LECTURE THREE – 3

P-I-D engine speed governors

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LEARNING OBJECTIVES

- To understand single-input-single-output control concepts
- To define and understand benefits & limitations of on/off control
- To define the full PID control law in its main forms
- To understand P / PI / PID / PD-control benefits and issues
- To be able to perform basic steady-state error analysis of linear systems using the Laplace Transform's Final Value Theorem
- To be able to perform perturbation analysis of marine propulsion engines in order to determine the rpm regulation problem
- To apply robust control theory and pole placement to design disturbance rejection PI and PID speed governors
- To apply robustness analysis for both parametric uncertainty and neglected dynamics
- To use available signals shipboard in order to improve robustness of PID engine controllers

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PID Control Fundamentals

Nik. Xiros

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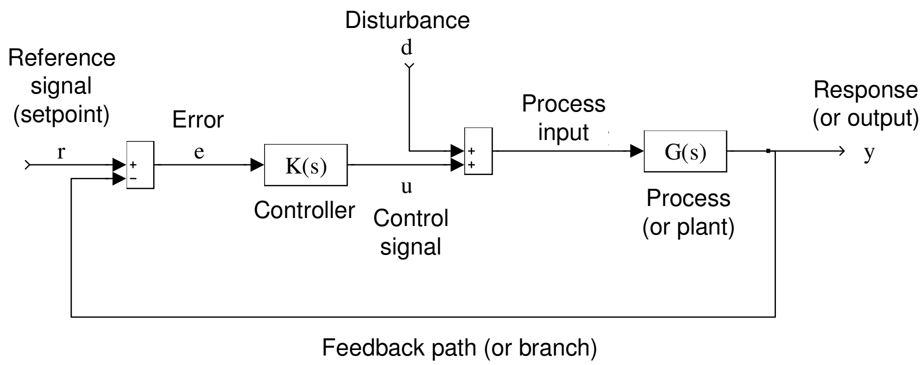
Single-Input-Single-Output system control



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Single-Input-Single-Output system control

$$\left.\begin{aligned} Y(s) &= G(s)\cdot\left[U(s)+D(s)\right] \\ U(s) &= K(s)\cdot E(s) = K(s)\cdot\left[R(s)-Y(s)\right] \end{aligned}\right\}\Rightarrow$$

$$\Rightarrow Y(s) = \frac{G(s)K(s)}{1+G(s)K(s)}\cdot R(s) + \frac{G(s)}{1+G(s)K(s)}\cdot D(s)$$

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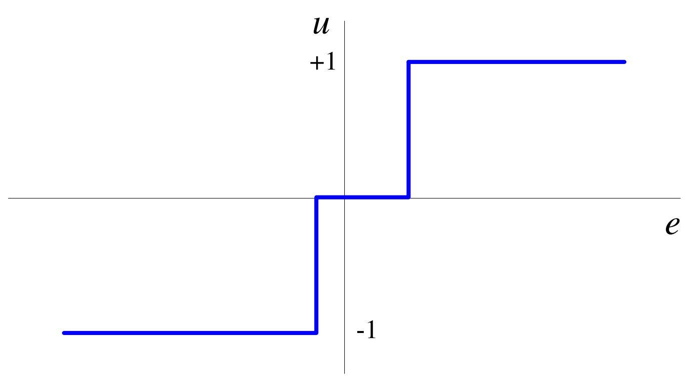
ON/OFF Control

$$u = \begin{cases} A\cdot \mathrm{sgn}(e), & |e| \geq Z_d \\ 0, & |e| < Z_d \end{cases} = \begin{cases} +A, & e \geq +Z_d \\ 0, & |e| < Z_d \\ -A, & e \leq -Z_d \end{cases}$$



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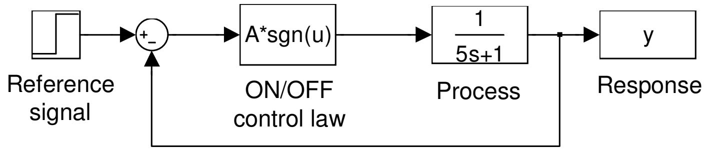
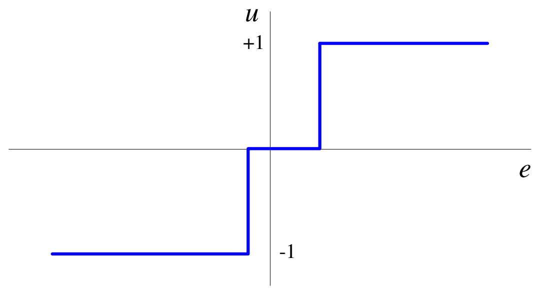
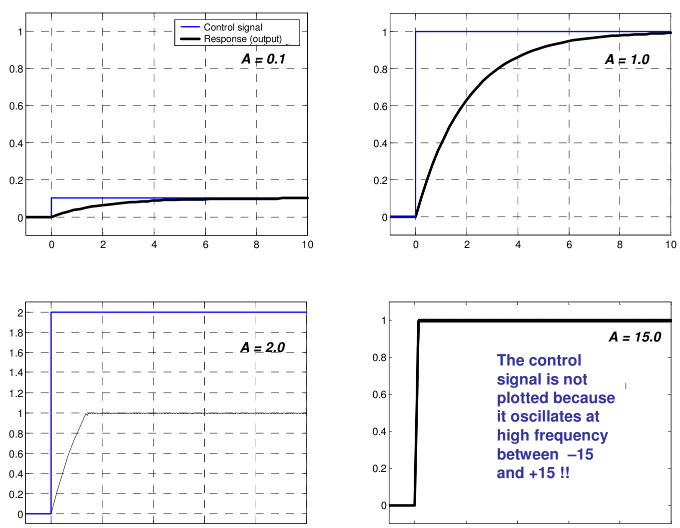
ON/OFF Control - *EXAMPLE*

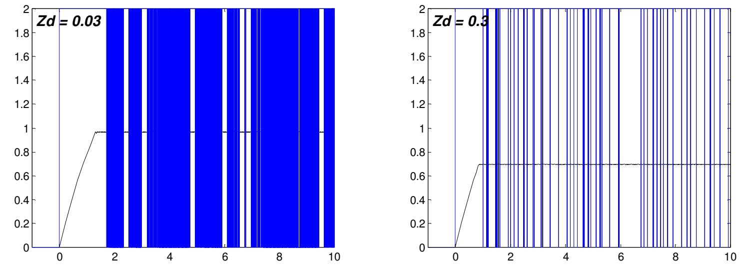


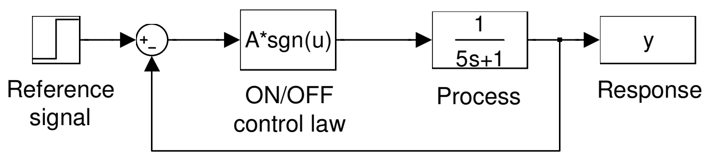


ON/OFF Control - *EXAMPLE*

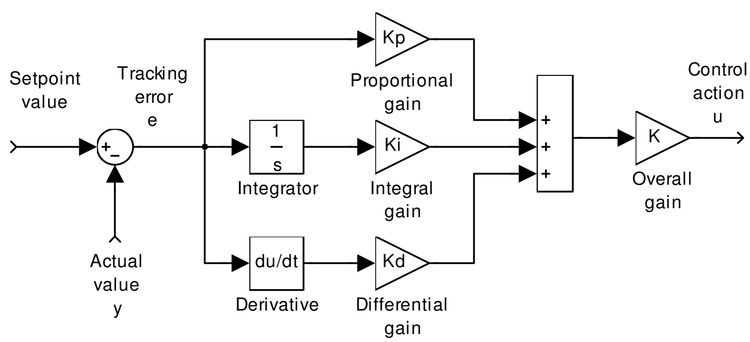


ON/OFF Control – *EXAMPLE*





The P-I-D Controller



$$u(t) = K_p \cdot e(t) + K_i \cdot \int_0^t e(\xi)d\xi + K_d \cdot \frac{d}{dt}e(t)$$

$$\Updownarrow$$

$$U(s) = K(s) \cdot E(s) = \left(K_p + \frac{K_i}{s} + K_d \cdot s \right) \cdot E(s) = \frac{K_i + K_p \cdot s + K_d \cdot s^2}{s} \cdot E(s)$$

Forms of the PID Control Law

- Analytic form

$$u(t) = K_p \cdot e(t) + K_i \cdot \int_0^t e(\xi) d\xi + K_d \cdot \frac{d}{dt} e(t)$$

$$\Updownarrow$$

$$U(s) = K(s) \cdot E(s) = \left(K_p + \frac{K_i}{s} + K_d \cdot s \right) \cdot E(s) = \frac{K_i + K_p \cdot s + K_d \cdot s^2}{s} \cdot E(s)$$

- Practical form

$$u(t) = K_p \cdot \left(-y(t) + \frac{1}{T_i} \cdot \int_0^t e(\xi) d\xi - T_d \cdot \frac{d}{dt} y(t) \right)$$

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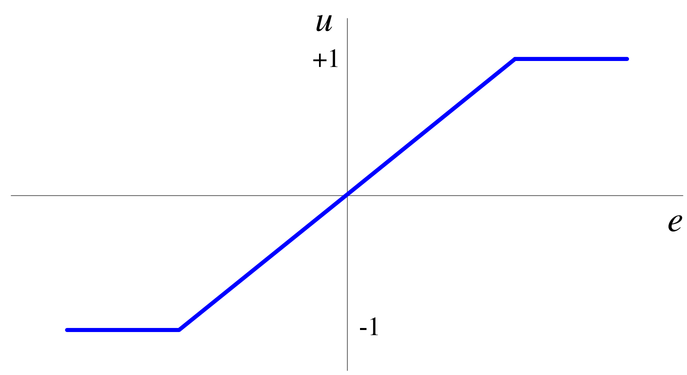
Proportional control

$$u(t) = \begin{cases} u_0 + K_P \cdot e_0, & e(t) > e_0 \\ u_0 + K_P \cdot e(t), & -e_0 \le e(t) \le e_0 \\ u_0 - K_P \cdot e_0, & e(t) < -e_0 \end{cases}$$



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Proportional control – *EXAMPLE 1*

Static plant equation $y = K_{SS} \cdot u = K_{SS} \cdot (u_0 + K_p \cdot e)$

Closed-loop equation $y = \dfrac{K_{SS} \cdot u_0 + K_{SS} K_p \cdot r}{1 + K_{SS} K_p} \Rightarrow e = \dfrac{r - K_{SS} \cdot u_0}{1 + K_{SS} K_p}$

Steady-state error $u_0 = r / K_{SS}$

$K_p \to \infty$

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Proportional control – *EXAMPLE 2*

1st-order plant $G(s) = \dfrac{K_{SS}}{\tau s + 1}$

Closed-loop equation

$$Y(s) = \frac{K_{SS} K_p}{\tau s + 1 + K_{SS} K_p} \cdot R(s) + \frac{K_{SS}}{\tau s + 1 + K_{SS} K_p} \cdot D(s)$$

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Proportional control – *Final Value Theorem*

$$d(t) = u_{step}(t) \Leftrightarrow D(s) = 1/s$$

$$\Downarrow$$

$$\lim_{t\to\infty}\big(y(t)\big) = \lim_{s\to 0}\big(sY(s)\big) = \lim_{s\to 0}\left(s \cdot \frac{K_{SS}}{\tau s + 1 + K_{SS}K_p} \cdot D(s) \right)$$

$$\Downarrow$$

$$\lim_{t\to\infty}\big(y(t)\big) = \lim_{s\to 0}\left(s \cdot \frac{K_{SS}}{\tau s + 1 + K_{SS}K_p} \cdot \frac{1}{s} \right) = \frac{K_{SS}}{1 + K_{SS}K_p}$$

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Proportional-Integral control

$$u(t) = K_p \cdot e(t) + K_i \cdot \int_0^t e(\xi)d\xi$$

$$\Updownarrow$$

$$U(s) = \left(K_p + \frac{K_i}{s} \right) \cdot E(s) = \frac{K_p \cdot s + K_i}{s} \cdot E(s)$$

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Proportional-Integral control

Generic linear plant $G(s) = \frac{p_n(s)}{p_0(s)}$

Closed-loop equation
$$Y(s) = \frac{(K_i + K_p \cdot s) \cdot p_n(s)}{s \cdot p_0(s) + (K_i + K_p \cdot s) \cdot p_n(s)} \cdot R(s) +$$
$$+ \frac{s \cdot p_n(s)}{s \cdot p_0(s) + (K_i + K_p \cdot s) \cdot p_n(s)} \cdot D(s)$$

Closed-loop Characteristic Polynomial
$$p_c(s) = s \cdot p_0(s) + (K_i + K_p \cdot s) \cdot p_n(s)$$

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Proportional-Integral control – *EXAMPLE*

Closed-loop equation for 1st order open-loop plant

$$Y(s) = \frac{K_{SS} \cdot (K_i + K_p s)}{s \cdot (1 + \tau s) + K_{SS} \cdot (K_i + K_p s)} \cdot R(s) +$$
$$+ \frac{K_{SS} \cdot s}{s \cdot (1 + \tau s) + K_{SS} \cdot (K_i + K_p s)} \cdot D(s)$$

Steady-state error using the Final Value Theorem

$$d(t) = u_{step}(t) \Leftrightarrow D(s) = 1/s$$
$$\Downarrow$$
$$y(t \to \infty) = \lim_{s \to 0}\left(s \cdot Y(s)\right) = \frac{K_{SS} \cdot 0}{0 \cdot (1 + \tau \cdot 0) + K_{SS} \cdot (K_i + K_p \cdot 0)} = 0$$

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Proportional-Integral-Differential control

Full PID controller equation

$$U(s) = \left(K_p + \frac{K_i}{s} + K_d s \right) \cdot E(s) = \frac{K_i + K_p s + K_d s^2}{s} \cdot E(s)$$

Closed-loop equation for 1st order open-loop plant

$$Y(s) = \frac{K_{SS}(K_i + K_p s + K_d s^2) \cdot R(s) + K_{SS} s \cdot D(s)}{\left(\tau + K_{SS} K_d\right) s^2 + \left(1 + K_{SS} K_p\right) s + K_{SS} K_i}$$

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Proportional-Differential control

Closed-loop equation for 1st order open-loop plant

$$Y(s) = \frac{K_{SS}(K_p + K_d s)}{\left(\tau + K_{SS} K_d\right) s + \left(1 + K_{SS} K_p\right)} \cdot R(s)$$

$$+ \frac{K_{SS}}{\left(\tau + K_{SS} K_d\right) s + \left(1 + K_{SS} K_p\right)} \cdot D(s)$$

Steady-state error analysis using Final Value Theorem

$$y(t \to \infty) = \frac{K_{SS}}{1 + K_{SS} K_p}$$

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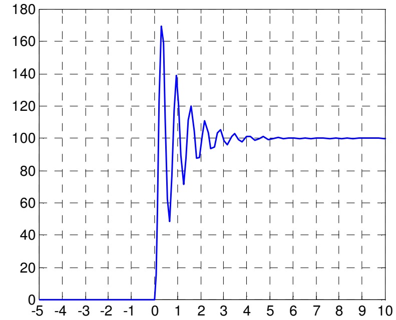
Issues with PI or PID control



$$\mathcal{L}\left\{\frac{d}{dt}y(t)\right\} = s\cdot Y(s) \overset{s=j\omega}{=} \left(j\omega\right)\cdot Y(j\omega) = \omega\cdot\left|Y(j\omega)\right|\cdot\exp\left(\angle Y(j\omega)+\pi/2\right)$$

Perturbation Analysis of Ship Powertrains

Nik. Xiros

Introduction

The speed regulation problem is one of the oldest and most important feedback control problems. Indeed the mechanical load to be turned by a prime mover consists of two distinct components:

a) the load's moment of inertia and

b) the load's resisting torque (moment) due to rotation.

Introduction

The objective of speed regulation is to balance and match the power delivery of the prime mover to the power demand of the turning load. The favorable results of introducing speed regulation are:

a) To maintain and restore the power balance of the prime mover to its load that is perturbed by the fluctuations of the resistance torque of the load and

b) To protect the prime mover from slipping outside of the admissible operational range.

Torque and power balance

Equation of motion $\dot{N}(t) = \frac{M(t) - L(t)}{J}$

Motor and load power $P_m(t) = P_{load}(t)$

Prime mover power $P_m(t) \propto N$

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Torque and power balance

$$P_m = P_{load}(t) = \frac{c_4}{\eta_S} N^3 \Rightarrow M = L = \frac{c_4}{\eta_S} N^2$$

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Engine torque perturbation analysis

CMCR:

$$P_{CMCR} = M_{CMCR} \cdot N_{CMCR} \Rightarrow \frac{W_{e,CMCR} \cdot N_{CMCR} \cdot Z_C}{k} = M_{CMCR} \cdot N_{CMCR} \Rightarrow$$

$$\Rightarrow M_{CMCR} \triangleq M_{100\%} = \frac{h_L Z_C}{k} \eta_{e,CMCR} m_{f,CMCR}$$

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Engine torque perturbation analysis

Engine torque $$M = \frac{h_L Z_C}{k} \eta_e m_f$$

Fuel index (rack) $$u_{\#\#} = \frac{M}{M_{CMCR}} = \frac{\eta_e m_f}{\eta_{e,CMCR} m_{f,CMCR}}$$

Fuel per cylinder & cycle $$m_f = \frac{\eta_{e,CMCR}}{\eta_e} m_{f,CMCR} u_{\#\#}$$

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Propeller torque perturbation analysis

$$L = L_0 + \Delta L = \frac{c_4}{\eta_S}\left(N_0 + n\right)^2 = \frac{c_4}{\eta_S}\cdot N^2 \Rightarrow$$

$$\Rightarrow \Delta L \approx \underbrace{2\frac{c_4}{\eta_S}N_0}_{C_{pr}}\cdot n \Rightarrow \Delta L \approx C_{pr}\cdot n$$

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Equation of motion for perturbation analysis

$$\dot{N}(t) = \frac{M(t) - L(t)}{J} \Rightarrow \frac{d}{dt}\left(N_0 + n\right) = \frac{M_0 + C_e u_{\#} - L_0 - C_{pr} n + d(t)}{J}$$

Derivation:

$$N = N_0 + n, \tfrac{d}{dt}N_0 = 0$$

$$M_{CMCR} u_{\#0} = M_0 = L_0 = \tfrac{c_4}{\eta_S} N_0^2$$

$$L = \tfrac{c_4}{\eta_S}\left(N_0 + n\right)^2 \overset{C_{pr} = 2\frac{c_4}{\eta_S}N_0}{\approx} L_0 + C_{pr} n$$

$$M = M_{CMCR} u_{\#\#} = M_{CMCR}\left(u_{\#0} + u_{\#}\right) \overset{M_{CMCR} = C_e}{=} M_0 + C_e u_{\#}$$

Result:

$$\boxed{\dot{n} = \tfrac{1}{J}\cdot\left(C_e \cdot u_{\#} - C_{pr}\cdot n + d(t)\right)}$$

The above is a linearized equation of motion for perturbation analysis. Torque disturbance $d(t)$ packs torque fluctuations, non-modeled and neglected terms etc. that cause deviation from the propeller curve.

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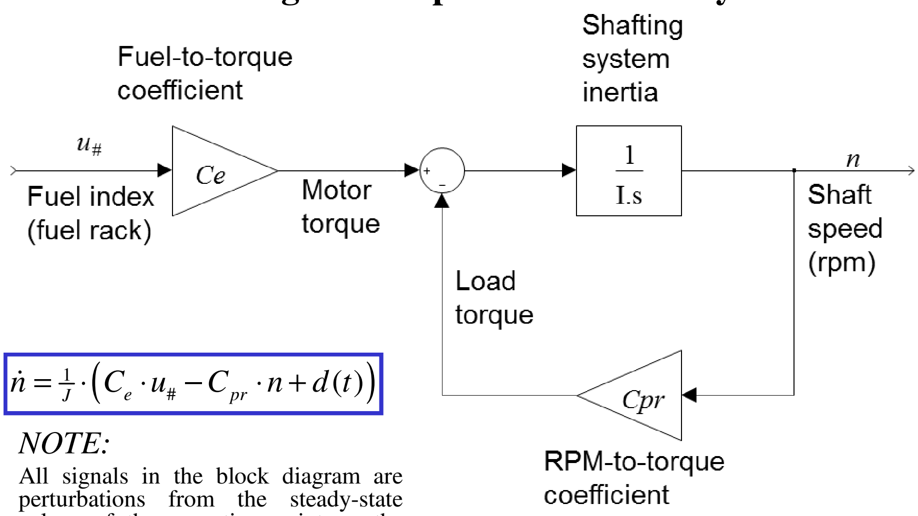
Block diagram for perturbation analysis



$$\dot{n} = \frac{1}{J} \cdot \left(C_e \cdot u_{\#} - C_{pr} \cdot n + d(t)\right)$$

NOTE:
All signals in the block diagram are perturbations from the steady-state values of the operating point on the propeller curve of the powertrain.

Robust PID Control of Marine Propulsion Engines

Nik. Xiros

ROBUST PID CONTROL

Marine engine control

- **Closed-loop transfer function with PID control law**

$$G_c(s) = \frac{n(s)}{k_Q(s)} = -\frac{N_0^2 \cdot s}{(I - CK_D)\cdot s^2 + (2K_{Q0}N_0 - CK_P)\cdot s - CK_I}$$

- **Specification for propeller load disturbance rejection**

$$\|G_c(s)\|_\infty \le G_0$$

- **Controller design equations**

$$-\frac{N_0^2}{I - CK_D} = K\omega_n^2, \quad \frac{2K_{Q0}N_0 - CK_P}{I - CK_D} = 2\zeta\omega_n, \quad -\frac{CK_I}{I - CK_D} = \omega_n^2$$

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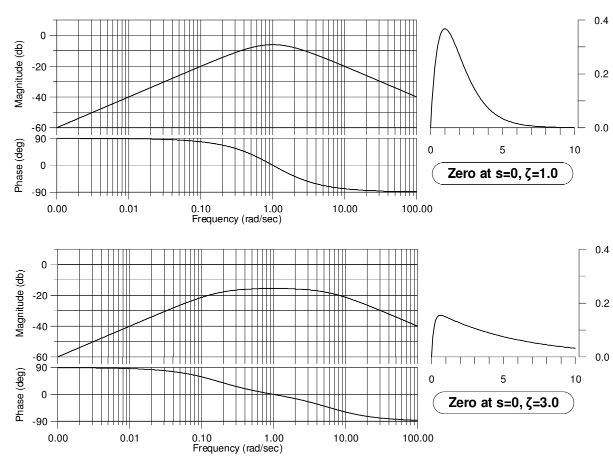
ROBUST PID CONTROL

Bode plots of 2nd order transfer function

$$G_c(s) = \frac{K\omega_n^2 \cdot s}{s^2 + 2\zeta\omega_n \cdot s + \omega_n^2}$$



➢ **Design decision:** $\zeta = 1.0 \Rightarrow \|G_c(s)\|_\infty = |G_c(j\omega_n)| = \frac{|K|\omega_n}{2}$

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ROBUST PID CONTROL

- **H∞ PI controller:**

$$\omega_n = \omega_{n,\mathrm{PI}} = \frac{N_0^2}{2G_0 I} \Rightarrow \left\| G_c(s) \right\|_\infty \le G_0$$

- **H∞ PID controller:**

$\omega_n < \omega_{n,\mathrm{PI}}$ **so that**

$$\tilde{G}_c(s) = \frac{f_R(s)}{k_Q(s)} = -\frac{N_0^2 \cdot \left(K_D s^2 + K_P s + K_I \right)}{(I - CK_D) \cdot s^2 + (2K_{Q0} N_0 - CK_P) \cdot s - CK_I},$$ **minimum phase**

&

Robustness against neglected fast dynamics…

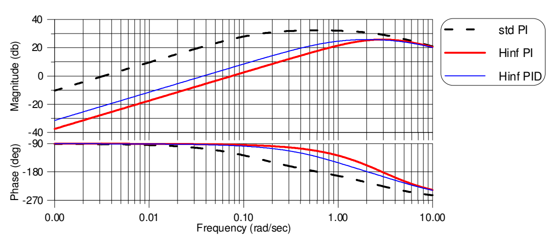
ROBUST PID CONTROL

PID control of "Shanghai Express" main engine

- **Controller tuning for std PI, Hinf PI και Hinf PID**

	$(-K_P)$ (%index/rpm)	$(-K_I)$ (%index/rpm/s)	$(-K_D)$ (%index·s/rpm)
std PI	5.00	1.00	0.00
Hinf PI	13.19	22.79	0.00
Hinf PID	13.19	11.67	2.53

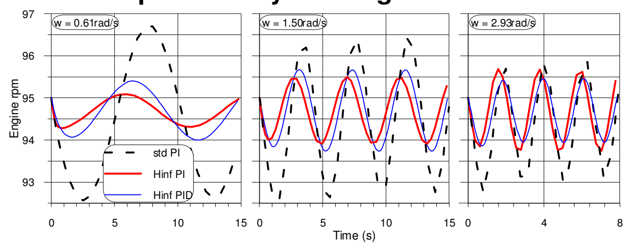
- **Closed-loop system Bode plots**



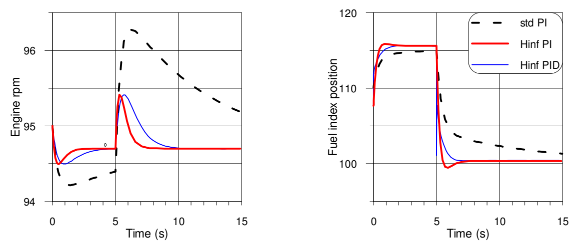
ROBUST PID CONTROL

PID control of “Shanghai Express” main engine

- **Sinusoidal response at system eigenvalues**



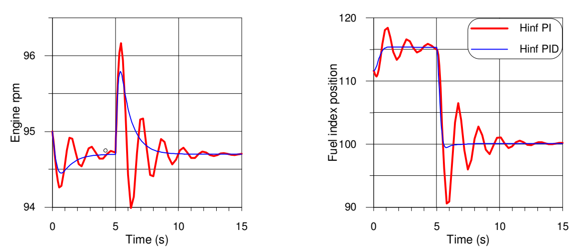
- **Reduced-order system step response**



ROBUST PID CONTROL

PID control of “Shanghai Express” main engine

- **Full order system step response**

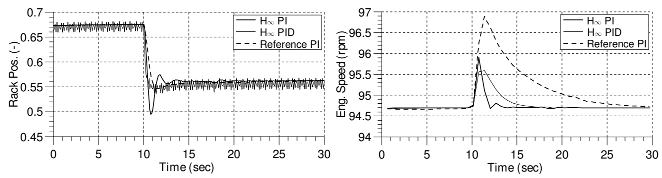


- **Approximate robustness analysis for a single open-loop neglected pole**

$$s_3 = -\frac{1}{\tau_{TC}} \cdot \left(1 - \frac{\tau_{TC}}{\tau_{prop}} - \frac{C}{I} \cdot K_D \right)$$

ROBUST PID CONTROL

- **Step response using *MoTher* – Process noise**



- **RPM rate estimate on the basis of shaft torque signal**

$$I_E \cdot \dot{n}_E(t) - I_P \cdot \dot{n}_P(t) \approx \frac{I_E - I_P}{I_E + I_P} \cdot \left(Q_E(t) - Q_P(t)\right),\ s = j\omega \to 0^+$$

$$\Downarrow$$

$$Q_P = \frac{I \cdot Q_{shaft} - I_P \cdot Q_E}{I_E} \Rightarrow \dot{N}(t) = \frac{C \cdot F_R(t) - Q_{fr} - Q_{shaft}(t)}{I_E}$$

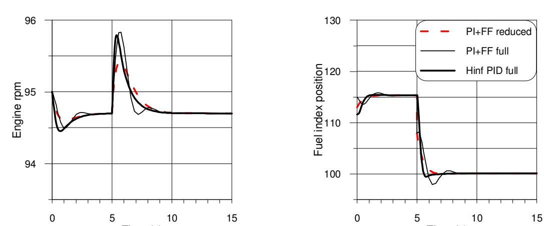
ROBUST PID CONTROL

- **Determination of Hinf PI+FF gains based on Hinf PID values**

$$K_P = \frac{I_E}{I_E - CK_{D0}} \cdot K_{P0}, K_I = \frac{I_E}{I_E - CK_{D0}} \cdot K_{I0}, K_{FF} = -\frac{K_{D0}}{I_E - CK_{D0}}$$

- **RPM regulation for "Shanghai Express"**

	($-K_P$) (%index/rpm)	($-K_I$) (%index/rpm/s)	K_{FF} (%index/kNm)
Hinf PI+FF	4.54	4.01	0.0226



CONCLUSIONS AT THE END OF THE 3rd SESSION

- Only full PID control provides with disturbance rejection and robustness against fast neglected dynamics
- D-term can be approximated by shaft torque signal

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