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Cross-Layer Design for Lifetime Maximization in Interference-Limited Wireless Sensor Networks

Ritesh Madan, Shuguang Cui, Sanjay Lall, and Andrea Goldsmith

***Abstract*— We consider the joint optimal design of physical, medium access control (MAC), and routing layers to maximize the lifetime of energy-constrained wireless sensor networks. The problem of computing a lifetime-optimal routing flow, link schedule, and link transmission powers is formulated as a non-linear optimization problem. We first restrict the link schedules to the class of interference-free time division multiple access (TDMA) schedules. In this special case we formulate the optimization problem as a mixed integer-convex program, which can be solved using standard techniques. For general non-orthogonal link schedules, we propose an iterative algorithm that alternates between adaptive link scheduling and computation of optimal link rates and transmission powers for a fixed link schedule. The performance of this algorithm is compared to other design approaches for several network topologies. The results illustrate the advantages of load balancing, multihop routing, frequency reuse, and interference mitigation in increasing the lifetime of energy-constrained networks. We also describe a partially distributed algorithm to compute optimal rates and transmission powers for a given link schedule.**

***Index Terms*— mathematical programming, optimization, cross-layer design, sensor networks, network lifetime**

I. Introduction

We consider a network of wireless sensor nodes distributed in a region. Each node has a limited energy supply and generates information at a fixed rate that needs to be communicated to a sink node. We assume that each node can vary its transmission power, modulation scheme, and duty cycle. *The focus of this paper is on the computation of optimal transmission powers, rates, and link schedule that maximize the network lifetime.* The network is considered to be alive while all nodes still have some energy; the lifetime is the earliest time at which a node runs out of energy.

We will use *transmission scheme/strategy* to refer to the data rates, transmission powers, and link schedule for a network. For energy-constrained wireless networks, we can increase the network lifetime by using transmission schemes that have the following characteristics.

1) *Multihop routing*: In wireless environments the received power typically falls off as the mth power of distance, with $2 \leq m \leq 6$. Hence, we can conserve transmission energy by using multihop routing [1], [2].
2) *Load Balancing*: If a node is on the routes of many source destination pairs, it will run out of energy very quickly. Hence, load balancing is necessary to avoid the creation of hot spots where some nodes die out quickly and cause the network to fail [3].
3) *Interference mitigation*: Links that strongly interfere with each other should be scheduled at different times to decrease the energy consumption on these links [4].
4) *Frequency reuse*: Weakly interfering links should be scheduled together so that each link has a longer duration of time to transmit the same amount of data. This reduces the average transmission power on each link [5].

We note that there is an inherent trade-off between using minimum energy routes and load balancing. Minimum energy routing may require some nodes to lie on routes of many source destination pairs, which may cause them to run out of energy quickly. The network lifetime can be increased by load balancing, where a part of the data may be transmitted over energy suboptimal paths. Also, there is a trade-off between scheduling each link for a larger amount of time to decrease energy consumption for data transmission, and interference mitigation by scheduling strongly interfering links at different times.

A. Prior Work

Cross-layer design for throughput maximization has received a lot of attention over the past few years. Achievable rate combinations for wireless networks were computed in [6]. Throughput maximization by joint design of power control, link scheduling, and routing layers was considered in, for example, [7], [8], [9], [10], [11].

The design challenges and the importance of cross-layer design for meeting application requirements in energy-constrained networks were described in [12]. For wireless sensor networks, we may not always need to operate on the boundary of the achievable rates region. Thus we have a choice among various transmission strategies (routing, power control, scheduling) that we can exploit to increase the lifetimes of such networks. An overview of the synergy between the various layers was given

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in [13]. A brief overview of recent work on optimization of different layers of a wireless network for minimizing the total energy consumption and maximizing the network lifetime is given below.

1) *Physical Layer*: For a given rate vector, power control can be used to conserve energy. An overview of power control was given in [14]. Energy-optimal modulation schemes for coded and uncoded systems were studied in [5], where given the number of bits to transmit with a certain deadline, an optimal constellation size was computed to minimize the total transmission and circuit energy. Also, it was shown that if we relax the constraint that the constellation size is an integer, the energy minimization problem can be approximated by a convex optimization problem.
2) *MAC*: The physical layer results in [5] were extended in [15] to compute the TDMA time slot lengths to minimize the total energy consumption in a network consisting of many transmitters and one receiver. However, note that the problem of deciding the slot allocation to links becomes combinatorial when we consider an arbitrary wireless network, with the possibility of interfering links being scheduled in the same time slot.
3) *Routing*: Routing algorithms for wireless networks have traditionally focused on minimizing the total energy consumption. However, as pointed out in [3], minimum energy routing can lead to some nodes in the network being drained of energy very quickly. Hence, instead of trying to minimize the total energy consumption, routing to maximize the network lifetime was considered in [16], [3]. Distributed algorithms to compute a routing scheme to maximize the network lifetime were proposed in [3], [17], [18], [19].
4) *Cross-layer*: Joint scheduling and power control to reduce energy consumption and increase single hop throughput was considered in [20]. Cross-layer design based on computation of optimal transmission powers, link schedule, and routing flow was described in [4]. The aim of that paper was to minimize the average transmission power over an infinite horizon. Also, the routing flow was computed in an incremental manner: it used the Lagrange multipliers obtained at each step by solving an optimization problem of possibly exponential complexity in the number of links. Energy efficient power control and scheduling, with no rate adaptation on links, for QoS provisioning were considered in [13]. Cross-layer design with emphasis on detailed modeling of circuit and transmission energy, and restriction of MAC to variable length TDMA was described in [21]. Joint routing, power control, and scheduling for a TDMA-CDMA network was considered in [22].

We consider a slotted time model, and guarantee the satisfaction of average rate requirements over a pre-defined frame duration; this is similar to the model in [13]. As in [4], [13], we use a realistic model of the interference between all links that transmit at the same time. This is more general than the interference model considered in [22], where the schedule for links in the same neighborhood was assumed to be orthogonal, and interference from distant links was neglected. For the special case of orthogonal link schedules, we compute the exact optimal transmission strategy. Also, unlike in [13], we consider *rate adaptation* on links for a fixed bit error rate (BER) requirement. As shown in [21], rate adaptation can lead to a significant decrease in energy consumption. However, allowing for rate adaptation on links makes the problem considerably more complex. We no longer have a linear constraint on transmission powers [13] to guarantee an SINR greater than a threshold required for a fixed rate and BER. Instead we have a non-linear and non-convex constraint on the rate and power of each link (see Section 2.4 for details). In addition, we consider joint routing along with link scheduling and power control (with rate adaptation). Also, instead of minimizing the total average power consumption over the network, we maximize network lifetime.

B. Outline

The next section lists the assumptions and formally defines the problem of computing the transmission powers, data rates, and link schedule to maximize the network lifetime, as an optimization problem. We show how to solve this problem exactly if we restrict the MAC layer to only TDMA schemes (interference-free case). In Section 3, we find a convex approximation to the problem of computing an optimal data rate and transmission power, for each link and time slot, for a fixed link schedule. Then we describe an algorithm that uses a heuristic to adapt the link schedule to the resulting optimal rates and powers. Section 4 describes the results of numerical studies of the algorithm for different network topologies. The results also illustrate the advantages of frequency reuse, interference mitigation, load balancing, and multihop routing in energy-constrained wireless networks. Section 5 describes a partially distributed algorithm to compute the link rates and transmission powers for a fixed link schedule. Section 6 gives the conclusions and possible directions for future work.

II. Problem Definition

A. System Model

We consider a static wireless network. We will make the following assumptions about the network.

1) A link exists from node i to node j, if the received power at node j when node i transmits at maximum power, is greater than a predefined threshold.
2) The channel over each link is an additive white Gaussian noise channel (AWGN), with fixed noise power. Also, we assume a deterministic path loss model where the power falls off as d^m for distance d, with $2 \leq m \leq 6$.
3) The maximum rate per unit bandwidth that can be supported over a link with SINR γ is

$$r = \log(1 + K\gamma)$$

where $K = -1.5/(\log(5\text{BER}))$. This is a good model for modulation schemes such as MQAM with constellation size greater than or equal to 4 [23]. For notational convenience we will consider r to be in nats/Hz/s, i.e. we use the natural logarithm. We will assume that a link can vary the constellation size over each subsequent time slot. Also, we relax the constraint that the constellation size is an integer, thus we allow r to take all values in $\mathbb{R}_+$.

4) At any given time, a node can either transmit to, or receive from, at most one other node in the network.
5) For the MAC layer, we assume that the network time shares between different transmission modes in a periodic fashion. The schedule is periodic with N time slots; during each slot the network uses one transmission mode - i.e. one set of powers and rates over each link. Also, the flow conservation equations need to be satisfied over every frame of N time slots.
6) If a node transmits at power P, the power consumption in the power amplifier circuit is given by $(1+\alpha)P$. The constant $\alpha > 0$ represents the inefficiency of the power amplifier. The power consumption values of the transmitter circuit (other than the power amplifier) and the receiver circuit are modeled as constants P_{ct} and P_{cr}, respectively [5].

B. Notation

We will use the following notation. Let

1) $\mathcal{G} = (V, L)$ denote the directed graph representing the network. V is the set of wireless nodes and L is the set of directed links.
2) $A \in \mathbb{R}^{|V|\times|L|}$ denote the incidence matrix of the graph $\mathcal{G}$. We have
$$A(v,l) = \begin{cases} 1 & \text{if } v \text{ is the transmitter of link } l \\ -1 & \text{if } v \text{ is the receiver of link } l \\ 0 & \text{otherwise} \end{cases}$$
Let us write
$$A = A^+ - A^-$$
such that $A^+(v,l), A^-(v,l) = 0$ if $A(v,l) = 0$, and A^+, A^- have only 0 and 1 entries.
3) N be the number of time slots in each frame of the periodic schedule.
4) L^n denote the set of links scheduled - i.e. allowed to transmit - during time slot $n \in \{1, \ldots, N\}$. Also, define the vectors $\mathbf{1}_t(P^n), \mathbf{1}_r(P^n) \in \mathbb{R}^{|V|}$ as follows:
$$(\mathbf{1}_t(P^n))_v = \begin{cases} 1 & \text{if } (a_v^+)^T P^n > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$(\mathbf{1}_r(P^n))_v = \begin{cases} 1 & \text{if } (a_v^-)^T P^n > 0 \\ 0 & \text{otherwise} \end{cases}$$
where $(a_v^+)^T$ and $(a_v^-)^T$ denote the vth row of the matrices A^+ and A^-, respectively. Thus the vectors give the sets of nodes that transmit and receive data, respectively, in each time slot.
5) P_l^n and r_l^n denote the transmission power and rate per unit bandwidth, respectively, over link l and slot n. $r^n, P^n \in \mathbb{R}^{|L|}$ will be used to denote the corresponding vectors for time slot n.
6) $P_l^{\max}$ be the maximum transmission power of the transmitting node of link l. The corresponding vector is $P^{\max} \in \mathbb{R}^{|L|}$.
7) $E \in \mathbb{R}^{|V|}$ be such that E_i denotes the initial amount of energy at node i.
8) P_{ct} and P_{cr} be the power consumption of the transmitter and the receiver circuits at a node, respectively. These values are assumed to be the same across all nodes.
9) $G \in \mathbb{R}^{|L|\times|L|}$ denote the link gain matrix of the network. G_{lk} denotes the power gain from the transmitter of link k to the receiver of link l.
10) s_i denote the rate at which information is generated at node i. This information needs to be communicated to the sink. Let $s \in \mathbb{R}^{|V|}$ be the vector whose entries are s_i.
11) N_0 denote the noise power; this is the total noise power over the bandwidth of operation.

C. Network Lifetime

Let T_v denote the lifetime of node v, that is the time at which it runs out of energy. Then the network lifetime is defined to be
$$T_{\text{net}} = \min_{v\in V, v\neq \text{sink}} T_v$$
This definition is the same as the one considered in [3]. Thus we consider a simplistic definition of network lifetime. We assume that all nodes are of equal importance and critical to the network operation. The optimization problem formulated in this paper can be interpreted as a problem to minimize the maximum ratio of power consumption to the initial energy at a node. Also, note that for each optimization problem (to maximize the network lifetime) formulated in this paper, we can formulate a similar corresponding optimization problem to minimize the total energy. Hence, the techniques in this paper can be used for energy minimization in the network as well.

D. Optimization Problem

The problem of maximizing the network lifetime can be written as the following optimization problem.

$$\begin{array}{ll} \text{max.} & T_{\text{net}} \\ \text{s.t.} & \frac{1}{N} A(r^1 + \ldots + r^N) = s \\ & r^n \succeq 0 \\ & \log\left(1 + K \frac{G_{ll} P_l^n}{\sum_{k\neq l} G_{lk} P_k^n + N_0}\right) \geq r_l^n \\ & 0 \preceq P^n \preceq P^{\max} \end{array}$$
$$\frac{T_{\text{net}}}{N} \sum_{n=1}^{N} \Big((1+\alpha) A^+ P^n + P_{ct} \mathbf{1}_t(P^n) + P_{cr} \mathbf{1}_r(P^n) \Big) \preceq E$$

for all $n = 1, \dots, N$ and $l \in L$. The variables are T_{net}, r_l^n, P_l^n, for $n \in \{1, \dots, N\}, l \in L$. Thus the solution gives optimal transmission modes during the N time slots of each frame, i.e. optimal transmission powers and rates over each link during each time slot. The constraints are explained below.

1) The first constraint is a set of flow conservation equations. The flow conservation equations are satisfied over each frame of N time slots.
2) The second constraint ensures the positivity of flows - the flow over a directed link can only be from the transmitter to the receiver.
3) The third constraint is a rate constraint over each link.
4) The fourth constraint forces the transmission power to be less than the maximum transmission power at each node.
5) The fifth constraint is an energy conservation inequality. The energy consumed by each node over time T_{net} should be less than or equal to the initial energy at the node.

As in [19], we can use a change of variable $q = 1/T_{\text{net}}$, to write the problem as an equivalent optimization problem.

Problem $\mathcal{P}1$:

$$
\begin{aligned}
\text{min.} \quad & q \\
\text{s.t.} \quad & A(r^1 + \ldots + r^N) = Ns \\
& \log\left(1 + \frac{G_{ll}P_l^n}{\sum_{k \neq l} G_{lk}P_k^n + N_0}\right) \geq r_l^n \\
& r^n \succeq 0, \quad 0 \preceq P^n \preceq P^{\max} \\
& \sum_{n=1}^{N}\Big((1+\alpha)A^+P^n + P_{ct}\mathbf{1}_t(P^n) + P_{cr}\mathbf{1}_r(P^n)\Big) \preceq qNE
\end{aligned}
$$

We have absorbed the system constant K into G_{ll}. Both the optimization problems considered in this Section are not convex optimization problems. We will first reduce the problem to a mixed integer-convex problem for TDMA link schedules. For the more general case of arbitrary link schedules, we will use an iterative approach that alternates between computation of rates and powers, and adaptation of the link schedule.

E. Optimal TDMA Schemes

The above problem when restricted to TDMA scheduling schemes can be rewritten as follows.

$$
\begin{aligned}
\text{min.} \quad & q \\
\text{s.t.} \quad & \sum_{l \in \mathcal{O}(v)} r_l n_l - \sum_{l \in \mathcal{I}(v)} r_l n_l = N s_v \\
& \log\left(1 + \frac{G_{ll}P_l}{N_0}\right) \geq r_l \\
& \sum_{l \in \mathcal{O}(v)} \big((1+\alpha)P_l + P_{ct}\big)n_l + \sum_{l \in \mathcal{I}(v)} P_{cr}n_l \leq qNE_v \\
& 0 \leq P \leq P_l^{\max}, \;\; r_l \geq 0 \\
& \sum_{l \in L} n_l \leq N, \quad n_l \in \{0, \dots, N\}
\end{aligned}
$$

for all $v \in V$. Here n_l is the number of slots allocated to link l, r_l is the rate of data transmission over link l and P_l is the transmission power over link l. Also, $\mathcal{O}(v), \mathcal{I}(v)$ denote the set of outgoing and incoming links, respectively, at node v. The variables are $q, r_l, n_l, P_l, l \in L$. The rate constraint equation is simplified in the absence of interference. Also, we have an additional constraint that the sum of the number of time slots allocated to the links should be less than or equal to the total number of time slots in a frame. An optimal strategy for a link uses constant transmission power over all the allocated time slots because there is no interference and the channel is time invariant. This was proved in [22] using the concavity of $f(x) = \log(1 + cx)$, for $c \geq 0, x \in \mathbb{R}$. Substituting

$$P_l = \frac{N_0}{G_{ll}}(e^{r_l} - 1)$$

and using a change of variables $x_l = r_l n_l$, we obtain an equivalent optimization problem in q, n_l, x_l.

$$
\begin{aligned}
\text{min.} \quad & q \\
\text{s.t.} \quad & \sum_{l \in \mathcal{O}(v)} x_l - \sum_{l \in \mathcal{I}(v)} x_l = N s_v \\
& x_l - n_l \log\left(1 + \frac{G_{ll}P_l^{\max}}{N_0}\right) \leq 0 \\
& \sum_{l \in \mathcal{O}(v)} \beta n_l \big(e^{\frac{x_l}{n_l}} - 1 + \frac{P_{ct}}{\beta}\big) + \sum_{l \in \mathcal{I}(v)} P_{cr} n_l \leq qNE_v \\
& x_l, n_l \geq 0 \\
& \sum_{l \in L} n_l \leq N, \quad n_l \in \{0, \dots, N\}
\end{aligned}
$$

where $\beta = \frac{N_0(1+\alpha)}{G_{ll}}$. The function $f(x, y) = \beta x e^{\frac{y}{x}}$ is convex over $x, y \geq 0$, for $\beta \geq 0$. Hence it is easy to see that the above problem is a *mixed integer-convex* problem. It can be solved using *branch and bound* methods (see, for example, [24], [25]). At each stage, a lower bound on optimal q can be computed by relaxing the integer constraint on n_l's, and an upper bound can be computed by rounding the solution of the relaxed problem. Even though branch and bound methods have worst case exponential complexity, for the examples computed in this paper, these methods were found to be quite efficient. If we relax n_l to take real values, the problem is convex and the computed solution gives an optimal variable-length TDMA scheme. The problem of deciding time slots for variable-length TDMA, integral constellation sizes and detailed modeling of circuit energy consumption was solved in [21].

III. Technical Approach

In this section we aim to find an algorithm to compute an optimal routing, scheduling and power control strategy with no restriction on link schedules. Thus we allow mutually interfering links to be scheduled to transmit in the same time slot. Since the problem formulation $\mathcal{P}1$ is not convex, it is difficult to solve. Hence, we take the following approach. For a fixed link schedule (i.e. fixed $L^n, n = 1, \dots, N$), we approximate the rate constraint as a convex constraint; this gives a convex set contained in the feasible set of the original problem. The resulting problem is a convex optimization problem that solves for optimal rates and powers for a given link schedule. During each iteration we compute the rates and powers for a given link schedule, and then adapt the link schedule to the computed rates and powers. The iterative process terminates when we reach a schedule for which there is no feasible set of link powers and rates that satisfies the flow conservation constraints.

A. Convex Optimization: Routing, Power Control

The rate constraint in the problem formulation $\mathcal{P}1$ is not convex. For a fixed link schedule $L^n, n = 1, \ldots, N$, we approximate the rate constraint for link $l \in L^n$ as follows.

$$r_l^n \leq \log\left(\frac{G_{ll}P_l^n}{\sum_{k\in L^n, k\neq l} G_{lk}P_k^n + N_0}\right) \tag{1}$$

This is a good approximation if the SINR over link l and time slot n is high. For low SINR, $\log(\gamma)$ is a lower bound on the achievable rate. Thus the feasible set corresponding to the optimization problem with the above approximation is a subset of the feasible set of the original optimization problem $\mathcal{P}1$. Thus the network lifetime computed under this approximation is a lower bound on the optimum network lifetime. Using a change of variables $Q_l^n = \log(P_l^n)$, we can rewrite the approximate rate constraint in (1) (see, for example, [26]) as follows.

$$\log\left(\frac{N_0}{G_{ll}}e^{r_l^n - Q_l^n} + \sum_{k\in L^n, k\neq l}\frac{G_{lk}}{G_{ll}}e^{r_l^n + Q_k^n - Q_l^n}\right) \leq 0$$

The function $\log(\sum_i a_i e^{x_i})$ is convex if $a_i \geq 0, x_i \in \mathbb{R}$ (see, for example, [27]). Composition with an affine function preserves convexity. Hence the function

$$\log\left(\frac{N_0}{G_{ll}}e^{r_l^n - Q_l^n} + \sum_{k\in L^n, k\neq l}\frac{G_{lk}}{G_{ll}}e^{r_l^n + Q_k^n - Q_l^n}\right)$$

is convex over r^n, Q^n. Thus we obtain the following convex optimization problem.

Problem $\mathcal{P}2$:

$$\begin{aligned}
\text{min.} \quad & q \\
\text{s.t.} \quad & A(r^1 + \ldots + r^N) = Ns \\
& r^n \geq 0, \;\; l \in L^n \\
& \log\left(\frac{N_0}{G_{ll}}e^{r_l^n - Q_l^n} + \sum_{k\in L^n, k\neq l}\frac{G_{lk}}{G_{ll}}e^{r_l^n + Q_k^n - Q_l^n}\right) \leq 0, \quad l \in L^n \\
& Q_l^n \leq \log(P_l^{\max}), \quad l \in L^n \\
& \sum_{n=1}^{N}\Big(\sum_{l\in\mathcal{O}(v)\cap L^n}\big((1+\alpha)e^{Q_l^n} + P_{ct}\big) + \sum_{l\in\mathcal{I}(v)\cap L^n} P_{cr}\Big) \leq qNE_v
\end{aligned}$$

for all $n \in \{1, \ldots, N\}, v \in V$. The variables are q, r_l^n, Q_l^n, for $l \in L^n, n = 1, \ldots, N$. Thus we solve the problem for optimal transmission powers and rates over each link, for a given link schedule. In general, it is not possible to characterize the computational complexity of solving a convex optimization problem. However, there exist efficient algorithms in practice to solve such problems. We used the *barrier method* described in [27] to solve the above problem. Also, note that the number of variables grow linearly with the number of links in the network and the number of time slots N.

B. Link Scheduling

The convex optimization problem $\mathcal{P}2$ is feasible only if the constraints $r_l^n \geq 0, l \in L^n, n = 1, \ldots, N$ are feasible. For the approximate rate constraint, it implies that each link has an SINR≥ 1 during the scheduled slots. If we schedule all the links during all the slots, the problem may be infeasible. There is no simple characterization of the set of link schedules for which the constraints $r_l^n \geq 0, l \in L^n, n = 1, \ldots, N$ are feasible. Hence, in order to use problem formulation $\mathcal{P}2$ to compute an optimal transmission scheme, we need to solve this problem for all possible link schedules. For a network of L links, and for a schedule frame that is N time slots long, there are 2^{NL} different link schedules. So the complexity of this approach is *doubly exponential* in the number of slots and the number of links. Thus the complexity of the problem increases rapidly with N. Also, the network overhead increases with an increase in N. However, when we compare the solution for say N slots and $2N$ slots, the solution corresponding to $2N$ slots gives a network lifetime greater than or equal to the lifetime corresponding to the solution for N slots. This is because we have more freedom in choosing the link schedules, and hence can find a transmission scheme that gives a larger network lifetime. In this paper, we take N to be a system constant.

We use a heuristic approach to iterate between link scheduling and computation of rates and powers. The links that carry a larger amount of traffic should be scheduled over a greater number of time slots - this decreases the average transmission power consumption over the links. Hence, the link schedule is adapted to the solution of problem $\mathcal{P}2$ at each iteration; and in turn the convex optimization problem is solved for the new link schedule.

Geometrically, the feasible set in problem $\mathcal{P}2$ is a convex subset of the feasible set in problem $\mathcal{P}1$. The heuristic approach proposed below solves a series of convex optimization problems with feasible regions given by different convex subsets of the original optimization problem $\mathcal{P}1$. Each convex subset corresponds to a link schedule and approximation of the rate constraints by convex constraints (as shown above).

C. Algorithm

The iterative approach used to compute an approximate optimal strategy is summarized in the flowchart in Fig. 1. The various steps are as follows.

1) Find an initial suboptimal, feasible schedule to begin with. A good candidate would be a schedule in which most of the links are activated at least once in each frame

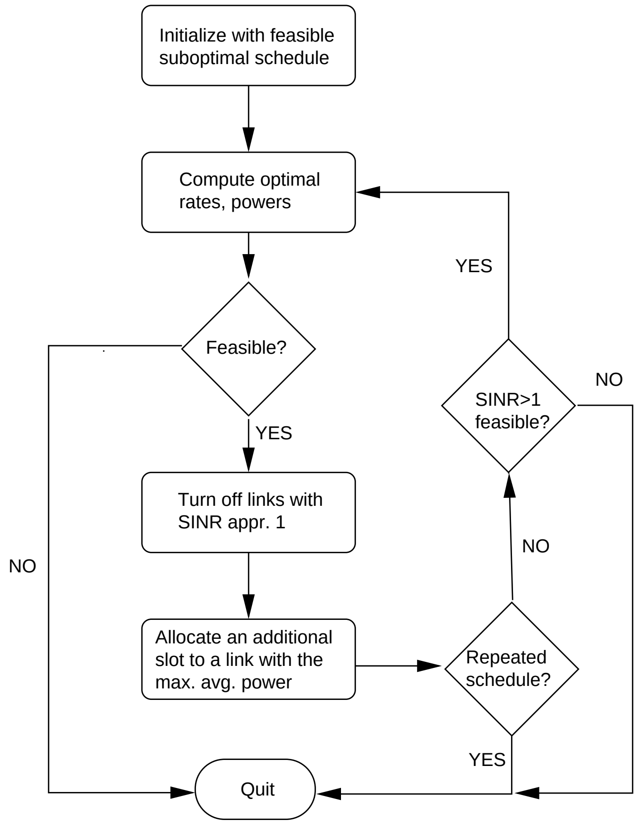


Fig. 1. Iterative approach to compute powers, rates and link schedule.

of N slots, and also links that are activated in the same slot only interfere weakly.

2) Solve problem $\mathcal{P}2$ to find an optimal routing flow and transmission powers during each slot under the high SINR approximation. If the problem is infeasible, quit.
3) Turn off links during slots in which they have an SINR close to 1. Since we approximated the rate as $r = \log(\text{SINR})$, links carry very little traffic over the slots in which they have an SINR of about 1.
4) Find a link that consumes the maximum average power over the entire frame. Schedule this link to be on during an additional time slot. The selected slot should be the one in which there is minimum interference to this link. If the resulting schedule is one that was used in a previous iteration, quit.
5) Check if SINR≥ 1 is feasible over all slots. If yes, go to (2), else quit.

For small networks, we can use a TDMA schedule for step (1). Hence, if we assume that the maximum transmission power constraint is loose, the initial schedule is always feasible. For large networks, we can use a graph coloring approach [9] to find a feasible schedule with low interference. The algorithm uses a greedy heuristic to adaptively schedule links at each iteration, and then re-solves the convex optimization problem $\mathcal{P}2$ to determine an optimal routing flow and transmission powers in each slot. As we will see in the following section, even such a simple greedy heuristic can give strategies with a higher network lifetime than that given by static approaches to scheduling (e.g. TDMA and time sharing between modes in which links separated by a distance greater than a certain minimum distance are scheduled together). The gains in network lifetime are due to energy-efficient multihop routing, frequency reuse, and load balancing.

Note that the algorithm is based on separating the combinatorial problem of link scheduling from the approximately convex problem of computing the transmission powers and rates over each link and slot for a fixed link schedule. Thus the algorithm is modular and hence can be easily modified to work with more complex link scheduling heuristics. Also, note that since the solution at each iteration is feasible, we can terminate the iterative process whenever we are satisfied with the computed solution.

IV. Numerical Results

Here we concentrate only on the transmission energy. Thus we take $P_{ct}, P_{cr} = 0$. Neglecting the circuit energy consumption only changes the optimal operating point of the network. The trends in the numerical results do not change if the network is interference-limited (in which case the transmission power dominates).

We assume $G_{ij} = \frac{k}{d_{ij}^m}$, where d_{ij} is the distance between the transmitter of link j and the receiver of link i, k is a constant that depends on system parameters like frequency and antenna gain, and m is the path loss exponent. For the computations that follow, we take $m = 4$, $k = 1$, $N_0 = 1$, $E_v = 50, \forall v \in V$. Thus if a transmitter transmits at unit power to a receiver at a distance of $1m$, then in the absence of any interference the receiver SINR is 1. Also, all the computations in this section are based on the approximate rate constraint (1). The solution obtained by the algorithm in the previous section can be refined using power control [14].

We will compare the performance of our algorithm with that of transmission schemes with specific scheduling at the MAC layer, outlined below.

1) *Uniform TDMA*: Each link in the network is scheduled for an equal number of time slots. In our computations we consider the number of slots per frame to be a multiple of the number of links.
2) *Optimal TDMA*: This refers to an optimal TDMA schedule computed by the mixed integer-convex problem formulation in Section 2.5.
3) *Spatially periodic time sharing*: This refers to a link layer scheduling scheme specific to one-dimensional (string and linear) topologies discussed below. A spatially periodic scheme with parameter T refers to a link schedule with T time slots per frame. In each time slot every Tth link is activated. Also, every link is activated once in every T slots. For example, $T = 2$ refers to a scheme which consists of two alternating transmission modes, with each transmission mode consisting of alternate links

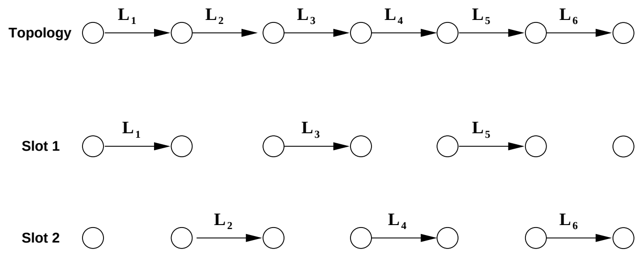


Fig. 2. Spatially Periodic Time Sharing with T=2

that are active. This is illustrated for a topology of six links in Fig. 2.

The algorithm in the previous section was initialized with a uniform TDMA link schedule for all the computations that follow.

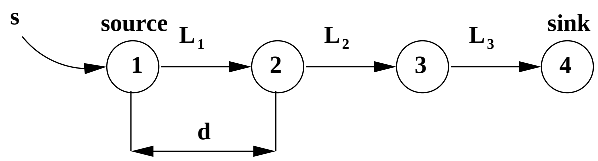


Fig. 3. String Topology

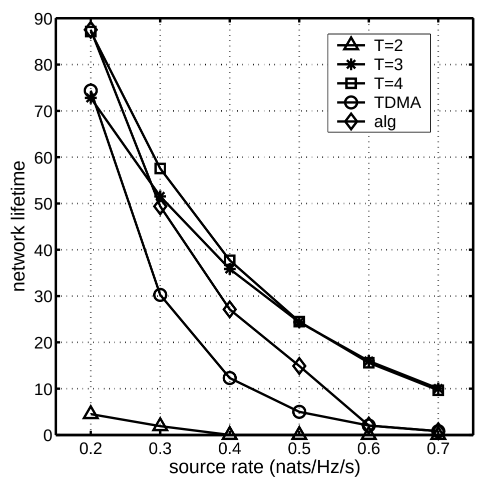


Fig. 4. String Topology Lifetime Computation

A. String Topology

A string topology consists of one source and one sink, connected by intermediate nodes that are arranged linearly. Each pair of intermediate nodes is separated by the same distance d, and connected by a directed link. The network carries information generated by the source to the sink. An example of a string topology of four nodes and three links is shown in Fig. 3. Here each link needs to support the same amount of average rate, which is the rate at which the source

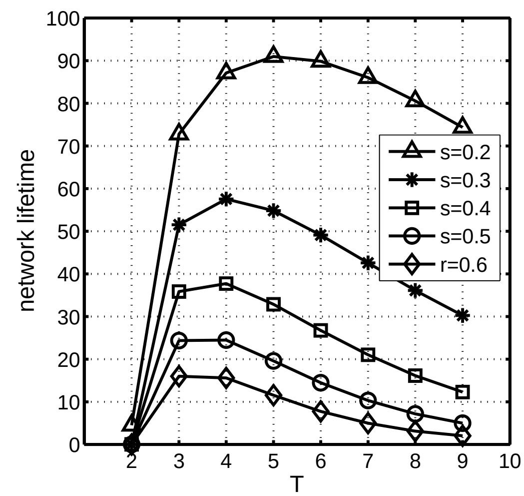


Fig. 5. String Topology - Spatially Periodic Schedules

generates information. For this topology there is only one routing path from the source to the sink. Hence, we only need to compute the link schedule and transmission powers.

We take $d = 1m$. Fig. 4 shows the network lifetime of a string topology of 10 nodes and 9 links achieved by our algorithm (curve labeled as "alg"), for different source rates. We used a frame length of 18 slots; the algorithm was initialized with a TDMA schedule in which every link was turned on for two time slots in each frame. The figure also shows the network lifetime under the TDMA scheme[1], and under different spatially periodic schedules (curves labeled with corresponding T values).

We can see that the algorithm performs well for low source rates, but as the source rate increases, it does increasingly worse compared to the spatially periodic scheme with optimal T. We would like to point out that for a string topology with many nodes, the spatially periodic schedule with an optimal value of T will be close to an optimal link schedule. This is because all links carry the same amount of traffic, and the nodes that are not close to either the source or the sink experience similar interference conditions. Hence, we do not need adaptive scheduling, we can use a fixed periodic link schedule with optimal T.

The network lifetime as a function of the value of T (for spatially periodic time sharing) is plotted for different source rates in Fig. 5. For a given source rate, the network lifetime first increases with T, and then decreases with T, with the optimal lifetime obtained at some value of T between 2 and 9. This illustrates the trade-off between

1) decrease in transmission energy by allowing each node to transmit data at a lower rate in each scheduled time slot

[1]Uniform TDMA is an optimal TDMA scheme since each link supports the same data rate over an AWGN channel with same noise power.

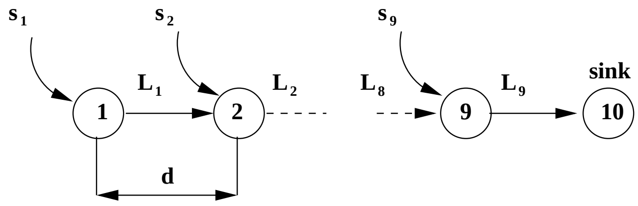


Fig. 6. Linear Topology

MAC	Network Lifetime
Uniform TDMA	0.14
Optimal TDMA	1.35
Spatially periodic (T=3)	9.6
Algorithm (no MAC constraint)	10.9

TABLE I

LINEAR TOPOLOGY - NETWORK LIFETIME FOR DIFFERENT MAC

2) increase in transmission energy due to interference caused by scheduling many links in the same time slot

The uniform TDMA scheme (T=9) performs poorly compared to the optimal spatially periodic schedule. Thus this topology illustrates the advantage of frequency reuse by simultaneous scheduling of links that do not interfere much.

B. *Linear Topology*

The linear topology is a simple generalization of the string topology. The nodes are again arranged linearly, but now each node is a source generating data at a possibly different rate. We computed the network lifetime using our algorithm for a linear topology of 10 nodes and 9 links with $d = 1m$. This topology is shown in Fig. 6. The source rates were take to be $s_1, \ldots, s_9 = 0.1$ nats/Hz/s, while the frame length was $N = 18$ slots.

Fig. 7(a) shows the best network lifetime achieved until each iteration. We can see that the lifetime increases as the link schedule adapts to the rates and transmission powers computed by solving problem $\mathcal{P}2$. We note that the increase in lifetime is not monotonic.

The network lifetime under different classes of link schedules is given in Table I. The network lifetime achieved by the best spatially periodic scheme ($T = 3$) was 12% lower than that achieved by our algorithm. As we can see from Fig. 7, the algorithm provides a greater number of slots to links that carry more traffic, and hence equalizes the average power consumption over the links with the four highest data rates. This is unlike the spatially periodic scheme which allocates the same number of time slots for each link. Also uniform TDMA ($T_{\text{net}} = 0.14$) and optimal TDMA ($T_{\text{net}} = 1.35$) perform poorly because they do not take advantage of frequency reuse. However, optimal TDMA is far superior than uniform TDMA because it allocates more slots to links with higher data rates.

link	no. of slots	avg. rate	avg. power
(1,3)	2	0.4	3.06
(3,5)	5	0.8	4.81
(2,5)	4	0.4	4.95
(4,5)	4	0.4	4.95

TABLE II

RHOMBUS TOPOLOGY - $s_i = 0.4, i = 1, 2, 3, 4.$

C. *Rhombus Topology*

The rhombus topology is shown in Fig. 8. There are four source nodes - nodes 1,2,3,4 with source rates s_1, s_2, s_3, s_4, respectively. Node 5 is the sink node. If we neglect interference and consider only the path loss, the minimum energy routes from the sources to the sink are (1,3,5), (2,3,5), (3,5) and (4,3,5) for sources 1,2,3, and 4, respectively. This topology illustrates the load balancing properties of the algorithm proposed in Section 3. A frame length of 16 time slots was used. The algorithm was initialized with a uniform TDMA schedule, with each link active for two of the 16 time slots in each frame.

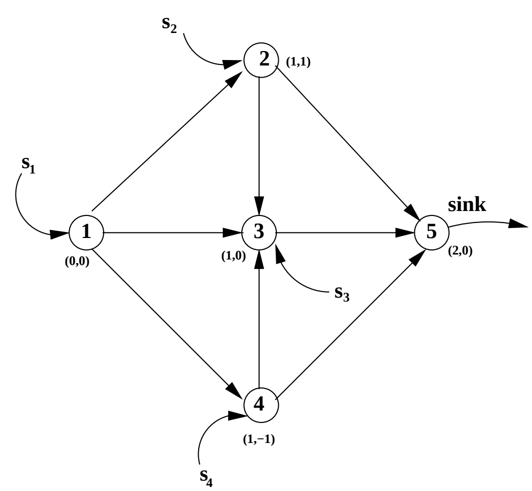
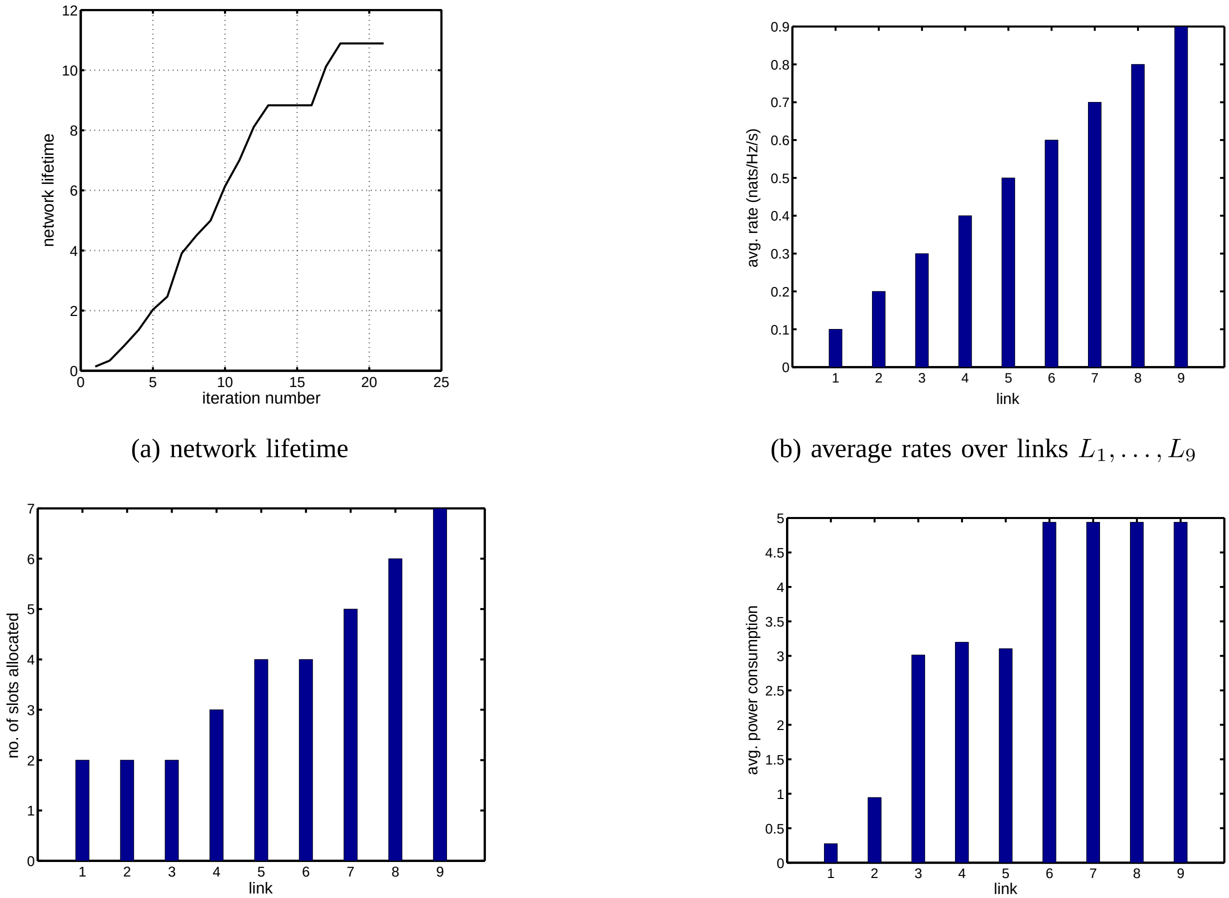


Fig. 8. Rhombus Topology

We used the algorithm to compute an efficient transmission strategy for two different sets of source rates.

1) *All sources on*: We used our algorithm to compute a transmission strategy for $s_i = 0.4$ nats/Hz/s , $i = 1, 2, 3, 4$. The rates, powers, and the schedule computed by the algorithm are shown in Table II. Link (3,5) carries more traffic than links (1,3), (2,5), and (4,5); also links (1,3), (3,5) have higher gain than the other two links. Hence, link (3,5) is allocated the highest number of slots, followed by links (2,5) and (4,5). We can see that the average power consumption of nodes 2,3,4 is about the same; hence these nodes die out at about the same time. Sources 2 and 4 send all their data directly to the sink rather than use the minimum energy routes; this avoids overloading node 3 with a large amount of data to transmit.
2) *Source 2 off*: An approximate optimal strategy was re-



(a) network lifetime

(b) average rates over links $L_1, \ldots, L_9$

(c) number of slots per frame for links $L_1, \ldots, L_9$

(d) avg. power consumption on links $L_1, \ldots, L_9$

Fig. 7. Network Lifetime Computation for a Linear Topology

link	no. of slots	avg. rate	avg. power
(1,2)	2	0.2	2.52
(1,3)	2	0.2	0.61
(2,5)	2	0.2	2.94
(3,5)	7	0.8	3.13
(4,3)	2	0.2	0.61
(4,5)	2	0.2	2.52

TABLE III

RHOMBUS TOPOLOGY - $s_2 = 0$, $s_i = 0.4$ NATS/HZ/S, $i = 1, 3, 4$.

computed with $s_2 = 0, s_i = 0.4$ nats/Hz/s , $i = 1, 3, 4$. The results are shown in Table III . Since node 2 does not generate data, a fraction of the data of node 1 is routed through node 2. Since only a fraction of node 1's data is routed through node 3, we can route some data of node 4 over the minimum energy route (4,3,5) through node 3. Hence, node 4 uses a multihop path for a fraction of its data. This equalizes the average power consumption of nodes 1,3 and 4. Also, since link (3,5) carries significantly higher data than other links, it is scheduled over more slots than the other links.

The value of the network lifetime for the strategy computed at each iteration is shown in Fig. 9. We can see from the figure that the network lifetime for the strategy computed by our algorithm is about 4.55 times that for the uniform TDMA schedule (corresponding to the first iteration), when all sources are on. When $s_2 = 0$, the value computed by the algorithm is about 2.57 times that for the uniform TDMA schedule. We can also consider minimum energy routing, with uniform TDMA for links that carry data. The minimum energy routing for this topology would use links (1,3),(2,3),(4,3),(3,5); the bottleneck node is node 3 which needs to transmit data of all the nodes to node 5. The network lifetime in this case is 0.33 when all sources are on, and 4.1 when $s_2 = 0$. The results are summarized in Table IV. We can see that optimal TDMA schemes computed using the mixed integer-convex program in Section 2.5 perform very well (in fact better than our algorithm which uses a heuristic for adapting link schedules) for the rhombus topology. This is because each link strongly interferes with all the other links in this topology, and hence we should not schedule multiple links in the same time slot. This is unlike a linear or a string topology where a link scheduled at one end does not cause significant interference to a link at the other end of the network.

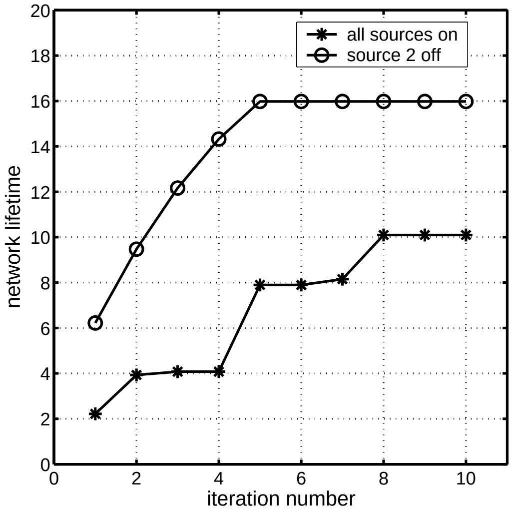


Fig. 9. Network Lifetime Computation for the Rhombus Topology

Transmission Scheme	T_{net} $s_2 = 0.4$	T_{net} $s_2 = 0$
Uniform TDMA	2.22	6.22
Optimal TDMA	11.23	16.96
Algorithm	10.10	16.00
Uniform TDMA, min. energy	0.33	4.1

TABLE IV

RHOMBUS TOPOLOGY - NETWORK LIFETIME UNDER DIFFERENT CLASSES OF TRANSMISSION SCHEMES ($s_1, s_3, s_4 = 0.4$ FOR BOTH CASES).

V. DISTRIBUTED COMPUTATION

In this section, we describe a partially distributed algorithm that computes optimal rates and powers to maximize the network lifetime for a *given link schedule*. Note that this is the computationally intensive part of the algorithm proposed in Section 3.3. Consider the convex optimization problem $\mathcal{P}2$ described in Section 3.1. We can relax the flow conservation and energy conservation constraints to form the partial Lagrangian (see, for example, [28]) and use the subgradient approach [29] to obtain a partially decentralized algorithm to solve the problem. We will illustrate this for the special case where $P_{ct}, P_{cr} = 0$. Let us use r, Q to denote the set of variables $r_l^n, Q_l^n, l \in L^n, n \in \{1, \ldots, N\}$. The partial Lagrangian is given by

$$\begin{aligned} &L(q, r, Q, \lambda, \nu) \\ &= q^2 + \sum_{v \in V} \lambda_v \left(\sum_{n=1}^{N} \sum_{l \in \mathcal{O}(v) \cap L^n} (1+\alpha) e^{Q_l^n} - N q E_v \right) \\ &+ \sum_{v \in V} \nu_v \left(\sum_{n=1}^{N} \left(\sum_{l \in \mathcal{O}(v) \cap L^n} r_l^n - \sum_{l \in \mathcal{I}(v) \cap L^n} r_l^n \right) - N s_v \right) \end{aligned}$$

where we have changed the primal objective function to q^2 (like in [19]) to make it strictly convex in q. Let $\mathcal{H}(l)$ and $\mathcal{T}(l)$ be the head and the tail nodes, respectively, of link l - i.e. l is a directed link from $\mathcal{H}(l)$ to $\mathcal{T}(l)$. Then, on rearranging the terms, we have

$$\begin{aligned} &L(q, r, Q, \lambda, \nu) \\ &\quad = \left(q^2 - qN \sum_{v \in V} \lambda_v E_v \right) - N \sum_{v \in V} \nu_v s_v \\ &\quad + \sum_{n=1}^{N} \sum_{l \in L^n} \left(r_l^n \left(\nu_{\mathcal{H}(l)} - \nu_{\mathcal{T}(l)} \right) + (1+\alpha) e^{Q_l^n} \lambda_{\mathcal{H}(l)} \right) \end{aligned}$$

The Lagrange dual function is

$$g(\lambda, \nu) = \inf_{q, r, Q} \left\{ L(q, r, Q, \lambda, \nu) \;\middle|\; \begin{array}{l} r_l^n \geq 0 \\ Q_l^n \leq \log(P_l^{\max}) \\ \phi(n) \leq 0 \end{array} \right\}$$

where $\phi(n)$ is a function of the transmission powers and rates over links scheduled during slot n, and is given by

$$\phi(n) = \log\left(\frac{N_0}{G_{ll}} e^{r_l^n - Q_l^n} + \sum_{k \in L^n, k \neq l} \frac{G_{lk}}{G_{ll}} e^{r_l^n + Q_k^n - Q_l^n} \right)$$

Hence the dual function can be written as

$$\begin{aligned} g(\lambda, \nu) = \inf_{q} \Big(q^2 - qN \sum_{v \in V} \lambda_v E_v \Big) - N \sum_{v \in V} \nu_v s_v \\ + g_1(\lambda, \nu) + \ldots + g_N(\lambda, \nu) \end{aligned}$$

where $g_n(\lambda, \nu)$ is the optimal value of the objective function of the following convex optimization problem, where the variables are the rates and transmission powers of links scheduled over slot n.

$$\begin{array}{ll} \text{min.} & \sum_{l \in L^n} \left(r_l^n \left(\nu_{\mathcal{H}(l)} - \nu_{\mathcal{T}(l)} \right) + (1+\alpha) e^{Q_l^n} \lambda_{\mathcal{H}(l)} \right) \\ \text{s.t.} & r_l^n \geq 0 \\ & Q_l^n \leq \log(P_l^{\max}) \\ & \phi(n) \leq 0 \end{array}$$

Let us denote the solution of this problem for given dual variables λ, ν as $r_l^{n*}(\lambda, \nu), Q_l^{n*}(\lambda, \nu), l \in L^n$. Thus we can use dual decomposition to evaluate the dual function efficiently by solving smaller convex optimization problems. Hence, we can find a partially distributed subgradient algorithm using the approach in [30], [28], [19] to solve the dual problem stated below.

$$\begin{array}{ll} \text{min.} & g(\lambda, \nu) \\ \text{s.t.} & \lambda \succeq 0 \end{array}$$

We assume that there exists a strictly feasible solution for problem $\mathcal{P}2$. Then, since the primal problem is convex, there is no duality gap (see, for example, [27]). The kth iteration of the resulting subgradient algorithm is given by

$$\begin{aligned} r_l^n &= r_l^{n*}(\lambda^{(k)}, \nu^{(k)}) \\ Q_l^n &= Q_l^{n*}(\lambda^{(k)}, \nu^{(k)}) \\ q &= q^*(\lambda^{(k)}) \\ \lambda_v^{(k+1)} &= \left(\lambda_v^{(k)} - \alpha_k h_v(\lambda^{(k)}, \nu^{(k)}) \right)_+ \\ \nu_v^{(k+1)} &= \nu_v^{(k)} - \alpha_k f_v(\lambda^{(k)}, \nu^{(k)}) \end{aligned}$$

where

$$q^*(\lambda^{(k)}) = \arg\min_q \Big(q^2 - qN \sum_{v \in V} \lambda_v E_v \Big)$$

and h_v, f_v are the subgradients of the negative dual function at $(\lambda^{(k)}, \nu^{(k)})$ and are given by (suppressing the dependence on λ, ν)

$$h_v = Nq^* E_v - \sum_{n=1}^{N} \sum_{l \in \mathcal{O}(v) \cap L^n} (1+\alpha) e^{Q_l^{n*}}$$

$$f_v = Ns_v - \sum_{n=1}^{N} \Bigg(\sum_{l \in \mathcal{O}(v) \cap L^n} r_l^{n*} - \sum_{l \in \mathcal{I}(v) \cap L^n} r_l^{n*} \Bigg)$$

for all $v \in V$. Here α_k is a positive scalar step size. Convergence is guaranteed if (see, for example, [29])

$$\alpha_k \to 0, \qquad \sum_{k=1}^{\infty} \alpha_k = \infty$$

Each iteration of the subgradient algorithm involves the following steps.

1) Computation of primal variables that minimize the Lagrangian for fixed λ, ν. This can be done in a partially decentralized manner as discussed above. For each time slot n, we can separately evaluate the primal variables $r_l^n, Q_l^n, l \in L^n$.
2) Update of dual variables by evaluating the subgradient of the dual function. Update of a dual variable corresponding to node v requires the value of q and the values of r_l^n, Q_l^n over all time slots on all outgoing and incoming links at node v. Thus all variables are local to node v, except the variable q.

VI. Conclusions

We considered the computation of transmission powers, rates and link schedule for an energy-constrained wireless network to jointly maximize the network lifetime. For the special case, where we restricted the link schedules to TDMA schemes, we obtained the exact optimal transmission scheme as the solution of a mixed integer-convex optimization problem. For the case of general link schedules, we proposed an iterative algorithm to approximate the optimal solution. Each iteration of the algorithm solved a convex optimization problem, where the feasible region was given by a convex subset of the feasible set in problem formulation $\mathcal{P}1$. The algorithm was found to perform well for the topologies considered in this paper. We also described an approach to decompose the computationally intensive part of the algorithm into smaller sub-problems.

The numerical studies emphasized the importance of cross-layer design for energy-constrained networks, and illustrated the advantages of multihop routing, load balancing, interference mitigation, and frequency reuse in increasing the network lifetime. Traditional approaches such as TDMA and minimum energy routing were found to perform poorly for certain topologies.

A. Future Work

Better heuristics for link scheduling, performance evaluation of this algorithm for large topologies, computation of upper bounds on the network lifetime using the problem formulation $\mathcal{P}1$, incorporation of detailed models of the circuit energy consumption, and distributed algorithms are directions that we plan to explore in future work.

Acknowledgment

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