A Computational Approach for Decentralized Control of Turbine Engines

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Abstract

We propose a heuristic approach to approximately compute the optimal decentralized control for linear systems. The method exploits the notion of quadratic invariance, which characterizes a class of convex problems in decentralized control design, and extends the application to general unstructured models. The plant model is approximated such that the decentralized information structure is quadratically invariant under the approximate plant. Then the optimal design is efficiently found via convex optimization, and it is applied back to the original full plant. A simple convex condition to prove the closed loop stability in this setup is presented. The method finds a satisfactory decentralized control design efficiently, and furthermore, the resulting design can be used as a good initial point for local optimization algorithms. A numerical example on a simplified turbine engine model is presented for demonstration.

1   Introduction

Multiple control units in decentralized architectures offer solutions to overcome the inevitable problems with the classical centralized control systems, by reducing the computation and communication burden imposed on the central control unit. We consider such decentralized control architectures where the control functions are distributed to several units, each with access to a different subset of the measurements or delayed information. The units may be able to communicate, typically over a data network, with associated random delays and limited bandwidth. Systematic and efficient design of decentralized policies in such architectures is a fundamental and central issue for networked control.

One of the critical factors limiting these technological developments is that the model-based control synthesis procedures which have been so effective at centralized control do not currently have counterparts for decentralized control. Although good heuristics are known in some cases, and certain special cases have been solved exactly, for the general problem there is currently no method that can in general numerically compute, for example, the optimal mean-square performance achievable by decentralized control, even for the highly specialized scenario of low dimensional linear time-invariant state-space systems. It no longer fits within the existing paradigm (*Riccati* equations, etc) for optimal centralized control problems; this is the key obstacle to the overall problem, and a tractable algorithm for finding the optimal controller, even the optimal linear controller, does not yet exist [4].

A number of local optimization algorithms have been suggested [6, 7, 8, 10], of which the convergence largely depends on the choice of initial feasible point. Branch-and-bound techniques were applied to find the globally optimal design [1, 14], however they usually result in extremely large computational load even for small-sized problems. A recent work [13] introduces the notion of *quadratic invariance (QI)* which characterizes the largest known class of tractable problems in structured control design problems. It shows that for a large range of practical problems, one can compute the minimum achievable mean-square error, and a controller which is optimal.

We propose a heuristic design procedure extending the application of quadratic invariance to general nonstructured plants. It reduces the synthesis problem to a quadratically-invariant one, where well-known computational techniques based on semidefinite programming may be used. Given a desired decentralized control structure, an approximate plant model which is quadratically invariant with the given structure is found. Then, the optimal solution for the approximate plant is easily computed, and it is expected to attain acceptable control performance when applied to the original full plant; in fact, a simple convex condition guarantees this. The design can be further improved by existing local search algorithms. In this paper, a heuristic coordinatewise search scheme is used to locally solve the bilinear matrix inequality (BMI) representation.

The suggested method was applied to a jet engine control design problem. Current engine control systems are typically a centralized control characterized by a Full Authority Digital Engine Control (FADEC) with point-to-

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point analog communications to sensors and actuators. The FADEC is a large and heavy computer system which is often fuel cooled to protect the control electronics. Furthermore, future systems with advanced control capability and enhanced health management functions will require additional sensor/actuator units and more frequent communication. Therefore, engine control systems that have distributed processing elements and decentralized control functions are anticipated [2, 3].

The numerical experiments demonstrate that this unique design procedure efficiently computes decent decentralized controllers, and the proposed methods can be served as a promising alternative to the existing local optimization techniques for designing decentralized controllers.

2 Decentralized Control Design

For centralized control architectures with widely accepted control objective functions such as the $\mathcal{H}_2$ norm or the $\mathcal{H}_\infty$ norm of the closed-loop system, there are several well known methods that solve the problem efficiently. Such problems can be expressed using the following generalized plant description.

$$\text{minimize} \quad \left\| P_{11} + P_{12}K(I - GK)^{-1}P_{21} \right\|$$
$$\text{subject to} \quad K \text{ stabilizes } P$$

where P_{11}, P_{12}, and P_{21} describe the input-output interconnection of the models. G represents the plant model and K is the controller to be designed.

The above objective function is not convex in the variable K, but the problem can be transformed to a convex one by change of variables according to $Q = K(I - GK)^{-1}$. The optimal control K^* is generally full, which represents the centralized control authority.

Synthesizing optimal decentralized controls requires additional constraints and this makes the problem far harder. Optimal decentralized control synthesis can be described as follows, by adding the structural constraint.

$$\text{minimize} \quad \left\| P_{11} + P_{12}K(I - GK)^{-1}P_{21} \right\|$$
$$\text{subject to} \quad K \text{ stabilizes } P$$
$$K \text{ satisfies the } \textit{information constraint}$$

The last constraint, which is generally a sparsity pattern constraint, describes the decentralized control architectures. The change of variables, which helped the centralized problem, is of no use in this case since it transforms the linear information constraints to complex nonconvex constraints. In general, finding the optimal control for such a decentralized setup is very hard, and no algorithm is known to efficiently solve the problem in polynomial time [4]. In the next section, we suggest a design method for an approximate solution to such problems, based on the recently introduced quadratic invariance idea.

2.1 Quadratic invariance method

Quadratic invariance characterizes a simple algebraic condition of the plant model and the controller model, under which the optimal decentralized control problem reduces to a convex problem.

Suppose $\mathcal{U}$ and $\mathcal{Y}$ are Banach spaces, and let F be the space of functions $K \colon \mathcal{Y} \to \mathcal{U}$. As a general representation of decentralization constraints, we call a subspace $S \subset F$ an information constraint.

We consider finding optimal linear controllers, and define the following class of information constraints.

Definition 1. *Suppose $G \colon \mathcal{U} \to \mathcal{Y}$ is linear, and S is an information constraint. S is called **quadratically invariant** under G if every element of S is linear, and*

$$KGK \in S \quad \text{for all } K \in S$$

We can show that for the linear decentralized control problem with a quadratically invariant information constraint, the optimal controller may be found via convex optimization. Further, this controller is optimal over the class of all controllers; *i.e.*, no nonlinear controller has better performance.

Theorem 2. *Suppose $G \colon \mathcal{U} \to \mathcal{Y}$ is linear, S is a closed quadratically invariant information constraint, and for every K in the subspace S the operator $I - GK$ is invertible. Then*

$$K \in S \quad \Longleftrightarrow \quad K(I - GK)^{-1} \in S$$

Proof. See [13] for details. ∎

The theorem says that the quadratic invariance guarantees the convexity of the information constraint set under the transformation according to $Q = K(I - GK)^{-1}$. This gives the equivalent problem.

$$\text{minimize} \quad \| P_{11} + P_{12}QP_{21} \|$$
$$\text{subject to} \quad Q \in \mathcal{RH}_\infty \cap S$$

This is now an infinite dimensional convex optimization problem, and the $\mathcal{H}_2$ norm case can be solved by standard methods [12]. This implies that if the system and the controller jointly satisfy some simple algebraic condition, the optimal decentralized control problem may be easily solved.

The notion of quadratic invariance is powerful for plants with some sparsity patterns. However it is not appropriate for application to general full models, *i.e.*, for full plant models, the only quadratically invariant class of controllers are full (centralized) controllers.

In approximately computing the optimal decentralized control laws for general linear systems, we suggest an intuitive heuristic procedure extensively applying the notion of quadratic invariance to the full models.

Quadratic invariance method:

1. Desired decentralized control structure S is specified, on the given full plant G. Say,

$$S = \begin{bmatrix} \bullet & \bullet \\ \circ & \bullet \end{bmatrix} \qquad G = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \end{bmatrix}$$

Note that S is not quadratically invariant under G and finding optimal $K^* \in S$ is hard.

2. Find a sparse approximation $\tilde{G}$ for the given model, such that S is quadratically invariant under $\tilde{G}$.

$$\tilde{G} = \begin{bmatrix} \bullet & \bullet \\ \circ & \bullet \end{bmatrix}$$

Now the optimal control $\tilde{K} \in S$ can be efficiently computed.

3. Find the optimal control $\tilde{K}$ for $\tilde{G}$.

$$\tilde{K} = \begin{bmatrix} \bullet & \bullet \\ \circ & \bullet \end{bmatrix} \in S \qquad \tilde{G} = \begin{bmatrix} \bullet & \bullet \\ \circ & \bullet \end{bmatrix}$$

4. Apply the computed control $\tilde{K}$ back to G.

$$\tilde{K} = \begin{bmatrix} \bullet & \bullet \\ \circ & \bullet \end{bmatrix} \qquad G = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \end{bmatrix}$$

Suppose that a control structure requirement, *e.g.*, diagonal, triangular, or some other sparsity patterns that meets the system design specification, is given. If it allows some sparse plant models under which the control structure is quadratically invariant, then we can approximate the plant model to such sparse ones and compute the optimal decentralized control for the approximate plant via off-the-shelve convex optimization tools.

The approximate plant model is obtained by truncating some elements of the original transfer function matrix. For example, suppose that the control structure S is required on the full plant G, where

$$S = \begin{bmatrix} \bullet & \bullet & \bullet \\ \circ & \circ & \bullet \\ \circ & \circ & \bullet \end{bmatrix} \qquad G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}$$

It can easily be checked that S is quadratically invariant under upper triangular plants. Hence the sparse approximation $\tilde{G}$ of G is simply

$$\tilde{G} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ 0 & g_{22} & g_{23} \\ 0 & 0 & g_{33} \end{bmatrix}$$

We may use the computed solution $\tilde{K}$ back to the original full plant G, and it is expected to attain acceptable control performance provided that the truncated sparse model dominates the full system dynamics, as in weakly coupled systems.

2.2 Stability condition

An intuitive conjecture is that for $\|G - \tilde{G}\|_\infty$ sufficiently small, applying the QI-designed optimal control $\tilde{K} \in S$ back to the original plant G would still stabilize the system. To claim this formally, let us partition G into the QI part $\tilde{G}$, and the off-QI part G_d.

$$G = \tilde{G} + G_d$$

where the control S is quadratically invariant under $\tilde{G}$, and trace $\tilde{G}^T G_d = 0$.

Lemma 3. *Consider a stable plant G, and let $\tilde{K}$ be the QI-designed optimal decentralized control on $\tilde{G}$. Then the feedback interconnection of $\tilde{K}$ on G is internally stable if $\|G_d \tilde{Q}\|_\infty < 1$, where $\tilde{Q} = \tilde{K}(I - \tilde{G}\tilde{K})^{-1}$.*

Proof. The feedback system is internally stable if and only if $\tilde{K}(I - G\tilde{K})^{-1}$ is stable. Also, the following identity holds.

$$\tilde{K}(I - G\tilde{K})^{-1} = \tilde{K}\big(I - (I - \tilde{G}\tilde{K})^{-1}G_d\tilde{K}\big)^{-1}(I - \tilde{G}\tilde{K})^{-1}$$
$$= \tilde{Q}(I - G_d\tilde{Q})^{-1}$$

Since G_d and $\tilde{Q}$ are stable, the above is stable if $\|G_d\tilde{Q}\|_\infty < 1$, as required. ∎

The above can be interpreted as the robust stability condition of the QI system with respect to the additive model uncertainty G_d.

We can derive a similar condition for unstable plants. Consider an unstable plant G with the unstable QI part $\tilde{G}$, and suppose that $K_n \in S$ stabilizes both G and $\tilde{G}$. Then the optimal decentralized control for the unstable $\tilde{G}$ is obtained from the optimal solution $\tilde{Q}$ associated with the following prestabilized plant.

$$\begin{aligned} \text{minimize} \quad & \big\| P_{11} + P_{12}K_n(I - \tilde{G}K_n)^{-1}P_{21} \\ & \quad + P_{12}(I - K_n\tilde{G})^{-1}Q(I - \tilde{G}K_n)^{-1}P_{21} \big\| \\ \text{subject to} \quad & Q \in \mathcal{RH}_\infty \cap S \end{aligned}$$

where the optimal control $\tilde{K}$ is determined by $\tilde{K} = K_n + \tilde{Q}(I + \tilde{G}_c\tilde{Q})^{-1}$, where $\tilde{G}_c = \tilde{G}(I - K_n\tilde{G})^{-1}$.

Corollary 4. *Suppose that an unstable plant G and the unstable QI part $\tilde{G}$ are stabilized by K_n, and let $\tilde{K}$ be the QI designed optimal decentralized control on $\tilde{G}$. Then the feedback interconnection of $\tilde{K}$ on G is internally stable if $\|(G_c - \tilde{G}_c)\tilde{Q}\|_\infty < 1$, where $G_c = G(I - K_nG)^{-1}$, $\tilde{G}_c = \tilde{G}(I - K_n\tilde{G})^{-1}$ and $\tilde{Q} = (\tilde{K} - K_n)\big(I - \tilde{G}_c(\tilde{K} - K_n)\big)^{-1}$.*

Proof. The prestabilized plant can be described by

$$\left[\begin{array}{c|c} P_{11} + P_{12}K_n(I - \tilde{G}K_n)^{-1}P_{21} & P_{12}(I - K_n\tilde{G})^{-1} \\ \hline (I - \tilde{G}K_n)^{-1}P_{21} & \tilde{G}_c \end{array} \right]$$

Since S is quadratically invariant under $\tilde{G}_c$[13], Lemma 3 can be directly extended to the prestabilized plant, which leads to the above statement. ∎

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Now the QI design optimization can be modified including this stability condition. For stable plants, the optimal solution $\tilde{Q}$ to the following problem

$$\begin{aligned} \text{minimize} \quad & \|P_{11} + P_{12}QP_{21}\| \\ \text{subject to} \quad & Q \in \mathcal{RH}_\infty \cap S \\ & \|G_d Q\|_\infty < 1 \end{aligned}$$

and $\tilde{K} = \tilde{Q}(I + \tilde{G}\tilde{Q})^{-1}$ guarantees the closed loop stability with the original plant. The counterpart for unstable plants can be derived equivalently using the prestabilized plant description.

The above is an infinite dimensional convex optimization problem. In this case, solving for the exact solution is not obvious as in [12] because of the last $\mathcal{H}_\infty$ norm condition. A series of convergent solutions may be found using the finite dimensional approximation techniques [5, 9].

2.3 Performance improvement by coordinatewise descent method

The design obtained by the quadratic invariance method is optimal for the approximated model. However, the design is not necessarily optimal for the original model; possibly not even locally optimal. Thus it can be further improved by local search methods such as an iterative algorithm presented below.

Express the plant P and the controller K in the following state-space realization.

$$P : \left[\begin{array}{c|cc} A & B_w & B \\ \hline C_z & D_{zw} & D_z \\ C & D_w & 0 \end{array}\right] \qquad K : \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

If we constrain the optimization over a set of fixed order controllers, the decentralized $\mathcal{H}_2$ control problem in the previous section can be equivalently written in the following BMI representation.

$$\begin{aligned} \text{minimize} \quad & \text{trace } \mathcal{Q} \\ \text{subject to} \quad & \left[\begin{array}{cc} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B} \\ \mathcal{B}^T\mathcal{P} & -I \end{array}\right] < 0 \\ & \left[\begin{array}{cc} \mathcal{P} & \mathcal{C}^T \\ \mathcal{C} & \mathcal{Q} \end{array}\right] > 0, \quad \mathcal{P} > 0, \quad \mathcal{D} = 0 \\ & \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \begin{array}{l} \text{satisfies the} \\ \textit{information constraint} \end{array} \end{aligned}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ describe the closed loop dynamics.

$$\begin{aligned} & \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] \\ = & \left[\begin{array}{cc|c} A + BD_KC & BC_K & B_w + BD_KD_w \\ B_KC & A_K & B_KD_w \\ \hline C_z + D_zD_KC & D_zC_K & D_{zw} + D_zD_KD_w \end{array}\right] \end{aligned}$$

Coordinatewise descent method:

1. Initialize a controller with the design obtained from the QI method.

2. Fix (A_K, B_K, C_K, D_K), and solve the resulting SDP in $\mathcal{P}$ and $\mathcal{Q}$.

3. Fix $\mathcal{P}$, and solve the resulting SDP in (A_K, B_K, C_K, D_K) and $\mathcal{Q}$.

4. Go to 2 and iterate the process until the progress reaches the termination criteria.

Note that the first matrix inequality is bilinear in (A_K, B_K, C_K, D_K) quadruple and $\mathcal{P}$. However for fixed (A_K, B_K, C_K, D_K), it is linear in $\mathcal{P}$ and $\mathcal{Q}$, reducing the problem to a semidefinite programming (SDP). Similarly the problem reduces to an SDP for fixed $\mathcal{P}$.

Based on this observation, we can improve the QI design by iteratively solving the two alternating LMIs. Note that this process guarantees to monotonically nonincrease the objective value from the initial design.

Since the convergence of such a local coordinatewise descent scheme is sensitive to the choice of the initial controller, the QI design as the initial point can result in a very useful design in practice.

3 Numerical Example

3.1 Decentralized control of a turbine engine

A linearized model of the GE F404 turbine engine at the rated thrust condition at 35,000 ft altitude was taken from [11], and then scaled for design convenience. The scaled model follows below, and the states, the measurements, and the control inputs are given in Table 1.

$$\begin{aligned} & \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & D_p \end{array}\right] \\ = & \left[\begin{array}{ccc|cc} -1.4600 & 3.3880 & 0 & 0.1840 & 0.4578 \\ 0.2219 & -2.2300 & 0 & 0.1630 & 0.0015 \\ 1.4670 & -4.8375 & -0.4000 & 1.5325 & -0.0978 \\ \hline 0 & 0 & 1.0000 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \end{array}\right] \end{aligned}$$

The above model has no pure integrator, thus needs to be augmented with additional integrators at the input terminal in order to achieve zero steady state error. The augmented system with $x^T = \begin{bmatrix} u_p^T & x_p^T \end{bmatrix}$ and $u = \dot{u}_p$ is shown below.

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|c} 0 & 0 & I \\ B_p & A_p & 0 \\ \hline D_p & C_p & 0 \end{array}\right]$$

The classical LQG/LTR approach suggests a target

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feedback loop of the following form.

$$\begin{aligned}\dot{x} &= Ax + Bu + Lw \\ y &= Cx + \sqrt{\mu}v\end{aligned} \qquad L = \begin{bmatrix} -(C_p A_p^{-1} B_p)^{-1} \\ C_p^T (C_p C_p^T)^{-1} \end{bmatrix}$$

where w and v can be interpreted as zero mean process and measurement noise with unit intensity, *i.e.*, $\mathbf{E}ww^T = I$, $\mathbf{E}vv^T = I$. L is chosen as above to match the singular values in all directions at low and high frequency regions.

The design problem with $z^T = \begin{bmatrix}(Cx)^T & \sqrt{\rho}u^T\end{bmatrix}$ can be written in the following generalized plant description. $\mu = 0.01$ and $\rho = 10^{-6}$ are chosen here.

$$\begin{bmatrix} z \\ \hline y \end{bmatrix} = \begin{bmatrix} C(sI-A)^{-1}L & 0 & C(sI-A)^{-1}B \\ 0 & 0 & \sqrt{\rho}I \\ \hline C(sI-A)^{-1}L & \sqrt{\mu}I & C(sI-A)^{-1}B \end{bmatrix} \begin{bmatrix} w \\ v \\ \hline u \end{bmatrix}$$

Two decentralized $\mathcal{H}_2$ controls (an upper triangular controller and a diagonal controller) were synthesized using the QI method. The centralized $\mathcal{H}_2$ control (LQG solution) is also presented here and compared for commanded step changes in turbine temperature. The stability measure, $\|G_d\tilde{Q}\|_\infty$, turns out to be well below 1 for both optimal QI designs (0.7183 for the upper triangular control and 0.9026 for the diagonal control). Therefore relaxing the stabililty constraint $\|G_d\tilde{Q}\|_\infty < 1$ does not change the optimal solution in these cases, and the solutions presented here were obtained as such. This reduces the computational complexity in the design optimization.

Simulation results are shown in Figure 1, where slight degradation in results between the centralized and the decentralized cases, including lack of disturbance rejection, is observed. Because the decentralized controls work with less information compared to the full centralized case, the observed performance degradation is not surprising at all.

Both of the QI designs can be further improved by the coordinatewise descent (CD) method, though only the diagonal controller is demonstrated here. The convergence profile and the response to turbine temperature command are shown in Figure 3 and Figure 2.

Achieved $\mathcal{H}_2$ norms are summarized in Table 2, which displays the obvious improvement in the QI+CD case compared to the QI only case.

	Variable	Description
x_p	N_2	Fan speed
	$N_{2.5}$	High pressure compressor speed
	$T_{t4.5}$	Turbine total temperature
y_p	$T_{t4.5}$	Turbine total temperature
	N_2	Fan speed
u_p	W_f	Fuel flow rate
	A_j	Nozzle area

Table 1: State variables (x_p), measurements (y_p), and control inputs (u_p) for the turbine engine model. All the variables are scaled.
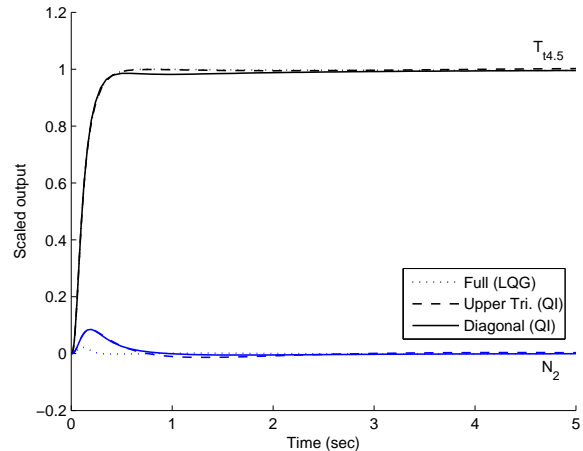

Figure 1: Response to turbine temperature command. An upper triangular controller and a diagonal controller, both designed by the QI method, are compared with the centralized LQG control.

Controller	Objective value $\left(\|\cdot\|_2^2\right)$
Full (LQG)	0.2990
Diagonal (QI)	0.3214
Diagonal (QI+CD)	0.3168

Table 2: Achieved objective values (squared $\mathcal{H}_2$ norms)

4 Concluding Remarks

We proposed a new design approach for decentralized control problems, and presented a simple condition to guarantee stability of the closed loop system.

The procedure in this paper makes use of a heuristic to reduce the synthesis problem to a quadratically-invariant one, for which well-known computational techniques exist. The plant model was approximated such that the decentralized information structure is quadratically invariant under the approximated plant model. Then the optimal design was efficiently found using convex optimization techniques, and it was applied to the original plant. A simple convex condition on the controller variables was shown to guarantee the stability of the suggested design method.

The designed controller was further improved by a coordinatewise descent method, which monotonically non-increase the objective value. Since the convergence of the coordinatewise descent method is sensitive to the performance of the initial controller, the QI design as an initial guess can be a clever choice.

Simulation results demonstrate that the proposed approach finds a decent control design for a simplified linear jet engine model efficiently. The designed decentral-
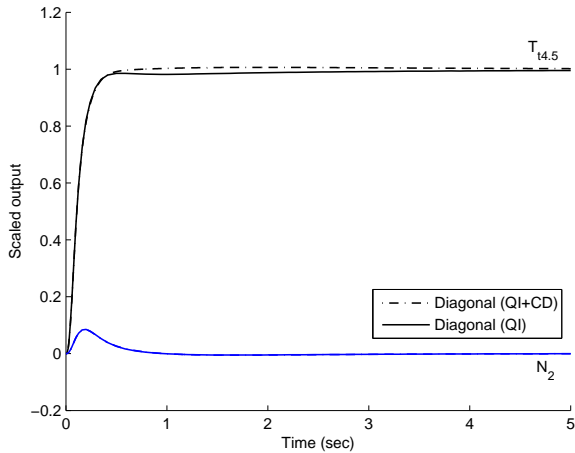
Figure 2: Response to turbine temprature command. The diagonal controller designed by the QI method (solid) is improved by the coordinatewise descent method (dash-dotted).
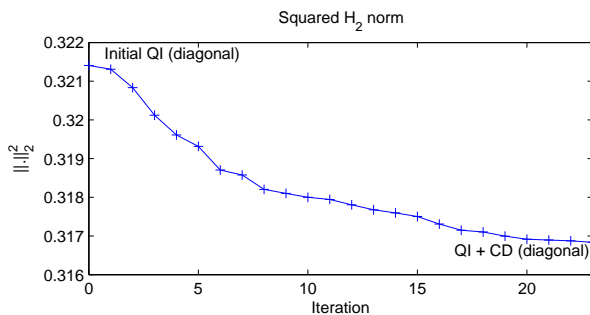

Figure 3: Performance improvement by the coordinatewise descent method.

ized controller exhibited approximately 10% performance degradation compared to the centralized control case.

Information delay throughout the communication networks can be considered in the same framework, and the performance degradation from it can be investigated too. Hence the proposed design technique is able to present promising candidates for future control systems in decentralized architectures with distributed intelligence.

Further studies may include computational techniques to manage the stability/performance condition in the design optimization. This will lend the proposed method to a systematic synthesis for a class of decentralized optimal control problems. Addressing the same problem in the robust control framework will be interesting too. More practically, realistic engine models including the impact of the communication delay should be considered.

References

[1] V. Balakrishnan and S. Boyd. Global optimization in control system analysis and design. In *Control and Dynamic Systems: Advances in Theory and Applications*, volume 53. Academic Press, 1992.

[2] A. Behbahani. Adaptive distributed intelligent control architecture for future propulsion systems. In *Proc. 61st Meeting of the Society for Machinery Failure Prevention Tech.*, Apr. 2007.

[3] A. Behbahani et al. Status, vision, and challenges of an intelligent distributed engine control architecture. In *Proc. SAE AeroTech Congress & Exhibition*, Sep. 2007.

[4] V. Blondel and J. Tsitsiklis. A survey of computational complexity results in systems and control. *Automatica*, 36:1249–1274, 2000.

[5] S. Boyd and C. Barratt. *Linear Controller Design: Limits of Performance*. Prentice-Hall, 1991.

[6] L. El Ghaoui and V. Balakrishnan. Synthesis of fixed-structure controllers via numerical optimization. In *Proc. 33rd IEEE Conference on Decision and Control*, volume 3, pages 2678–2683, Dec. 1994.

[7] J. Han and R. Skelton. An LMI optimization approach for structured linear controllers. In *Proc. 42nd IEEE Conference on Decision and Control*, volume 5, pages 5143–5148, Dec. 2003.

[8] A. Hassibi, J. How, and S. Boyd. A path-following method for solving BMI problems in control. In *Proc. 1999 American Control Conference*, volume 2, pages 1385–1389, Jun. 1999.

[9] H. Hindi, B. Hassibi, and S. Boyd. Multi-objective H_2/H_∞-optimal control via finite dimensional Q-parametrization and linear matrix inequalities. In *Proc. 1998 American Control Conference*, volume 5, pages 3244–3249, Jun. 1998.

[10] T. Iwasaki. The dual iteration for fixed order control. In *Proc. 1997 American Control Conference*, volume 1, pages 62–66, Jun. 1997.

[11] P. Kapasouris and M. Athans. Multivariable control systems with saturating actuators antireset windup strategies. In *Proc. 1985 American Control Conference*, pages 1579–1584, Jun. 1985.

[12] M. Rotkowitz and S. Lall. On computation of optimal controllers subject to quadratically invariant sparsity constraints. In *Proc. 2004 American Control Conference*, volume 6, pages 5659–5664, Jun. 2004.

[13] M. Rotkowitz and S. Lall. A characterization of convex problems in decentralized control. *IEEE Transactions on Automatic Control*, 51(2):274–286, Feb. 2006.

[14] J. VanAntwerp, R. Braatz, and N. Sahinidis. Globally optimal robust process control. *Journal of Process Control*, 9:375–383, Oct. 1999.