Using technology to support effective mathematics teaching and learning: What counts?

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Abstract

What counts when it comes to using digital technologies in school mathematics? Is technology there to help students get 'the answer' more quickly and accurately, or to improve the way they learn mathematics? The way people answer this question is illuminating and can reveal deeply held beliefs about the nature of mathematics and how it is best taught and learned. This presentation considers the extent to which technology-related research, policy and practice might usefully inform each other in supporting effective mathematics teaching and learning in Australian schools. The first part of the presentation considers key messages from research on learning and teaching mathematics with digital technologies. The second part offers some snapshots of practice to illustrate what effective classroom practice can look like when technologies are used in creative ways to enrich students' mathematics learning. The third part analyses the technology messages contained in the draft *Australian curriculum – Mathematics* and the challenges of aligning curriculum policy with research and practice.

Introduction

Digital technologies have been available in school mathematics classrooms since the introduction of simple four-function calculators in the 1970s. Since then, computers equipped with increasingly sophisticated software, graphics calculators that have morphed into 'all-purpose' hand-held devices integrating graphical, symbolic manipulation, statistical and dynamic geometry packages, and web-based applications offering virtual learning environments have changed the mathematics teaching and learning terrain. Or have they? This presentation considers the extent to which technology-related research, policy and practice might usefully inform each other in supporting effective mathematics teaching and learning in Australian schools.

The first part of the presentation considers key messages from research on learning and teaching mathematics with digital technologies. The second part offers some snapshots of practice to illustrate what effective classroom practice can look like when technologies are used in creative ways to enrich students' mathematics learning. The third part analyses the technology messages contained in the draft *Australian curriculum – Mathematics* and the challenges of aligning curriculum policy with research and practice.

Key messages from research on learning and teaching mathematics with digital technologies

Fears are sometimes expressed that the use of technology, especially hand-held calculators, will have a negative effect on students' mathematics achievement. However, meta-analyses of published research studies have consistently found that calculator use, compared with non-calculator use, has either positive or neutral effects on students' operational, computational, conceptual and problem-solving skills (Ellington, 2003; Hembree & Dessart, 1986; Penglase & Arnold, 1996). A difficulty with these meta-analyses, however, is that they select studies that compare treatment (calculator) and control (non-calculator) groups of students, with the assumption that the two groups experience otherwise identical learning conditions. Experimental designs such as this do not take into account the possibility that technology fundamentally changes students' mathematical practices and even the nature of the mathematical knowledge they learn at school.

Technology and mathematical knowledge

In their contribution to the *17th ICMI Study on Mathematics Education and Technology*, Olive and Makar (2010) analysed the influence of technology on the nature of mathematical knowledge as experienced by school students. They argued as follows:

> If one considers mathematics to be a fixed body of knowledge to be learned, then the role of technology in this process would be primarily that of an efficiency tool, i.e. helping the learner to do the mathematics more efficiently. However, if we consider the technological tools as providing access to new understandings of relations, processes, and purposes, then the role of technology relates to a conceptual construction kit. (p. 138)

Their words encapsulate the contrasting purposes of technology that were foreshadowed in the opening paragraph of this paper. For learners, mathematical knowledge is not fixed but fluid, constantly being created as the learners interact with ideas, people and their environment. When technology is part of this environment, it becomes more than a substitute for mathematical work done with pencil and paper. Consider, for example, the way in which dynamic geometry software allows students to transform a geometric object by 'dragging' any of its constituent parts to investigate its invariant properties. Through this experimental approach, students make predictions and test conjectures in the process of generating mathematical knowledge that is new for them.

Technology and Mathematical Practices

Learning mathematics is as much about *doing* as it is about *knowing*. How knowing and doing come together is evident in the mathematical practices of the classroom. For example, school mathematical practices that, in the past, were restricted to memorising and reproducing learned procedures can be contrasted with mathematical practices endorsed by most modern curriculum documents, such as conjecturing, justifying and generalising. Technology can change the nature of school mathematics by engaging students in more active mathematical practices such as experimenting, investigating and problem solving that bring depth to their learning and encourage them to ask questions rather than only looking for answers (Farrell, 1996; Makar & Confrey, 2006).

Olive and Makar (2010) argue that mathematical knowledge and mathematical practices are inextricably linked, and that this connection can be strengthened by the use of technologies. They developed an adaptation of Steinbring's (2005) 'didactic triangle' that in its original form represents the learning ecology as interactions between student, teacher and mathematical knowledge. Introducing technology into this system transforms the learning ecology so that the triangle becomes a tetrahedron, with the four vertices of student, teacher, task and technology creating 'a space within which new mathematical knowledge and practices may emerge' (p. 168).

Within this space, students and teachers may imagine their relationship with technologies in different ways. Goos, Galbraith, Renshaw and Geiger (2003) developed four metaphors to describe how technologies can transform teaching and learning roles. Technology can be a *master* if students' and teachers' knowledge and competence are limited to a narrow range of operations. Students may become dependent on the technology if they are unable to evaluate the accuracy of the output it generates. Technology is a *servant* if used by students or teachers only as a fast, reliable replacement for pen and paper calculations without changing the nature of classroom activities. Technology is a *partner* when it provides access to new kinds of tasks or new ways of approaching existing tasks to develop understanding, explore different perspectives, or mediate mathematical discussion. Technology becomes an *extension of self* when seamlessly integrated into the practices of the mathematics classroom.

Pierce and Stacey (2010) offer an alternative representation of the ways in which technology can transform mathematical practices. Their *pedagogical map* classifies ten types of pedagogical opportunities afforded by a wide range of mathematical analysis software. Opportunities arise at three levels that represent the teacher's thinking about:

- the *tasks* they will set their students (using technology to improve speed, accuracy, access to a variety of mathematical representations)
- *classroom interactions* (using technology to improve the display of mathematical solution processes and support students' collaborative work)
- the *subject* (using technology to support new goals or teaching methods for a mathematics course).

Snapshots of classroom mathematical practice

Two snapshots are presented here to illustrate how technology can be used creatively to support new mathematical practices.

Changing tasks and classroom interactions

Geiger (2009) used the master-servant-partner-extension-of-self framework to analyse a classroom episode in which he asked his Year 11 students to use the dynamic geometry facility on their

CAS calculators to draw a line $\sqrt{45}$ units long. His aim was to encourage students to think about the geometric representation of irrational numbers. The anticipated solution involved using the Pythagorean relationship $6^2 + 3^2 = (\sqrt{45})^2$ to construct a right-angled triangle with sides 6 and 3 units long and hypotenuse $\sqrt{45}$ units long. Figure 1 summarises the flow of the episode and how technology was used.

In this episode, technology was initially used as a *servant* to perform numerical calculations that did not lead to the desired geometric solution. It became a *partner* when students passed their calculators around the group or displayed their work to the whole class to offer ideas for comment and critique. As a *partner* it gave the student who found the solution the confidence he needed to introduce his conjectured solution into a heated small group debate. In terms of Pierce and Stacey's (2010) pedagogical map, this episode illustrates opportunities provided by a task that link numerical and geometric representations to support *classroom interactions* where students share and discuss their thinking.

Table 1: Draw a line $\sqrt{45}$ units long

Classroom interaction	Role of technology
Students find the square roots of various numbers.	Servant
Students pass calculators back and forth to share and critique each other's thinking.	Partner
Teacher invites student to present calculator work to whole class. Audience identifies misconceptions about how calculators display decimal versions of irrational numbers.	Master (prior group work) then partner (whole class display and discussion)
Teacher hint: think about triangles. Students search for Pythagorean formulation without geometric representation.	Servant
Teacher redirects students to consider geometry, not just numbers. Student interrupts group discussion to propose geometric solution; passes his calculator around group to share and defend his solution.	Partner

Changing course goals and teaching methods

Geiger, Faragher and Goos (in press) investigated how CAS technologies support students' learning and social interactions when they are engaged in mathematical modelling tasks. In this snapshot, Year 12 students worked on the following question:

> When will a population of 50,000 bacteria become extinct if the decay rate is 4% per day?

One pair of students developed an initial exponential model for the population y at any time x, $y = 50000 \times (0.96)^x$. They then equated the model to zero in order to represent the point at which the bacteria would be extinct, with the intention of using CAS to solve this equation. When they entered the equation into their CAS calculator, however, it unexpectedly responded with a *false* message. The students thought this response was a result of a mistake with the syntax of their command. When they asked their teacher for help, he confirmed their syntax was correct, but said they should think harder about their assumptions.

Eventually, the teacher directed the problem to the whole class and one student spotted the problem: 'You can't have an exponential equal to zero'. This resulted in a whole class discussion of the assumption that extinction meant a population of zero, which they decided was inappropriate. The class then agreed on the position that extinction was 'any number less than one'. Students used CAS to solve this new equation and obtain a solution.

In this episode the teacher exploited the 'confrontation' created by the CAS output to promote productive interaction among the class (technology as *partner*). Using this pedagogical opportunity allowed the teacher to refocus *course goals and teaching methods* on promoting thinking about the mathematical modelling process rather than on practice of skills.

Aligning curriculum with research and practice?

The brief research summary and classroom snapshots presented above show how digital technologies provide a 'conceptual construction kit' (Olive & Makar, 2010, p. 138) that can transform students' mathematical knowledge and practices. To what extent does the *Australian curriculum – Mathematics* support this transformative view of technology?

The shape paper that provided the initial outline of the K–12 mathematics curriculum (National Curriculum Board, 2009) made it clear that technologies should be embedded in the curriculum 'so that they are not seen as optional tools' (p. 12). Digital technologies were seen as offering new ways to learn and teach mathematics that helped deepen students' mathematical understanding. It was also acknowledged that students should learn to choose intelligently between technology, mental, and pencil and paper methods.

The draft consultation version 1.0 of the K–10 mathematics curriculum expected 'that mathematics classrooms will make use of all available ICT in teaching and learning situations'. The intention is that use of ICT is to be referred to in content descriptions and achievement standards. Yet this is done superficially and inconsistently throughout the curriculum, with technology often being treated as an add-on that replicates by-hand methods. This is seen, for example, in the following content description from the Year 8 Number and Algebra strand: 'Plot graphs of linear functions and use these to find solutions of equations *including use of ICT*' (emphasis added).

In the corresponding consultation versions of the four senior secondary mathematics courses, the aims for all courses refer to students choosing and using a range of technologies. Nevertheless, each course contains a common technology statement – 'Technology can aid in developing skills and allay the tedium of repeated calculations' – that betrays a limited view of its role. Across the courses, variable messages about the use of technology are conveyed in words like 'assumed' and 'vital' in Essential and General Mathematics to 'should be widely used in this topic', 'can be used to illustrate practically every aspect of this topic', or no mention at all for some topics in Mathematical Methods and Specialist Mathematics.

In both the K–10 and senior secondary mathematics curricula, uses of technology, where made explicit, are mostly consistent with the *servant* metaphor of Goos et al. (2003), despite the more transformative intentions evident in the initial shaping paper. Pedagogical opportunities afforded by the curriculum are restricted to the level of *tasks* in Pierce and Stacey's (2010) taxonomy, in that technology may be used to make computation and graphing quicker and more accurate and possibly to link representations.

Although the technology messages contained in the *Australian curriculum – Mathematics* do not do justice to what research tells us about effective teaching and learning of mathematics, it is almost inevitable that there are gaps between an intended curriculum and the curriculum enacted by teachers and students in the classroom. Many teachers are already using technology effectively to enhance students' understanding and enjoyment of mathematics. In their hands lies the task of enacting a truly futures-oriented curriculum that will prepare students for intelligent, adaptive and critical citizenship in a technology-rich world.

References

Ellington, A. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education, 34*, 433–463.

Farrell, A. M. (1996). Roles and behaviors in technology-integrated precalculus classrooms. *Journal of Mathematical Behavior, 15*, 35–53.

Geiger, V. (2009). *Learning mathematics with technology from a social perspective: A study of secondary students' individual and collaborative practices in a technologically rich mathematics classroom*. Unpublished doctoral dissertation, The University of Queensland, Brisbane, Australia.

Geiger, V., Faragher, R., & Goos, M. (in press). CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modelling in secondary school classrooms. *Mathematics Education Research Journal.*

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003) Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior, 22*, 73–89.

Hembree, R., & Dessart, D. (1986). Effects of hand-held calculators in pre-college mathematics education: A meta-analysis. *Journal for Research in Mathematics Education, 17*, 83–99.

Makar, K., & Confrey, J. (2006). Dynamic statistical software: How are learners using it to conduct data-based investigations? In C. Hoyles, J. Lagrange, L. H. Son, & N. Sinclair (Eds.), *Proceedings of the 17th Study Conference of the International Commission on Mathematical Instruction*. Hanoi Institute of Technology and Didirem Université Paris 7.

National Curriculum Board (2009). *Shape of the Australian curriculum: Mathematics*. Retrieved May 29, 2010 from http://www.acara.edu.au/verve/_resources/Australian_Curriculum_-_Maths.pdf

Olive, J., & Makar, K., with V. Hoyos, L. K. Kor, O. Kosheleva, & R. Straesser (2010). Mathematical knowledge and practices resulting from access to digital technologies. In C. Hoyles & J. Lagrange (Eds.), *Mathematics education and technology – Rethinking the terrain. The 17th ICMI Study* (pp. 133–177). New York: Springer.

Penglase, M., & Arnold, S. (1996). The graphics calculator in mathematics education: A critical review of recent research. *Mathematics Education Research Journal, 8*, 58–90.

Pierce, R., & Stacey, K. (2010). Mapping pedagogical opportunities provided by mathematics analysis software. *International Journal of Computers for Mathematical Learning, 15*(1), 1–20.

Steinbring, H. (2005). *The construction of new mathematical knowledge in classroom interaction: An epistemological perspective.* New York: Springer.