Mixed Integer-Linear Programming for Link Scheduling in Interference-Limited Networks

Ritesh Madan, Shuguang Cui, Sanjay Lall, and Andrea J. Goldsmith

***Abstract*— We consider the problem of link scheduling in wireless networks with interference. The problem of computing a link schedule to minimize the power consumption with the constraint that each link supports a given average data rate is formulated as an optimization problem. A simple analytical solution is obtained for time division multiple access (TDMA) scheduling to minimize a linear combination of the average power consumption on the links; this schedule gives the minimum total power consumption for regular topologies. We then propose a method to compute energy efficient link schedules. It consists of the following two steps. (1) computation of a schedule (using a mixed integer-linear programming formulation) that minimizes the maximum sum of the cross-link gains from interfering links to an active link, subject to the constraint that each link is active for a time proportional to the optimal slot length computed for TDMA schedules. (2) power control to find the minimum transmission power vector that supports the desired average data rates on the links for the computed schedule. We also briefly consider the reverse problem of computing a link schedule that maximizes the minimum link rate, for given fixed link transmission powers.**

***Index Terms*— link scheduling, resource allocation, mixed integer-linear programming, energy efficiency, minimum rate**

I. Introduction

We consider wireless networks that are bandwidth and energy limited. Optimization of such networks to maximize data rates or to minimize the average power consumption has received tremendous attention recently. In this paper, we consider the problem of link scheduling to allocate spectral resources to links in an interference-limited wireless network. We first consider the problem of minimization of the total power consumption in the network subject to rate constraints over the links. We state the optimization problem formally and reason that the problem is difficult to solve. We then propose a two step approach to solve this problem approximately. The computationally intensive part of our approach involves the solution of a mixed integer-linear program. We also formulate a mixed integer-linear program to maximize the minimum link transmission rate, when the link transmission powers are fixed.

An overview of link scheduling to minimize the transmission power for given desired signal to interference and noise ratios (SINR), is given in [1]. The methods described in this article only consider simultaneous scheduling of all links for all time. At the other extreme, we can use TDMA to schedule only one link at any given time; thus TDMA divides the resources between the links in an orthogonal manner. Optimal TDMA scheduling and power control strategies to either minimize energy consumption for a given rate, or to maximize a concave function of the rates for given power constraints, can be computed using methods described in [2]. In this paper, we consider link schedules which are a generalization of the two classes of schedules described above.

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A. System Model and Problem Formulation

We model only the transmission power and ignore the power consumption in the underlying hardware. This is a good assumption in interference-limited wireless networks, where the transmission power dominates over the power consumption in the circuit components. The channel over each link is assumed to be an additive white Gaussian noise (AWGN) channel. For an SINR γ and MQAM modulation scheme with a constellation size greater than or equal to 2, the maximum transmission rate over a channel can be approximated by [3]

$$R = B\log(1 + K\gamma)$$

where B is the total transmission bandwidth and $K = -1.5/\log(5\text{BER})$ (see [3]). For notational convenience we take $K = 1$; a different value of K will not change the analysis or the numerical results in this paper. Let $r = R/B$ denote the rate per unit bandwidth in bits/s/Hz; in the rest of the paper the data rates will be normalized with respect to the bandwidth. We assume that each link can vary its data rate according to the SINR over the channel.

We model the wireless network as a directed graph $\mathcal{G} = (V, L)$, where V is the set of wireless nodes, and L is the set of links. All links are assumed to operate in the same frequency band; the spectral resources are allocated to the links in the time domain using scheduling. We model the interference from each active link to all other links in the network by a link gain matrix G, where G_{lk} is the gain from the transmitter of link k to the receiver of link l, for $k \neq l$. The diagonal entries G_{ll} represent the gain over each link l.

Consider link $l \in L$ that supports an average data rate r_l, but transmits only a fraction τ_l of the time. Hence, during transmission, the data rate over the link is r_l/τ_l, and the minimum required transmission power is

$$\frac{N + I_l}{G_{ll}}(2^{r_l/\tau_l} - 1)$$

where N is the total noise power in the frequency band of bandwidth B, and I_l is the total interference power due to other links active at the same time. Thus the average power consumption over the link is

$$P(r_l, \tau_l) = \frac{N + I_l}{G_{ll}} \tau_l (2^{r_l/\tau_l} - 1) \tag{1}$$

which is a decreasing function of τ_l. If the total noise and interference power does not depend on τ_l, then the minimum average power strategy for the link is to use $\tau_l = 1$, i.e. transmit continuously with rate r_l.

We consider the following two models for the medium access control layer.

1) *TDMA*: Variable length time slots in each frame, and only one active link per time slot.
2) *Slotted-time*: Equal length time slots in each frame. More than one link can be active in a given time slot.

The solution for the case of TDMA schedules will be used to motivate our proposed computational approach for the slotted MAC layer. For the slotted model, each link is constrained to be active for at least one slot in each frame of fixed length T. So, for example, if we assume a frame of 5 slots, and if a link is active for one slot per frame, then the link can transmit data for one-fifth of the time. In this paper, we study two related link scheduling problems. We first fix the number of slots and formulate mixed integer-linear programs to compute schedules (1) that are energy efficient and support the given average data rates over the links and (2) that maximize the minimum data rate for fixed transmission powers. Then we vary the number of slots to see how the division of spectral resources affects the performance. For a large number of slots, there will be few active links in each slot and hence the interference will be low. However, each link will then get a smaller amount of time to transmit its data. Thus there is a tradeoff between the energy savings by activating a link for a longer time and energy savings by scheduling few links at any given time to mitigate interference.

II. Energy Efficient Scheduling

Here, we consider the special case where the routing flow is fixed in interference-limited wireless networks. This would be the case, for example, in a cluster with a star topology, in a sensor network. Consider a slotted time model, where each frame has M slots. Let p_l^m denote the link transmission power over link $l \in L$ and slot $m \in \{1, \ldots, M\}$. Then the interference to link l in slot m is given by

$$I_l^m = \sum_{k \in L, k \neq l} G_{lk} p_k^m$$

Hence, the SINR over link l and slot m is

$$\gamma_l^m = \frac{G_{ll} p_l^m}{\sum_{k \in L, k \neq l} G_{lk} p_k^m + N}$$

Thus the rate of transmission r_l^m in slot m over link l satisfies

$$r_l^m \leq \log\left(1 + \frac{G_{ll} p_l^m}{\sum_{k \in L, k \neq l} G_{lk} p_k^m + N}\right)$$

This can be rewritten as

$$G_{ll} p_l^m \geq (2^{r_l^m} - 1)\left(\sum_{k \in L, k \neq l} G_{lk} p_k^m + N\right) \tag{2}$$

We have the constraint that the average rate over link l should be greater than or equal to the specified rate r_l, i.e.

$$\frac{1}{M} \sum_{m=1}^{M} r_l^m \geq r_l \tag{3}$$

The objective is to minimize the total power consumption in the network subject to the average rate constraints. This can be written as the following optimization problem in variables p_l^m for all $l \in L, m = 1, \ldots, M$.

$$\begin{aligned} \text{minimize} \quad & \sum_{l \in L} \sum_{m=1}^{M} p_l^m \\ \text{subject to} \quad & G_{ll} p_l^m \geq (2^{r_l^m} - 1)\left(\sum_{k \in L, k \neq l} G_{lk} p_k^m + N\right) \\ & r_l \leq \frac{1}{M} \sum_{m=1}^{M} r_l^m \end{aligned}$$

for all $l \in L, m = 1, \ldots, M$. This is a special case of the optimization problem considered in [4]. As discussed in that paper, this problem is hard to solve. Here, we propose an approach that involves the solution of a mixed integer-linear program and the computation of a power control strategy that involves a matrix inversion. The approach is motivated by the special case in which the topology is regular, i.e. G_{ll} is the same for all links $l \in L$. We base our approach on the following two observations and try to explore the resulting tradeoff.

- If we schedule links $k \in L$ along with link l in slot m, such that the G_{lk} terms are high, then the power p_l^m needed to support some rate r_l^m is high (see inequality (2)). Hence, we should try to schedule strongly interfering links as much spaced out in time as possible.
- But at the same time, if we schedule a link for only a short amount of time, the rate r_l^m during a slot m in which a link is on will be high (see inequality (3)), leading to a high power consumption.

We first study variable-length TDMA schedules to minimize a linear combination of the power consumption on all the links. We then motivate our approach for general link schedules for the slotted model using the solution obtained for the TDMA case. Specifically, we consider generic link schedules with the property that each link is activated for a time proportional to the optimal slot length in a variable-length TDMA schedule.

A. Optimal TDMA Scheduling

We will motivate our method by first considering optimal variable-length TDMA schedules to minimize a weighted sum of the power consumption on all the links in the network. The choice of the objective function is one which allows for an analytical solution to the optimization problem. We will use the analytical solution to motivate our proposed heuristic in the next section. Consider link l that supports an average rate of r_l and transmits for a fraction τ_l of the time. Then the average power consumption over link l for a TDMA scheme (which has zero interference) is given by equation (1)

$$P(r_l, \tau_l) = \frac{N\tau_l}{G_{ll}}\Big(2^{\frac{r_l}{\tau_l}} - 1\Big) \tag{4}$$

We consider the following optimization problem.

$$\begin{aligned} \text{minimize} \quad & \sum_{l \in L} G_{ll} P(r_l, \tau_l) \\ \text{subject to} \quad & \tau_l \geq 0, \quad \forall l \in L \\ & \sum_{l \in L} \tau_l = 1 \end{aligned} \tag{5}$$

The objective function is a strictly increasing linear function of the average power consumption on each of the links. If the G_{ll}'s are equal for all links $l \in L$ (for example, in regular networks), then the problem formulation minimizes the total power consumption in the network. Also, the objective function is such that it favors the links with high link gains G_{ll}'s. The variables are the τ_l's. Note that we consider variable-length TDMA schemes, and hence there are no integer constraints. Then we have the following proposition.

Proposition 2.1: Assume that for all links $l \in L$, $r_l > 0$. The optimal variable-length TDMA schedule that solves problem (5), is given by $\tau_l = kr_l$, where τ_l is the fraction of time for which link l is active, and k is a constant of proportionality such that $\sum_{l \in L} \tau_l = 1$.

Proof: Using equation (4), the objective function can be written as

$$\sum_{l \in L} G_{ll} P(r_l, \tau_l) = \sum_{l \in L} N\tau_l (2^{\frac{r_l}{\tau_l}} - 1)$$

The Karush-Kuhn-Tucker (KKT) conditions (see, for example, [5]) for problem (5) can be written as follows.

$$\sum_{l \in L} \tau_l = 1, \;\; \tau \succeq 0$$
$$\lambda_l \geq 0, \;\; \lambda_l \tau_l = 0, \quad \forall l \in L$$
$$N 2^{\frac{r_l}{\tau_l}} \left(1 - \frac{r_l}{\tau_l}\right) + (\nu - N) + \lambda_l = 0, \quad \forall l \in L$$

Any τ and (λ, ν) satisfying these conditions are primal and dual optimal, with zero duality gap. Using the fact that for an optimal solution, $\tau_l > 0$ (and hence $\lambda_l = 0$) for all l, the KKT conditions are satisfied if

$$\sum_{l \in L} \tau_l = 1, \;\; \tau \succeq 0$$
$$N 2^{\frac{r_l}{\tau_l}} \left(1 - \frac{r_l}{\tau_l}\right) = N - \nu, \quad \forall l \in L$$

We can satisfy these conditions by choosing $\tau_l = kr_l$, such that $\sum_{l \in L} \tau_l = 1$.

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Intuitively, a link that supports a higher average rate is scheduled for a larger fraction of time. For a regular topology where all the diagonal entries of the matrix G are equal, i.e. G_{ll} is the same for all $l \in L$, this gives a TDMA schedule with minimum total average power consumption in the network. For a topology with different values of G_{ll}'s, we can bound the total power consumption of this scheme in terms of the minimum possible total power consumption for TDMA schemes. Let $\tilde{P}_l$ be the average power consumption on link l corresponding to the solution of problem (5). Let P_l^* be the average power consumption on link l corresponding to the solution of the following optimization problem to minimize the total power consumption over all the links for TDMA schemes.

$$\begin{aligned} \text{minimize} \quad & \sum_{l \in L} P(r_l, \tau_l) \\ \text{subject to} \quad & \tau_l \geq 0, \quad \forall l \in L \\ & \sum_{l \in L} \tau_l = 1 \end{aligned} \tag{6}$$

Proposition 2.2: We can bound the total power consumption in the network corresponding to the solution of problem (5) as follows.

$$\sum_{l \in L} \tilde{P}_l \leq \frac{\max_{l \in L} G_{ll}}{\min_{l \in L} G_{ll}} \sum_{l \in L} P_l^*$$

Proof: We have

$$\begin{aligned} \sum_{l \in L} P_l^* &\geq \frac{1}{\max_{l \in L} G_{ll}} \sum_{l \in L} G_{ll} P_l^* \\ &\geq \frac{1}{\max_{l \in L} G_{ll}} \sum_{l \in L} G_{ll} \tilde{P}_l \\ &\geq \frac{\min_{l \in L} G_{ll}}{\max_{l \in L} G_{ll}} \sum_{l \in L} \tilde{P}_l \end{aligned}$$

where the second inequality follows from the fact that $\tilde{P}_l$ corresponds to the solution to problem (5). ■

B. Minimum Cross-Link Interference Gain Schedules

We are interested in computing general link schedules and transmission powers which minimize the total power consumption in the network. In this section, we propose a method to solve this problem in an approximate manner. Suppose, given the link rates r_l's, we want to design a periodic slotted schedule to support the rates. Let ρ be such that $r_l = \tau_l \rho, \forall l \in L$, where the τ_l's are integers. We can always find such a ρ to an arbitrary degree of accuracy. We consider link schedules where each link l is activated for τ_l number of slots. Thus the time allocated to each link is in proportion to the optimal slot length in a variable-length TDMA schedule. In addition, we now consider frequency reuse which allows more than one link to be active in each slot. Also, each active link transmits at the same data rate ρ.

Let x_l^m be a scheduling variable such that $x_l^m = 1$ if link l is active in slot m, and 0 otherwise. For a fixed number of time slots M per frame and a schedule $x = \{x_l^m, l \in L, m = 1, \ldots, M\}$, the maximum sum of the *cross-link gains* from the interfering links to an active link is

$$\lambda(x) = \max_{\{l,m:x_l^m=1\}} \sum_{k \in L, k \neq l} G_{lk} x_k^m$$

The problem of computing a feasible schedule that minimizes the maximum total cross-link gain to an active link can be written as the following mixed-integer linear program.

$$\begin{array}{ll} \text{minimize} & \lambda \\ \text{subject to} & \sum_{k\in L, k\neq l} G_{lk}x_k^m \leq \lambda + D(1-x_l^m) \\ & \sum_{m=1}^{M} x_l^m = \tau_l, \\ & x_l^m \in \{0,1\} \end{array} \tag{7}$$

for all $l \in L, m = 1, \ldots, M$, and D an arbitrary large constant. The variables are λ and the x_l^m's. The first set of constraints ensure that the maximum total cross-link gain to an active link is less than or equal to λ. The second set of constraints activate each link for a number of slots proportional to the TDMA slot lengths. For a path loss model, where the received power decays monotonically with the distance from the transmitter, the above problem formulation can be thought of as spacing the active links in all time slots "as far apart as possible", subject to the scheduling constraints. We can interpret the above optimization problem as follows.

Proposition 2.3: Consider a regular topology where $G_{ll} = g$ for all links $l \in L$. If all the links are constrained to use the same transmission power P, when active, the schedule computed by the mixed integer-linear program (7) minimizes the total power consumption among all schedules with M slots and each link l active for a number of slots proportional to r_l.

Proof: Consider a schedule $\{x_l^m\}$ that satisfies

$$\sum_{k\in L, k\neq l} G_{lk}x_k^m \leq \lambda + D(1-x_l^m)$$

Also, let λ be such that this relation is satisfied with equality for at least one m and l such that $x_l^m = 1$. For every link l and slot m such that $x_l^m = 1$, let $\lambda_l(m)$ be given by

$$\lambda_l(m) = \sum_{k\in L, k\neq l} G_{lk}x_k^m$$

Then for every link l and slot m such that $x_l^m = 1$, the common transmission power satisfies

$$\begin{aligned} & \log\left(1 + \frac{gP}{\sum_{k\in L,k\neq l} G_{lk}x_k^m P + N}\right) \geq \rho \\ \Rightarrow \quad & \frac{gP}{\sum_{k\in L,k\neq l} \lambda_l(m)P + N} \geq 2^\rho - 1 \\ \Rightarrow \quad & P \geq \frac{N(2^\rho - 1)}{g - (2^\rho - 1)\lambda_l(m)} \end{aligned}$$

where we assume that for all $\{l, m : x_l^m = 1\}$,

$$g - (2^\rho - 1)\lambda_l(m) > 0$$

Since

$$\lambda = \max_{\{l,m:x_l^m=1\}} (\lambda_l(m))$$

the minimum possible common transmission power is

$$P = \frac{N(2^\rho - 1)}{g - (2^\rho - 1)\lambda}$$

Thus P is a increasing function of λ for $g - (2^\rho - 1)\lambda > 0$. Since the solution to the optimization problem in (7) minimizes λ, it minimizes the value of P. This also minimizes the total average power consumption in the network, which is given by

$$\frac{1}{M}\sum_{l\in L} P\tau_l$$

If the links are allowed to use different transmission powers, then we can still use the problem formulation as a heuristic to compute a schedule with low interference.

C. Power Control

For a schedule computed as above, we can use optimal power control (see, for example, [1], and the references therein) to obtain the minimum transmission power vector for each slot such that each active link transmits at the common rate ρ. We summarize the relevant results from [1] below.

Let L^m be the set of links that transmit data in slot m, at rate ρ. For notational convenience, let us renumber these links as $1, 2, \ldots, |L^m|$. Then each link needs a minimum SINR of $\gamma_0 = (2^\rho - 1)$. Let the vector $u \in \mathbb{R}^{|L^m|}$ be such that

$$u_i = \frac{\gamma_0 N}{G_{ii}}$$

Also, consider the matrix $F \in \mathbb{R}^{|L^m|\times|L^m|}$ given by

$$F_{ij} = \frac{\gamma_0 G_{ij}}{G_{ii}}\mathbf{1}_{i\neq j}$$

where $\mathbf{1}_{i\neq j} = 1$, if $i \neq j$, and 0 otherwise. The rate ρ over each link $l \in L^m$ is feasible if $\rho_F < 1$, where ρ_F is the Perron-Frobenious eigenvalue of F. Then the Pareto-optimal transmission power vector for slot m is given by

$$(I - F)^{-1}u$$

III. Numerical Examples

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A simple two-link example illustrates the tradeoff between energy savings by interference mitigation and those by scheduling each link for a longer fraction of time. Consider two links that transmit data at the same average rate r. Let the gain matrix be given by

$$G = \begin{bmatrix} 1 & 0.12 \\ 0.12 & 1 \end{bmatrix}$$

The total average power consumption was computed for the following two link schedules.

1) *Simultaneous Transmission*: Both links transmit simultaneously at rate r.
2) *TDMA*: Only one link transmits at any given time. The optimal time-sharing is for each link to transmit half the time (see Section III.B) at rate $2r$.

The results are as shown in Fig. 1. We can see that simultaneous scheduling of the two links is a better strategy for rates lower than a certain threshold data rate. For higher data rates, TDMA is better; in fact for high enough data rates simultaneous scheduling does not even admit a feasible

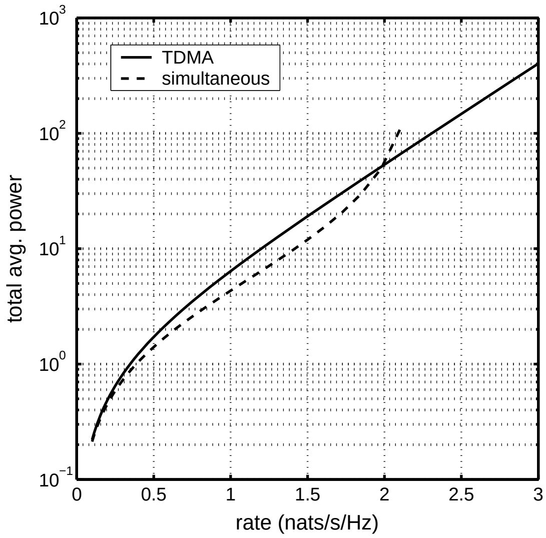

Fig. 1. Simple two link example

transmission power vector. The data rate at which the power consumption curves for the two schemes intersect depends on the cross-link gains G_{12}, G_{21}.

Consider the grid topology shown in Fig. 2(a). Each link in the grid supports the same average rate $r = 0.15$ bits/s/Hz. This may correspond to, for example, an aggregation pattern with data fusion and redundant transmission for robust multipath routing in sensor networks. We assume a path loss model, where the received power decays as the fourth power of distance from the transmitter. We used a branch and bound method (see, for example, [6]) to compute a schedule that minimizes the maximum total cross-link gain for a fixed M (see problem (7)). The computation was repeated for different number of slots M per frame, for a constant frame-length. The resulting schedule for $M = 5$ is shown in Fig. 2(a). The number next to each link gives the slot index over which it is scheduled. For the schedule thus obtained for each value of M, we computed the optimal transmission power on each link to support an average data rate of $x = 0.15$ bits/s/Hz. The variation of the total power consumption and the maximum total cross-link gain with M is shown in Fig. 2(b). The values are normalized so that we can plot both the cross-link gains and the power consumption on the same graph. The rate vector is infeasible for the schedule computed for $M = 2, 3$ slots. For $M \geq 4$, we see that the power consumption first increases with M because the increase in power due to less transmission time dominates the power savings due to interference mitigation. However, for $M = 8$, the schedule computed using the mixed integer-linear program in (7) mitigates the interference to such a large extent that the power savings due to interference mitigation dominates. For $M \geq 8$, the power savings due to less interference are not significant, and hence power consumption increases with M. The values of M at which the interference decreases sharply depend on the topology under consideration. Also, note that $M = 12$ corresponds to a TDMA schedule, where each link is active for one-twelfth of the time.

IV. Scheduling to Maximize Minimum Rate

Here, we briefly consider the reverse problem of computing a link schedule to maximize the average transmission rate, given the link transmission powers. We assume that the transmission power over each link l is fixed at P_l. Also, each link is constrained to be active for exactly one slot per frame; more than one link can be active at a given time. Such a scenario may correspond to, for example, a star topology network. Then for a fixed number of slots M per frame, the problem of maximizing the minimum average link rate can be formulated as the following mixed integer-linear program in the variables α, x_l^m's.

$$\begin{array}{ll} \text{minimize} & \alpha \\ \text{subject to} & \displaystyle\sum_{k \in L, k \neq l} G_{lk} P_k x_k^m + N \leq G_{ll} P_l \alpha + D(1 - x_l^m) \\ & \displaystyle\sum_{m=1}^{M} x_l^m = 1 \\ & x_l^m \in \{0, 1\} \end{array}$$

for $l \in L, m = 1, \ldots, M$. The first set of constraints ensure that the SINR for each scheduled link is greater than or equal to $1/\alpha$. D is again a constant large enough to ensure that the inequality is always satisfied for inactive links. The second set of constraints enforce each link to be active in exactly one slot. We also impose binary integer constraints on the x_l^m's. For a given α satisfying the above constraints, the average rate on each link satisfies

$$r_l(M, \alpha) \geq \frac{1}{M} \log\left(1 + \frac{1}{\alpha}\right), \quad \forall l \in L$$

Thus $r_l(M, \alpha)$ is a decreasing function of α. Hence, by minimizing α, we can maximize the minimum average link rate. We can find the optimal value of M by repeating this computation for different values of M.

V. Conclusions

In this paper, we proposed computational approaches to design efficient wireless networks. Such design approaches may be useful in the design of, for example, mesh, infrastructure and sensor networks. The computational approach for energy-efficient design is optimal for regular topologies when each link transmits at the same transmission power and a common transmission rate.

The mixed integer-linear programs considered in this paper have a worst case exponential complexity when branch and bound methods are used to compute the solution. One possible direction for further work is the exploration of semi-definite programming techniques in [8] to compute approximate solutions in an efficient manner. Also, extending our results to a continuous time model like the one in [7], is another possible direction for future research.

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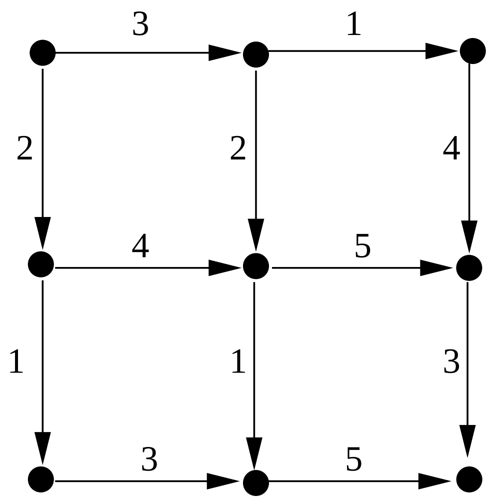
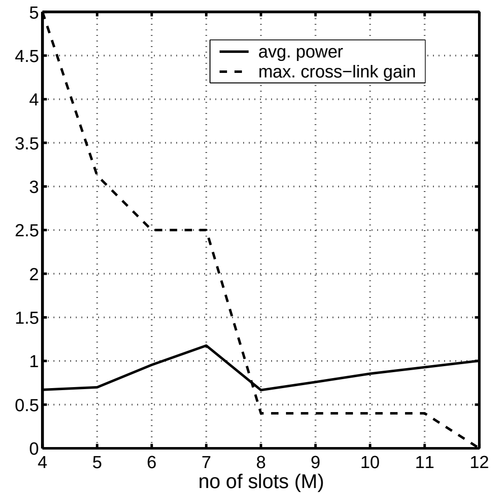


(a) grid topology - M=5



(b) power consumption vs. M

Fig. 2. Numerical Results - Power consumption in normalized units.

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