A Clock and Ephemeris Algorithm for Dual Frequency SBAS

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ABSTRACT

In the next years, the new GPS and Galileo signals (L1, L5) will allow civil users to remove the ionospheric delay in the pseudoranges. This will have a large impact on the Satellite Based Augmentation Systems (SBAS), as the ionospheric delay is currently the largest error. Once this source of error is removed, the Vertical Protection Levels will decrease substantially, and other error sources will dominate. The remaining terms in the error bound were much less critical than the ionospheric delay error bound, so they have received less attention. It is therefore likely that they can still be optimized. This is true in particular for the User Differential Range Error (UDRE) algorithm which computes the clock and ephemeris error bounds. In addition, new SBAS messages will be broadcast in the L5 channel, and their content is still not fixed. Therefore, it is a good opportunity to determine whether changes can be made both in the UDRE and Message Type 28 (MT28) computation and transmission to increase overall SBAS performance.

In this work, we propose an algorithm to compute the error bounds on the clock and ephemeris in SBAS. As opposed to the current Wide Area Augmentation System (WAAS) UDRE algorithm, this algorithm computes the UDRE and MT28 simultaneously and takes into account receiver failures explicitly. We will evaluate the performance of the algorithm and compare it to the current UDRE and MT28 algorithm to determine whether its implementation for dual frequency SBAS would be worthwhile.

INTRODUCTION

L1-L5 WAAS is being developed [1] to take advantage of the second civil signal in the L5 frequency band. This second signal will allow receivers to estimate and cancel the effect of the pseudorange delay induced by the ionosphere. Since this delay is the most important source of uncertainty in single frequency SBAS [2], it is the largest contributor to the user position error bound. Once the ionospheric error bound is removed, the largest contributor will be the term bounding three different sources of error: the clock and ephemeris error, the code-carrier coherence (CCC), and the signal deformation (SQM) [1]. This term is designated in the Minimum Operational Performance Standards [3] as σ_{flt}. In the case of WAAS, this term is the product of the UDRE and a shaping matrix contained in Message Type 28 [4]. The broadcast index UDRE is the maximum of the output of the UDRE algorithm and the floor imposed by both the CCC and SQM monitors.

WAAS today provides vertical guidance in the conterminous U.S and Alaska with very high availability. However, if we want to either achieve better levels of service or be more robust against depleted constellations, it will be necessary to reduce the Protection Levels. This could be done by modifying the Vertical Protection Level [5], [6] or by reducing the term σ_{flt}. With the development of WAAS dual frequency, there is an opportunity to upgrade the algorithms. In this paper, we outline the broad lines of a clock and ephemeris algorithm that has the potential to reduce the WAAS error bounds significantly. This algorithm uses ideas similar to the ones described in [11], but differs in some key points. In the first section we will outline the threats and the message constraints that the current WAAS clock and ephemeris algorithm accounts for, and must be accounted in a new algorithm. The second part will show how each of these constraints can be accounted for. In the third part we will summarize the algorithm. The fourth part will show the potential benefits of the new algorithm as compared to the current one. Finally, we will add a few remarks on the implementation of this algorithm.

THREAT MODEL AND MESSAGE CONSTRAINTS

The current clock and ephemeris algorithm has evolved to account for:

- Nominal error from the network receivers
- Nominal biases (antenna biases)
- The use of corrections that are generated outside the safety processor
- Possibly undetected errors in the network receivers (one station is assumed to return erroneous measurements)

PROBLEM STATEMENT

The pseudorange error due to the clock and ephemeris for a user with a line of sight u_{LOS} is given by:

$$u_{LOS}^T \left(x_{Broadcast} - x \right)$$

The 4 by 1 vector x represents the true satellite clock and ephemeris, and $x_{Broadcast}$ represents the clock and ephemeris computed by the receiver after applying the SBAS corrections [7]. The problem consists on finding an upper bound on this expression for each satellite-user pair. The error bound needs to be of the form:

$$L\left(u_{LOS}\right) = K\sigma_{flt} = K\sigma_{UDRE}\sqrt{u_{LOS}^T \mathrm{Cov}_{MT28}\ u_{LOS}}$$

The matrix Cov_{MT28} is a 4 by 4 matrix that is sent every 120 s per satellite. Every 6 s, it is possible to modify it by multiplying it by σ_{UDRE}. K is the factor assumed by the receiver and is 5.33.

Measurements and prior distribution

Every second, the ground network receivers collect pseudorange measurements to all GPS satellites in view in L1 CA and L2 semi-codeless. These measurements are processed to obtain an ionospheric delay free carrier-smoothed estimate of each pseudorange [8]. Let y be the vector of smoothed measurements from the ground receivers to one satellite corresponding to one epoch. After linearization, the relationship between the ground pseudorange measurements and the satellite's true clock and ephemeris can be represented by:

$$y = Gx + n$$

G is a matrix where each row represents the line of sight to one of the WAAS stations. The vector n is the noise affecting each measurement. This noise is characterized by a Gaussian whose standard deviation is give by the Code Noise and Multipath (CNMP) curve [8], as well as an antenna bias in the order of tens of centimeters which is deterministic, but very difficult to calibrate [9]. The noise is modeled as a gaussian random vector with covariance W^{-1} and bias b.

$$n \sim N\left(b, W^{-1}\right)$$

The bias b is unknown but its magnitude is bounded by b_{max}:

$$|b| \le b_{\max}$$

In addition to the measurements, WAAS assumes a conservative prior distribution of the position of the satellite, which we note x_{prior}. The inverse of the covariance of the prior is given by P, and its magnitude can be found in [4].

ERROR BOUND DERIVATION

Error bound on the estimation error in nominal conditions

The approach taken in this work is to estimate the clock and ephemeris of the satellite using the above equations. If we neglect for the moment the nominal biases b, the optimal estimate is given by a minimum mean square estimator:

$$x_{Estimated} = x_{prior} + \left(P + G^T W G\right)^{-1} G^T W \left(y - Gx_{prior}\right)$$

The covariance of the estimation error is given by:

$$Cov = \left(P + G^T W G\right)^{-1}$$

So we have:

$$P\left(\left|u_{LOS}^T\left(x_{Estimated} - x\right)\right| \ge K_{HMI}\sqrt{u_{LOS}^T Cov\ u_{LOS}}\right) = 2Q\left(-K_{HMI}\right)$$

where Q is the cdf of a normal unit Gaussian. K is related to the integrity allocation $PHMI_{alloc}$ through the equation:

$$PHMI_{alloc} = 2Q\left(-K_{HMI}\right)$$

The error bound is then given by:

$$L_1\left(u_{LOS}\right) = K_{HMI}\sqrt{u_{LOS}^T Cov\ u_{LOS}}$$

This error bound fits within the message format. In the next sections we will modify this error bound to account for the constraints cited above.

Taking into account the nominal biases

The previous equation does not take into account the nominal biases. The error bound must be increased to account for them. For a user's line of sight, the contribution of the biases is given by:

$$u_{LOS}^T H b$$

Where:

$$H = \left(P + G^T W G\right)^{-1} G^T W$$

An upper bound on the error is then given by:

$$\max_{|b|\le b_{\max}} u_{LOS}^T Hb$$

However, this bias term does not fit within the message. An upper bound of this bias is given by the Cauchy-Schwartz inequality:

$$\left|u_{LOS}^T Hb\right| = \left|u_{LOS}^T HW^{-\frac{1}{2}} W^{\frac{1}{2}} b\right| \le \sqrt{u_{LOS}^T HW^{-1} Hu_{LOS}} \sqrt{b^T Wb}$$

Since we have:

$$HW^{-1}H = \left(P + G^T WG\right)^{-1} G^T WG \left(P + G^T WG\right)^{-1} \prec \left(P + G^T WG\right)^{-1} = Cov$$

we end up with:

$$\left|u_{LOS}^T Hb\right| \le \sqrt{u_{LOS}^T Cov\ u_{LOS}^T} \sqrt{b^T Wb}$$

The next step is to compute an upper bound of the scalar $\sqrt{b^T Wb}$. For this we compute:

$$K_{bias} = \max_{|b|\le b_{\max}} \sqrt{b^T Wb}$$

W is diagonal, so the upper bound is given by:

$$K_{bias} = \max_{|b|\le b_{\max}} \sqrt{b^T Wb} = \sqrt{b_{\max}^T W b_{\max}}$$

The error bound is now given by:

$$L_2\left(u_{LOS}\right) = \left(K_{HMI} + K_{bias}\right) \sqrt{u_{LOS}^T Cov\ u_{LOS}}$$

Taking into account the broadcast clock and ephemeris

The error bound computed in the previous section does not yet account for the fact that the user uses $x_{Broadcast}$ instead of $x_{Estimated}$. $x_{Broadcast}$ is computed by the Corrections Processor, whereas $x_{Estimated}$ is computed in the Safety Processor. For a more detailed description of the system architecture, please refer to [10]. For the purpose of this work it suffices to say that $x_{Broadcast}$ is a more accurate estimate than $x_{Estimated}$ under nominal conditions. The role of the Safety Processor is to make sure that the error bound associated to $x_{Broadcast}$ is valid under all circumstances. This is done by accounting for the difference between $x_{Broadcast}$ and $x_{Estimated}$.

$$u_{LOS}^T\left(x_{Broadcast} - x\right) = u_{LOS}^T\left(x_{Broadcast} - x_{Estimated}\right) + u_{LOS}^T\left(x_{Estimated} - x\right)$$

Again the first term does not fit within the message. We proceed again using the Cauchy-Schwartz inequality:

$$\left|u_{LOS}^T\left(x_{Broadcast} - x_{Estimated}\right)\right| = \left|u_{LOS}^T Cov^{\frac{1}{2}} Cov^{-\frac{1}{2}}\left(x_{Broadcast} - x_{Estimated}\right)\right|$$

$$\le \sqrt{\left(x_{Broadcast} - x_{Estimated}\right)^T Cov^{-1}\left(x_{Broadcast} - x_{Estimated}\right)} \sqrt{u_{LOS}^T Cov\ u_{LOS}}$$

Because the error bound must be valid for 120 s, an upper bound of the first term is necessary. That is we find K_{fa} such that with a probability consistent with the false alarm requirement we have:

$$\sqrt{\left(x_{Broadcast} - x_{Estimated}\right)^T Cov^{-1}\left(x_{Broadcast} - x_{Estimated}\right)} \le K_{pfa}$$

After this additional term, the error bound is given by:

$$L_3\left(u_{LOS}\right) = \left(K_{HMI} + K_{bias} + K_{pfa}\right) \sqrt{u_{LOS}^T Cov\ u_{LOS}}$$

Taking into account undetected measurement errors

The error bound computed in the previous sections would be valid if all measurements were trusted. However, there exists the possibility that measurements used to assess the integrity might be corrupted. Although this happens very rarely, the WAAS threat model assumes that at all times one of the measurements might be erroneous. This can be taken into account by computing the pair:

$$\left(x_{Estimated}^{(k)}, Cov^{(k)}\right)$$

for each subset k where measurement k has been excluded. The problem consists now in finding a matrix Cov_{ob} such that for all lines of sight over the footprint:

$$u_{LOS}^T Cov^{(k)}\ u_{LOS} \le u_{LOS}^T Cov_{ob}\ u_{LOS}$$

such that $u_{LOS}^T Cov_{ob}\ u_{LOS}$ is as small as possible. The exact optimization problem could be then written:

$$\text{minimize} \int_{u_{LOS}\ \text{over footprint}} u_{LOS}^T Cov_{ob}\ u_{LOS}$$

such that $u_{LOS}^T Cov^{(k)}\ u_{LOS} \le u_{LOS}^T Cov_{ob}\ u_{LOS}$ for all u_{LOS} and k

The objective function is a linear function of Cov_{ob}, so it can be written as the trace multiplied by a matrix A. The constraint can be relaxed by extending the constraint to any vector u (not only a line of sight). The resulting problem is written:

$$\text{minimize trace}\left(Cov_{ob}A\right)$$
$$\text{such that } Cov^{(k)} \leq Cov_{ob} \text{ for all } k$$

Under this form, this problem is a Second Order Cone Program (SOCP) [11]. It is a convex problem and can be solved efficiently.

Exploiting the structure of the set of matrices $Cov^{(k)}$

Although the problem above can be solved for any set of definite positive matrices, it is worthwhile exploiting their structure, in particular the fact that they differ from the all-in-view covariance by a rank two matrix in the general case and by a rank one matrix if the weighting matrix is diagonal. In the diagonal case, which is assumed throughout the paper, we have:

$$Cov^{(k)} = \left(P + G^{(k)T}W^{(k)}G^{(k)}\right)^{-1} = \left(Cov^{-1} - g_k w_{kk} g_k^T\right)^{-1}$$
$$= Cov + \frac{Cov\ g_k w_{kk} g_k^T\ Cov}{1 - g_k^T w_{kk} Cov\ g_k g_k^T}$$

The problem above can therefore be simplified to:

$$\text{minimize trace}\left(\Delta Cov_{ob}A\right)$$
$$\text{such that } h_k h_k^T \leq \Delta Cov_{ob} \text{ for all } k$$

where:

$$h_k = \sqrt{\frac{w_{kk}}{1 - g_k^T w_{kk} Cov\ g_k g_k^T}} Cov\ g_k$$

Heuristics to find Cov_{ob}

In this section, we describe the method that was used to compute Cov_{ob} at each step and for each satellite. Using the notations above, the following steps are performed:

1. For each k compute:

   $$r_k = h_k^T Cov^{-1} h_k$$

   By construction we have:

   $$h_k h_k^T \leq r_k Cov$$

2. Find the set I_{large} of r_k that exceeds a threshold $\tau = 0.10$
3. Define C_0 as τCov. For k outside of I_{large} we have:

   $$h_k h_k^T \leq \tau Cov$$

   Sort the r_k in I_{large} in decreasing order. We renumber them to be r_1 to r_p.
4. For k from 1 to p we perform the following operations:

   $$\alpha_k = 1 - \frac{1}{h_k^T C_{k-1}^{-1} h_k}$$
   $$C_k = C_{k-1} + \max\left(0, \alpha_k\right) h_k h_k^T$$

The resulting matrix C_p is an upper bound of the matrices $h_k h_k^T$. The final matrix is then:

$$Cov_{ob} = Cov + C_p$$

The error bound computed by the user must be such that:

$$L\left(u_{LOS}\right) = K\sigma_{flt} = \left(K_{HMI} + K_{bias} + K_{pfa}\right)\sqrt{u_{LOS}^T Cov_{ob}\ u_{LOS}}$$

We found that the sub-optimal approach produced error bounds less than 5% larger than the optimal one.

UDRE Floor implementation

As indicated above, σ_{flt} has a floor imposed by the CCC and SQM monitors. Let us assume that the floor is given by σ_{floor}. We must then find Cov_{ob+fl} to account for the floor. Cov_{ob+fl} must be such that:

$$\sigma_{floor} \leq \frac{K_{HMI} + K_{bias} + K_{pfa}}{K}\sqrt{u_{LOS}^T Cov_{ob+fl}\ u_{LOS}}$$
$$\sqrt{u_{LOS}^T Cov_{ob}\ u_{LOS}} \leq \sqrt{u_{LOS}^T Cov_{ob+fl}\ u_{LOS}}$$

To meet this inequality, it is sufficient to have:

$$\frac{\sigma_{floor}^2}{2} I \leq \left(\frac{K_{HMI} + K_{bias} + K_{pfa}}{K}\right)^2 Cov_{ob+fl}$$
$$Cov_{ob} \leq Cov_{ob+fl}$$

To solve the above problem, we form the singular value decomposition of Cov_{ob} :

$$Cov_{ob} = U^T D_{ob} U$$

Changing basis, the above constraints are equivalent to:

$$\frac{\sigma_{floor}^2}{2}\left(\frac{K_{HMI}+K_{bias}+K_{pfa}}{K}\right)^{-2} I \le UCov_{ob+fl}U^T$$

$$D_{ob} \le UCov_{ob+fl}U^T$$

We define the diagonal matrix D_{ob+fl} as:

$$D_{ob+fl,ii} = \max\left(D_{ob,ii}, \frac{\sigma_{floor}^2}{2}\left(\frac{K_{HMI}+K_{bias}+K_{pfa}}{K}\right)^{-2}\right)$$

Finally, the matrix Cov_{ob+fl} defined as:

$$Cov_{ob+fl} = U^T D_{ob+fl} U$$

meets the conditions above.

Composing the message and discretization

The covariance and UDRE broadcast by WAAS must be such that we have:

$$K\sigma_{flt} = \left(K_{HMI}+K_{bias}+K_{pfa}\right)\sqrt{u_{LOS}^T Cov_{ob+fl}\ u_{LOS}}$$

As mentioned above, the user forms σ_{flt} by combining the UDRE and MT28. The broadcast σ_{UDRE} and Cov_{MT28} must be such that:

$$K\sigma_{UDRE}\sqrt{u_{LOS}^T \mathrm{Cov}_{MT28}\ u_{LOS}} \ge \left(K_{HMI}+K_{bias}+K_{pfa}\right)\sqrt{u_{LOS}^T Cov_{ob+fl}\ u_{LOS}}$$

A sufficient condition is:

$$\sigma_{UDRE}^2 \mathrm{Cov}_{MT28} \ge \left(\frac{K_{HMI}+K_{bias}+K_{pfa}}{K}\right)^2 Cov_{ob+fl}$$

Cov_{MT28} is computed by choosing a value for σ_{UDRE} and discretizing the matrix:

$$\left(\frac{K_{HMI}+K_{bias}+K_{pfa}}{K\sigma_{UDRE}}\right)^2 Cov_{ob+fl}$$

The discretization of MT28 is described in [4]. This discretization introduces a small penalty, so σ_{UDRE} should be chosen to minimize it. In the implementation simulated below, a value of 0.91 m (which corresponds to the UDRE index of 5) was chosen.

Summary of the algorithm

Here are the main steps of the algorithm:

1. Compute $\left(x_{Estimated}^{(k)}, Cov^{(k)}\right)$ for each subset
2. Check that $\sqrt{\left(x_{Broadcast}-x_{Estimated}^{(k)}\right)^T Cov^{(k)-1}\left(x_{Broadcast}-x_{Estimated}^{(k)}\right)} \le K_{pfa}$
3. Compute $K_{bias} = \sqrt{b_{\max}^T W b_{\max}}$
4. Compute the matrix Cov_{ob+fl} as indicated above
5. Choose σ_{UDRE} and compute:
   $$Cov_{MT28,pd} = \left(\frac{K_{HMI}+K_{bias}+K_{pfa}}{K\sigma_{UDRE}}\right)^2 Cov_{ob+fl}$$
6. Discretize $Cov_{MT28,od}$ to obtain Cov_{MT28}

AVAILABILITY EVALUATION

In this section, we compare the performance of the proposed algorithm with the current UDRE algorithm adapted to L1 – L5. A description of the basic elements of the current algorithm can be found in [11]. The Service Volume Analysis tool MAAST was used to simulate the performance of the algorithm for 24 hours every 300 s over North America. The 24 satellite GPS constellation specified in [3] was assumed.

Parameter settings

The magnitude of W is determined by the CNMP curve and the clock calibration error. K_{bias} is computed in real time and is a function of W and the maximum biases [8], [9]. The resulting factor is between 3 and 4. K_{HMI} is determined by the integrity allocation. In this work, the value 5.5 was assumed and is an upper bound of what would need to be assumed. The floor for σ_{flt} was taken to be 0.68 m (UDRE index 4).

Results

The histogram shown in Figure 1 shows the ratio between σ_{flt} computed using the proposed algorithm and the current algorithm. The new values are up to 50% smaller. Figure 2 and 3 show the 99% VPL quantile for the current and new algorithm respectively. There is a significant improvement, which suggests that such an approach could help WAAS achieve lower VPLs and, as a consequence, new levels of service.

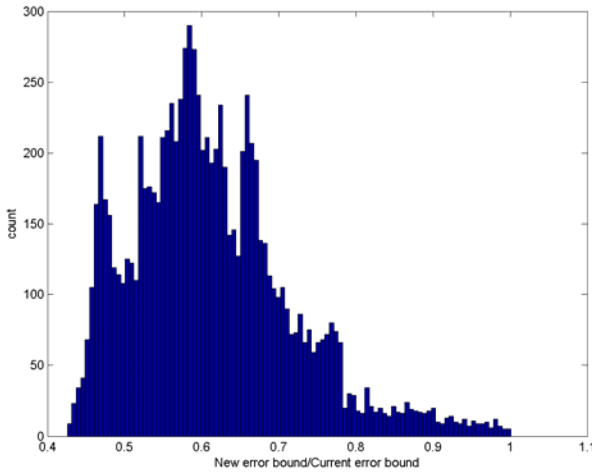

Figure 1. *Histogram of New Error Bound/ Current Error Bound*

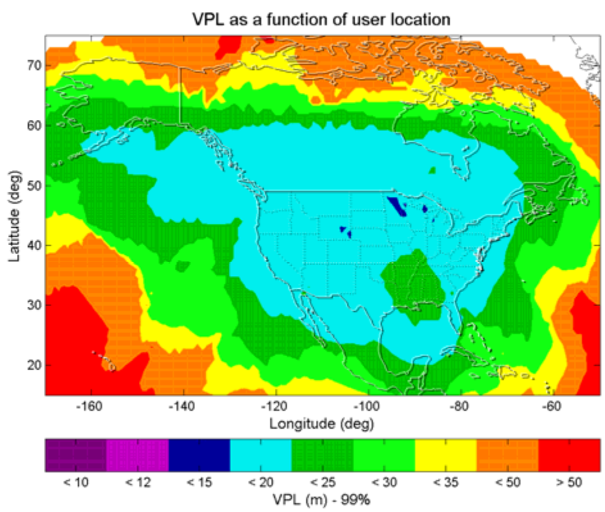

Figure 2. *99% VPL quantile for the current algorithm*

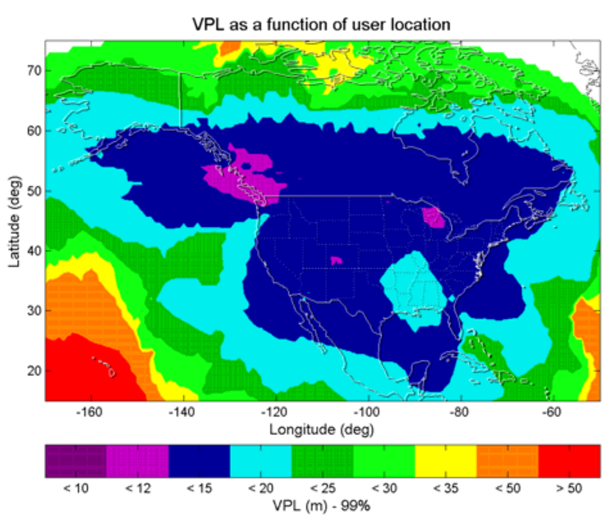

Figure 3. *99% VPL quantile for the proposed algorithm*

ADDITIONAL REMARKS

In this paper we have only presented the outline of the algorithm, which is sufficient to evaluate its potential. For its implementation, many decisions remain to be taken. For example, it will be necessary to check the consistency of the measurements before computing $x_{Estimated}$. If they are not consistent, it will have to be decided whether Fault Detection and Exclusion should be performed, or the satellite declared unfit for WAAS. Another point that will need to be specified is the external UDRE monitor. As mentioned above, the covariance can only be sent every 120s, but the multiplying factor σ_{UDRE} can be sent every 6 s. Future work should address the optimal way of updating σ_{UDRE} and the fast correction.

CONCLUSION

In dual frequency WAAS, σ_{flt} which includes the clock and ephemeris error, is the largest contributor to the error bound. In this work, we propose a clock and ephemeris algorithm that could reduce VPLs by 20%. The algorithm computes a covariance that bounds both the user estimation error in the presence of biases and receiver faults. The improvements presented here do not depend on a change in the message standards because the information produced by this algorithm fits in the current ones.

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