00	106.20	.00	107.21
Max Torque	-.15	90.58	-.15	97.53	-.12	98.41	-.14	99.33
Max Force	.00	99.65	.00	99.96	.00	99.76	.00	93.88
Total Work	.00	109.01	.00	106.24	.00	111.03	.00	113.45
Max Lin Work	.00	111.31	.00	108.65	.00	112.53	.00	95.53
Max Ang Work	.00	85.72	-.03	82.50	.00	89.22	.00	92.07
Peak Power	-.04	83.85	-.07	85.63	-.06	94.10	-.07	98.96
Peak Lin Power	-.05	95.16	-.06	80.19	-.05	99.59	-.06	98.23
Peak Ang Power	-.05	90.60	-.10	84.41	-.08	95.70	-.13	97.08
Time Ratio	---	1.07	---	1.11	---	1.02	---	.98

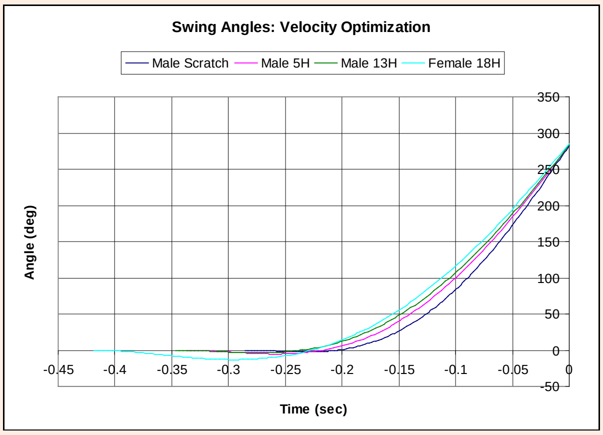

Figure 7. Swing angular trajectories for the club head velocity optimization for the subjects.

The result of the secondary (kinetic minimization) optimization for subject 1 is given in Table 4 and has the same format as Table 3. The optimized hub paths and swing angular trajectories for the various kinetic quantities during the downswing for subject 1 are shown in Figures 8 and 9. Graphical representations of MATLAB generated optimized swing trajectories for this subject are shown in the Appendix (Figures B1 through B4). In addition, secondary optimized hub paths for all subjects are shown in Figures C1 through C4 in the Appendix. These graphs are included for additional comparative purposes, however they do not inform the discussions relative to this optimization for the sake of brevity.

Discussion

Original subject swings

The kinematic and kinetic data presented in Table 2 compare well with previously reported values for all quantities (Budney and Bellow, 1979; 1982; Cochran and Stobbs, 1969; Jorgensen, 1970; 1999; Nesbit and Serrano, 2005; Williams, 1966; Vaughn, 1981). The four subjects present diverse swing kinematic and kinetic characteristics (see Table 2 and Figures 4 and 5). The range of the kinematic and kinetic quantities among the subjects is considerable and quantitatively emphasize the differences in their swing mechanics and club trajectories, a finding supported in Nesbit (2005). Of note is the individual nature of the swing hub path (Figure 4), and the profiles of the swing angle of the club (Figure 5). Thus the goal of analyzing a diverse set of swing styles from a mechanics point of view was achieved.

Relative to the original hub paths, differences are noted in the amount of vertical and horizontal ranges-of-motion, the radius-of-curvature profiles of the path, and the point in the downswing when the path changes direction. In general, the radius-of-curvature of the hub path was initially large (0.63–0.83 m) at the initiation of the downswing. From this local maximum value, it reduced steadily to a local minimum (0.40–0.50 m) near the midpoint in the downswing. From this point until near 10 degrees before impact, the radius increases to a second

Table 4. Secondary optimization data for subject 1.

Kinetic Optimization	Kinetic Quantity (% of original)							
Data Type	**Torque**	**Time**	**Force**	**Time**	**Work**	**Time**	**Power**	**Time**
Club Head Vel	99.65	0.00	99.38	0.00	101.42	-0.01	100.38	0.00
Max Torque	**78.36**	-0.24	99.63	-0.16	89.14	-0.14	87.96	-0.15
Max Force	99.08	0.00	**86.41**	0.00	92.17	-0.01	88.76	-0.02
Total Work	98.63	0.00	100.42	0.00	**95.03**	-0.01	97.68	0.00
Max Lin Work	108.83	0.00	94.71	-0.01	89.94	0.00	96.64	0.00
Max Ang Work	87.15	0.00	105.87	0.00	92.97	-0.06	88.35	0.00
Peak Power	92.65	-0.05	96.52	-0.04	84.33	-0.06	**89.34**	-0.04
Peak Lin Power	94.77	-0.05	77.16	-0.06	84.78	-0.06	86.07	-0.05
Peak Ang Power	37.73	-0.05	99.12	-0.05	83.14	-0.09	97.70	-0.06
Time Ratio	1.03	---	1.02	---	1.12	---	1.15	---

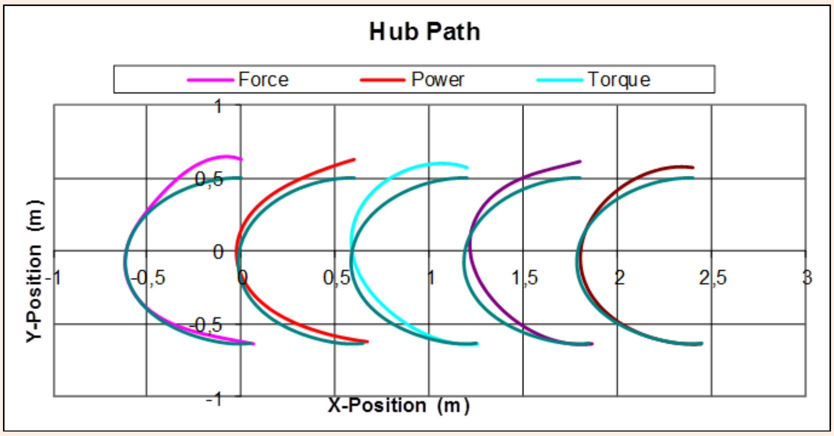

Figure 8. **Comparison of kinetic optimized hub path to actual hub paths for subject 1.**

local maximum (0.54-0.96 m). During the remainder of the downswing, the radius decreases sharply.

Relative to the swing angle profiles, subjects 1, 2, and 3 present nearly bi-linear profiles with nearly equal ranges-of-angular motion. The club is near the horizontal position in the downswing for these three subjects when the slopes change, indicating the onset of a more rapid outward movement of the club. On the other hand, subject 4 presents a more constant linear profile indicative of a uniform outward movement of the club during the downswing, and a smaller range-of-motion.

For all subjects, peak velocity occurs at impact. Relative to the kinetic inputs, torque peaks first in the downswing when the club is near the horizontal position for subjects 1, 2, and 3, and about 60 degrees before impact for subject 4. The total power peaked next at about 45 degrees before impact for all subjects. The angular power component peaked slightly before the linear power component. Force and total work peaked at or near impact. The angular work component peaked just prior to impact, and the linear work component peaked at impact.

The relative smoothness/aggressiveness of these subjects' swing style seems to be reflected in the relative magnitudes of the peak swing torque and linear power measures. The differences among the subjects are much greater than would be expected based upon peak club head velocities. For subjects 1 and 2, identified as having aggressive swing styles, these quantities are nearly 1.5 to 2.0 times higher than for subjects 3 and 4, the subjects with the smoother swing style. This kinetic assessment of swing style is only relevant to this study.

It is evident that the magnitudes of all the kinetic quantities are somewhat related to maximum club head velocity, however they do not scale directly with the exception of total work (to velocity squared). This is an expected result predicted by Newton's Laws, however the manner in which a subject generates and transfers this total work to the club is a complex combination of an individual's force and torque strength capacities, their respective linear and angular ranges-of-motion, and their ability to maintain high values of interaction forces and torques as the velocity of the swing increases. Thus, there appears to be several viable kinetic pathways to achieving club head velocity. Since the club is driven and controlled by these kinetic inputs of force, work, and power, the resulting geometry of the hub path, and the kinematics of

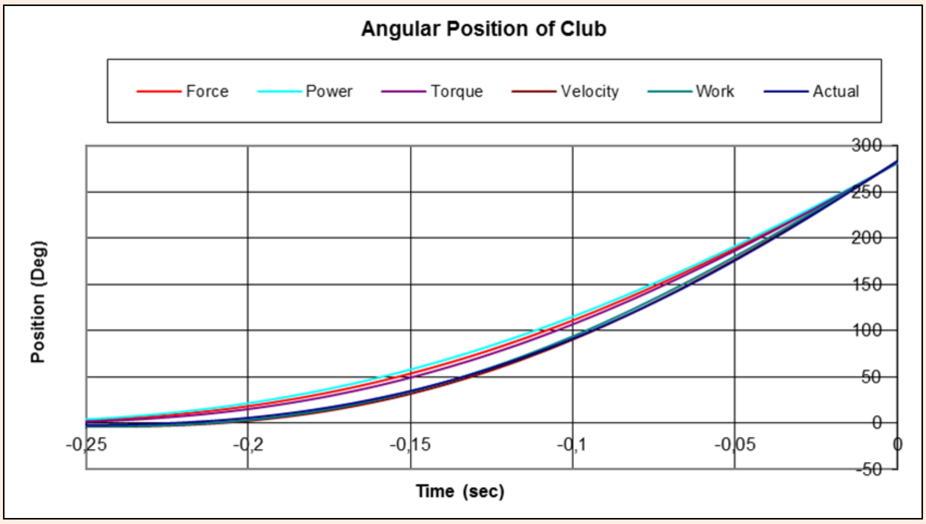

Figure 9. **Kinetic optimized swing trajectories for subject 1.**

the swing angle reflects the complexity and individuality of these kinetic inputs. Thus one would surmise that there in not one ideal geometry of the hub path, or kinematic profile of the swing angle that would yield the highest club head velocity, but several possibilities that would reflect the kinetic capabilities of the individual. The results of the optimization analyses support this supposition.

Primary (maximize velocity) optimization

Referring to Table 3, it appears that increases in maximum club head velocity are possible through modification of the hub path geometry and angular swing profile for all subjects within the kinetic limitations of each subject. The manner of the modification, the eventual limiting kinetic parameter, and the possible degree of improvement are subject dependent. In general the optimization identified the following as beneficial for potentially increasing club head velocity; a slightly longer duration of the downswing, an increased vertical range-of-motion of the hub path, a reduced initial outward movement of the club, and a smoother (reduced peak acceleration) progression to impact. In several cases, the peak kinetic values occurred sooner in the downswing, and remained higher (and more consistent) during a longer portion of the swing. The result was that for all four subjects the modified swings resulted in higher overall total work.

For subject 1, the most skilled subject of the group and one of the subjects identified as having an aggressive swing style, the optimization analysis predicted that a potential 4.40% increase in club head velocity was possible. Relative to the original hub path, the modified hub path has a higher initial radius (flatter profile) which reduces during the first half of the downswing similarly to the original path. Both the modified and original hub paths reach relative minimum radius values (sharper profile) when the hands are near the halfway point in the downswing, although the modified path reaches this point sooner than the original, and the minimum value is not as low as the original hub path. From this point until the hands are at the 7:00 position in the swing hub, the radius of both hub paths increase markedly. The optimized path reaches a higher maximum radius value than the original swing at this point in the downswing, and it does so sooner than the original. From this point until impact, both the optimized hub path and original hub path exhibit a large reduction in radius, however this reduction is much more pronounced in the original swing. There appears to be a trade-off occurring near impact where the rapid reduction in hub radius that is evident in the original swing done to facilitate the transfer of energy to the club (Miura, 2001), is sacrificed in an effort to control the linear force by increasing the radius thus reducing the dominant centrifugal loading at this point in the downswing. As a result the velocity profile of the optimized swing reveals a lower tangential acceleration near impact compared to the original swing. The optimized wrist angular trajectory for this subject, though closely following his original trajectory, delays the angular movement of the club relative to the hub path compared to the original swing. The resulting club head velocity profile during the downswing starts slower, equals the original value when the club is near horizontal, then experiences a greater acceleration until impact resulting in a higher overall club head velocity.

Kinetically, the optimized swing is limited by the max linear force for this subject. The remaining kinetic quantities are lower than the original implying that the optimized hub path and swing profile together require less instantaneous effort from this golfer to execute than the original swing. The timing of maximum values are similar between the original and optimal swings with the exception of the swing torque. This subject was able to generate considerably higher linear and overall work, while generating lower angular work. This subject who had the highest skill level and greatest club head velocity among the subjects, had the lowest potential increase in club head velocity, and was generally nearest to his kinetic limitations in most categories compared to the other subjects. This finding may imply that this subject was nearest to the maximum club head velocity he could achieve given his kinetic capabilities and swing style.

The other subjects had greater potential to increase their club head velocities and associated total work, though the required degree of modification to the swing hub and wrist trajectories were more pronounced. The limiting parameter was linear force for subjects 2 and 3, and wrist torque for subject 4. The general modifications relative to the original swings were similar to those for subject 1 and included initially slowing and delaying the wrist angular motion, increasing the hub path radius at beginning and end of the downswing, reaching the first local radii minimum sooner in the downswing, and increasing the midpoint hub radius. It was found that increasing the radius of the hub path near impact potentially benefits subjects 1, 2, and 3 the most due to the limiting parameter of linear force. Reaching the maximum wrist torque sooner in the downswing was indicated for all subjects, and maximum (linear and angular) power for subjects 2, 3, and 4. For the other kinetic quantities, the timing of the optimized swing maximums occurred fairly close to when they occurred for the subject swings though there were exceptions.

The kinetic quantities that seem to indicate swing style are power (and components), and the duration of the downswing. For the optimized swings, the power quantities were lower relative to their respective maximums for the aggressive swing style (subjects 1 and 2) than for the subjects with the smoother style (subjects 3 and 4). In addition, the optimized swings were much longer for the aggressive style relative to the smooth style. These findings would imply a smoothing of the aggressive swing style, and conversely more aggression from the smooth swing subjects to be beneficial.

Secondary (kinetic minimization) optimization

Referring to Table 4, it appears possible for subject 1 to considerably reduce the targeted maximum kinetic input value while maintaining the original club head velocity, and remaining at or below maximum for all other ‘non-targeted’ kinetic inputs. These secondary optimizations were accomplished through the coordinated modification of both the hub path geometry, and angular swing trajectory. The manner of the modifications and the associated limiting kinetic quantity, are strongly dependent upon the kinetic quantity being minimized. For all optimizations, the vertical range-of-motion of the hub path was increased, and the duration of the swing was longer, in some cases markedly. Of possible practical interest is that these optimized swings visually present how a subject may compensate for a relative lack of a particular kinetic capability thus providing teachers with visual clues to particular kinetic weaknesses of subjects.

The swing torque optimization yielded the largest reduction in the targeted kinetic quantity. This finding may be due to the fact that this subject's swing appears to be dominated by the linear kinetic quantities. The profile of the optimized swing torque curve (not shown) is more trapezoidal in shape compared to the original profile and yielded a similar area under the curve as reflected in the relative value of the angular work. The timing of this maximum torque occurred much sooner in the downswing compared to the original torque profile. The limiting kinetic quantity was linear force, and the subject overcame the reduction in torque by doing more linear work. The hub path geometry was modified by increasing the vertical range-of-motion, decreasing the radius during the initial portion of the downswing, and then increasing it during the mid and lower portions of the downswing. The radius at the bottom of the downswing is slightly less than the original path at that point. This torque optimized hub path deviated the most compared to the original hub path relative to the other kinetic optimizations. Its geometry effectively controlled the outward movement of the club primary with centrifugal forces, thus less swing torque was required. The resulting swing angular profile reflects more outward movement of the club at initiation of the downswing, and a more uniform/smoother (lower angular acceleration) angular motion overall. The resulting swing was slightly longer in duration, looping in appearance, and approximated a “free hinge” swing style movement.

The linear force optimization yielded a hub path profile that is more linear than the original especially nearing impact, and deviates considerably from the original during the first and last thirds of the downswing. The centre portion is nearly identical. Minimizing the linear force is mainly about minimizing the centrifugal loading on the club near impact (Nesbit, 2005) so this proposed modification is expected. The resultant force vector was oriented tangential to the hub path for longer in the downswing which aided in maintaining the linear work, and ultimately the max impact velocity. The timing of the peak force occurred at impact similar to the original swing and again reflects the centrifugal loading of the club. The limiting kinetic quantity for this optimization was swing torque. An increase in relative angular work compensated for the reduction in linear work. This swing angular profile also reflects more outward movement of the club at initiation of the downswing, and a more uniform/smoother angular motion overall, although not to the same degree as for the torque optimization. The resulting swing was slightly longer in duration, with an exaggerated “chopping wood” appearance.

The work optimization yielded a hub path that most closely resembled the original swing for this subject, and a swing angular profile that was indistinguishable from the original profile. There was a reduction in all the kinetic quantities, and there does not appear to be one specific limiting kinetic quantity. The duration of the downswing is considerably longer than the original. With the exception of the increased time, the optimization more or less reproduced the original swing, which could be interpreted as verifying the swing mechanics of this subject. The suggested swing modification again supports a smoother less aggressive style for this subject and the result is a more efficient swing where a higher percentage of the work produced by the golfer is realized as kinetic energy of the club.

The power optimization yielded a hub path with a nearly constant radius, and a swing angular profile with the lowest maximum acceleration and the flattest shape. The duration of this optimized downswing is also considerably longer than the original. The reduced power requirement of this swing is primarily a result of a reduced linear power component. Similar to the work optimization, there was a reduction in all the kinetic quantities, and there does not appear to be one specific limiting kinetic quantity. The overall work was maintained as was the club head velocity, and there was a small increase in efficiency. The appearance of this swing would be sweeping with little apparent effort, with a gradual increase in the outward movement of the club and swing speed, again suggesting a smoother swing style.

Practical implications

The following observations and practical implications are offered based upon the findings of this study:

- The path of the hands and the swing angular trajectory during the downswing reflect a complex interaction of all the kinetic outputs from the golfer: force, torque, linear and angular work, and linear and angular power. It is not of constant radius.
- The path of the hands plays an important role in the control of the club trajectory, and generation of club head velocity.
- Assessment and manipulation of the hand path may be more effective at improving a golfer's swing than affecting the movements of individual body segments and joints.
- The path of the hands is influenced by both the kinetic capabilities of the subject and the characteristics of the club used.
- It is possible to increase the club head velocity at impact for a subject within their individual kinetic output maximums solely through the manipulation of the hand path.
- All kinetic inputs affect club head velocity to some subject dependent degree. Increasing the maximum capacity of any one kinetic input (while maintaining the others) appears to effectively increase club head velocity.
- The path of the hands influences the swing angular trajectory of the club more so than the profile of the swing torque, thus the path of the hands should be ma-

nipulated as the primary means to influence the swing angular trajectory. This effect is due to the relative dominance of the centrifugal loading on the club compared to the output torque of the subject.
- The path of the hands can reveal relative kinetic output strengths/weaknesses of a subject.
- Increasing the vertical range-of-motion of the hand path potentially increases the club head velocity for all subjects.
- Increasing the time of the downswing potentially increases the club head velocity for most subjects, especially those with fast/aggressive swing styles. Increasing shaft flexibility may increase swing time for this swing style.
- Transitioning from a horizontal U-shaped hub path during the downswing to a more horizontal V-shaped hub path was suggested for all subjects. This yielded a greater increase in velocity for the subjects with the slow/smooth swing style.
- A slow/smooth swing style is most improved by either increasing the output torque capacity of the subject (relative strength of the wrists), or by using a shorter length club (reduction in swing inertia).
- A fast/aggressive swing style is most improved by increasing the output linear force capacity of the subject (relative strength of the arms), or by using a lighter club (reduction in club mass).
- The ability of a subject to effectively transfer the kinetic energy generated in the body to the club depends upon the strength of the arms and wrists by a 3/2 ratio. This energy transfer is increased by delaying the outward movement of the club through the proper configuration of the hand path, not through a conscience effort to control swing torque.

Limitations/context of study and suggested future work

The results presented in this paper must be considered within the context of the simplifications made to the computer model (2D treatment of the swing, rigid shaft, no rotations about the shaft axis), the few number of subjects (4), the manner in which the subjects were tested (indoors with a net), the method of trial selection (by the subjects), and the number of trials analyzed (only one trial per subject). While all of these aspects of the study are justified with earlier investigations of golf mechanics, the precise effects upon the results are not known.

The optimization algorithms searched within a subject's kinetic output limits as revealed through the analysis of a typical full-effort downswing. While these values were never exceeded, it was assumed that they could occur at any point during the downswing, not necessarily were they occurred in the actual subject swing. The actual physiology of the subject may further constrain the maximum kinetic values to be a function of the point where they occur in the downswing and/or the relative position/orientation of the body segments. In addition, the range-of-motion, strength, power, and speed limitations of the individual joints may bound what is obtainable for the subject. These factors may limit the degree of improvement possible for the subject as suggested by the optimization findings.

In order to further assess the viability of these findings to a particular subject, the optimized hub path and swing angular trajectories should be extrapolated (mapped on) to the individual joints of the subject. These joint trajectories and associated torque profiles can be compared to the joint trajectories and torque profiles obtained from the recorded swing to assess the potential for actual improvement for the subject. This is a difficult problem because the mapping may not yield a unique set of joint motions, and because the closed-loop nature of the arms-club-upper body configuration is difficult to model (Nesbit, 2005). However, the effort will result in valuable information for potentially improving the golf swing of particular subjects within their individual physiology and capabilities.

Conclusion

The objective of this study was to optimize the golf swing through the manipulation of the hub path geometry, and angular swing trajectories. There were two optimization goals; the primary goal was to maximize the club head velocity at impact, while the secondary goal was to minimize the individual kinetic inputs while maintaining the original club head velocity. The constraining factors for each optimization were the kinetic limitations of the golfer as revealed through the analysis of a typical swing. The primary optimization was applied to four diverse subjects, and the secondary optimization was applied to a single subject from this group.

The primary optimization analysis determined that there is potential to considerably increase the maximum club head velocity at impact within a particular subject's kinetic limits through the coordinated modification to their respective hub path geometries and angular swing trajectories. The manner of the modification, the limiting kinetic parameter, and the amount of potential velocity increase were subject dependent. The secondary optimization analysis was successful in identifying hub path geometries and angular swing trajectories that resulted in substantial reductions in the targeted kinetic input. For this optimization analysis, the manner of the modification, and the limiting kinetic parameter, were dependent upon the kinetic quantity being minimized. Both optimization analyses provided insight to the important and complex effects of hub path geometries and wrist swing trajectories on the kinetic inputs from the golfer.

From a practical point of view, the results of this study should further emphasize the importance and individuality of a golfer's hub path geometry and angular trajectories in generating club head velocity within their respective kinetic limitations. Whether it is possible for a subject to realize these modifications to produce the results implied by the optimizations is not known. Regardless, these findings provide insight to possible means for improving the golf swing which has important implications for golf instruction, and injury prevention.

Acknowledgements

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Key points

- The hand path trajectory is an important characteristic of the golf swing and greatly affects club head velocity and golfer/club energy transfer.
- It is possible to increase the energy transfer from the golfer to the club by modifying the hand path and swing trajectories without increasing the kinetic output demands on the golfer.
- It is possible to identify relative kinetic output strengths and weakness of a golfer through assessment of the hand path and swing trajectories.
- Increasing any one of the kinetic outputs of the golfer can potentially increase the club head velocity at impact.
- The hand path trajectory has important influences over the club swing trajectory.

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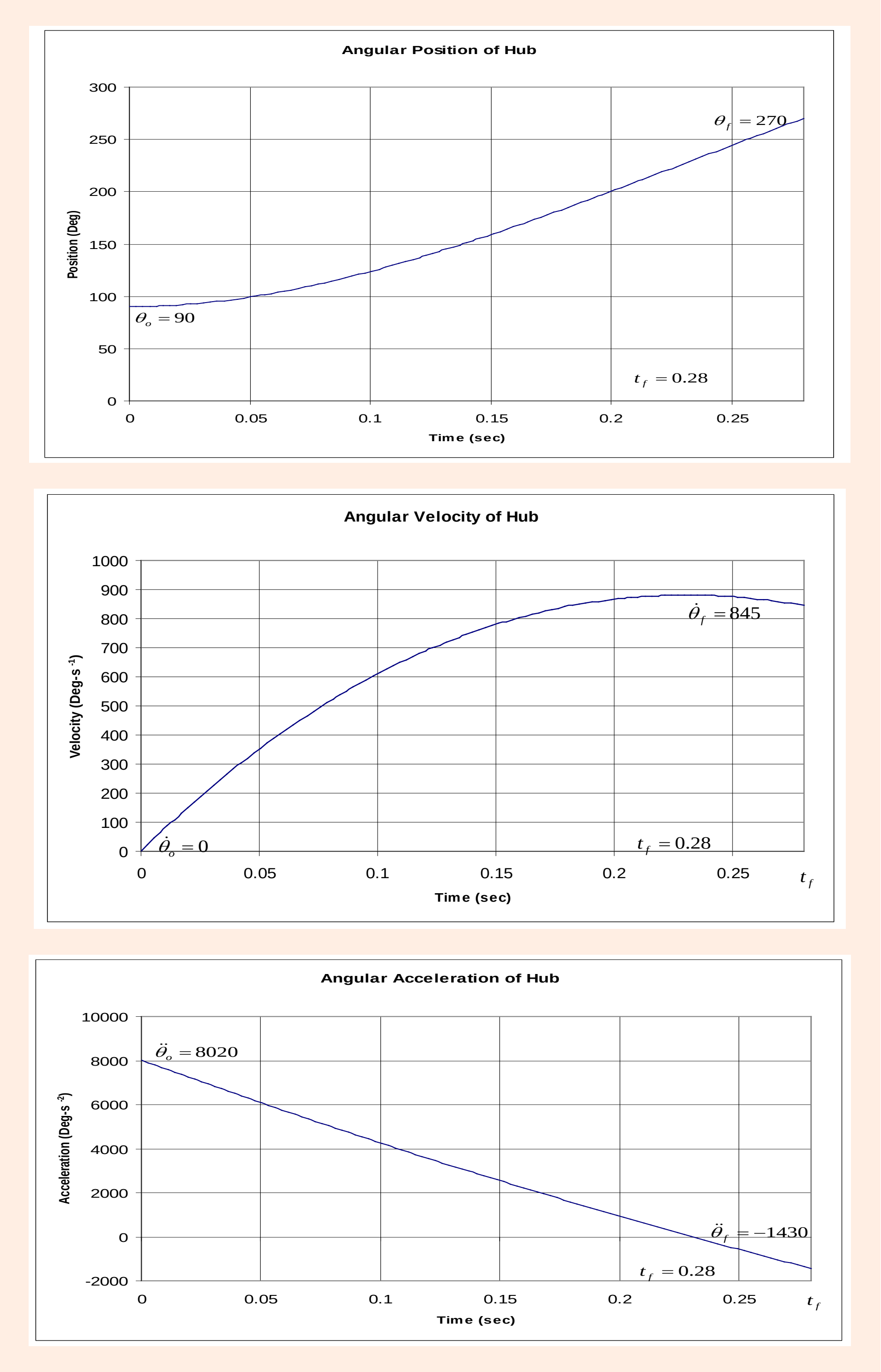
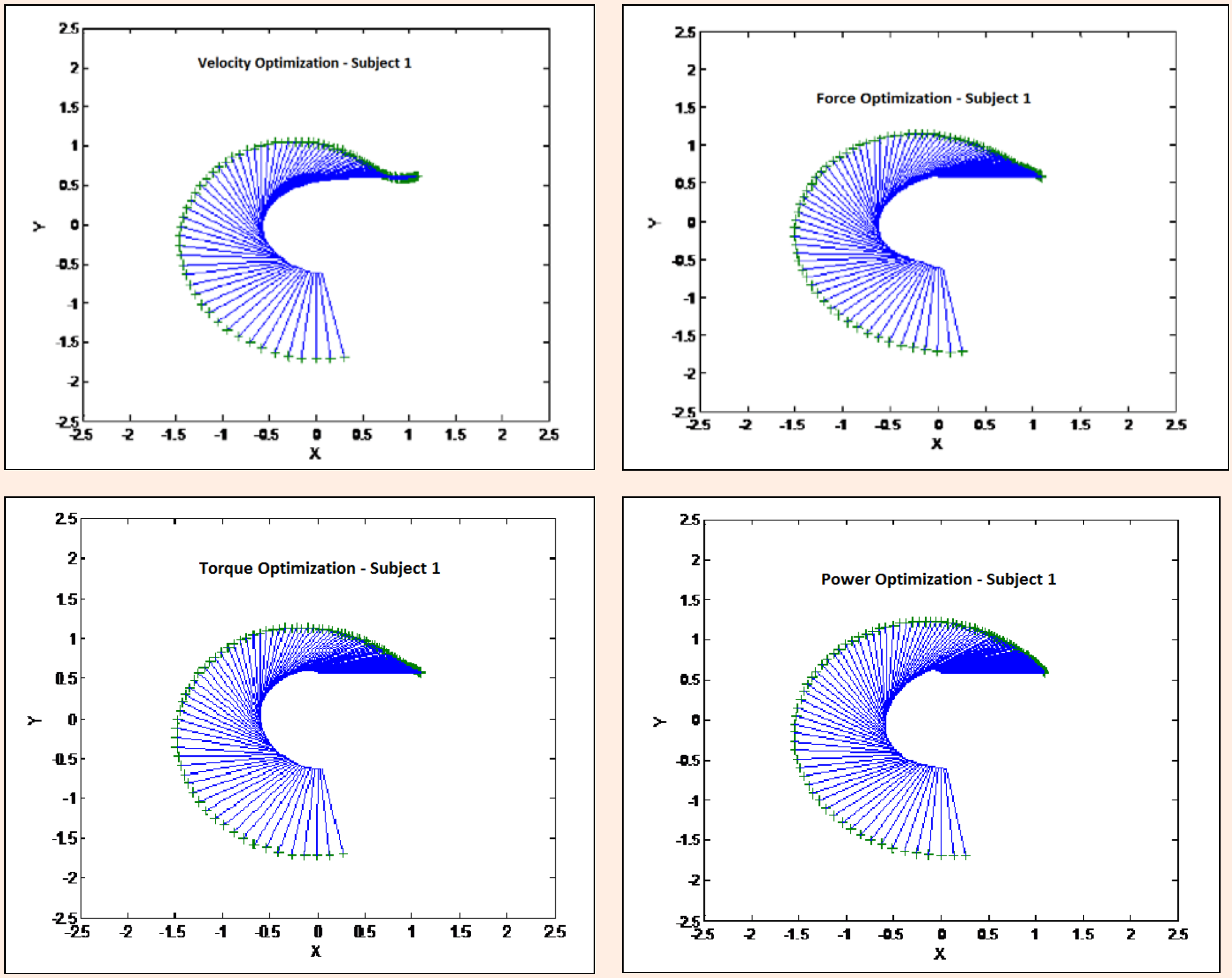
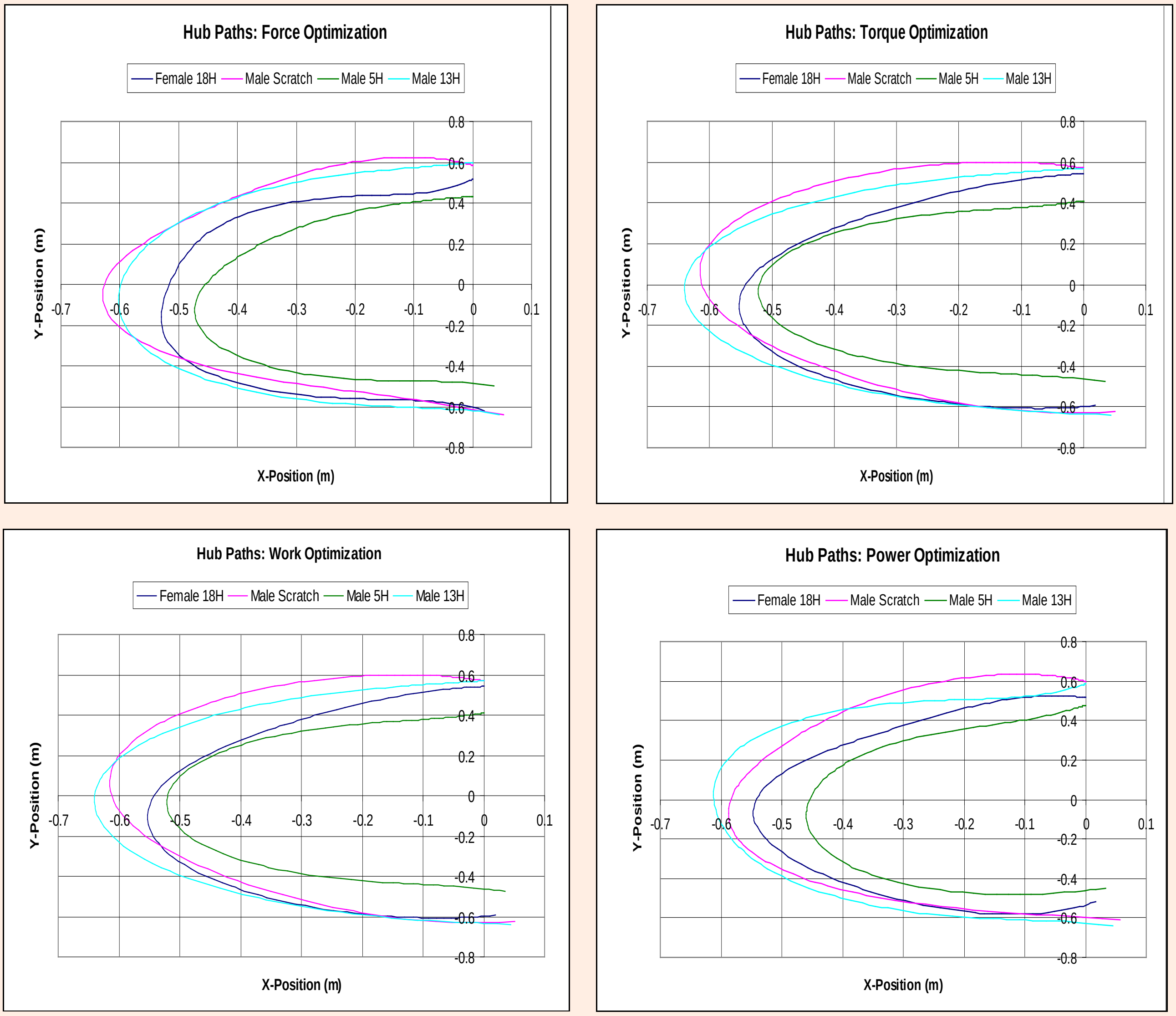
APPENDIX



Figures A1 through A3. **Typical angular positions, velocity, and acceleration profiles.**



Figures B1 through B4. Kinetic optimized swings for subject 1.



Figures C1 through C4. **Kinetic optimized hub paths for all subjects.**