A Simple Algorithm for Dual Frequency Ground Monitoring Compatible with ARAIM

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ABSTRACT

Dual frequency Absolute RAIM has the potential of providing global coverage of vertical guidance. However, a constellation of thirty or more satellites will be necessary to achieve this goal. In this work, a hedge for Absolute RAIM users at a low cost for the integrity provider is investigated. The concept relies on a simple real time ground monitoring algorithm and on the generalization of the ARAIM algorithms. The ground monitoring described here consists of a simple snapshot algorithm where all unknowns are estimated simultaneously – although exploiting explicitly the ARAIM assumptions. For each satellite, a bound on the worst case error is broadcast. This error bound is integrated by the user in the ARAIM solution. This is achieved by generalizing the Multiple Hypothesis Solution Separation ARAIM algorithm. Preliminary performance results are shown for a reference station network in North America as a function of the reference network and the constellation. Finally, the importance of the ARAIM assumptions and the necessity of offline monitoring are stressed.

INTRODUCTION

With the upcoming deployment of dual frequency GPS satellites and the Galileo constellation, there is the possibility that Absolute Receiver Autonomous Integrity Monitoring (as defined in [1]) will provide worldwide coverage of vertical guidance. Indeed, GNSS receivers will be able to remove the ranging error due to the ionosphere through the use of dual frequency. Also, the clock and ephemeris errors will be greatly reduced through the use of better clocks and improved orbit estimation. ARAIM for worldwide vertical navigation is a very attractive option, as it would reduce the need for a ground monitoring infrastructure (but it would not eliminate it, as will be seen in the last section of this paper). Unfortunately, with less than 30 satellites ARAIM would not provide sufficient coverage [1].

In this work we investigate an integrity concept where a very simple ground monitor helps the ARAIM user mitigate the possible failures. This concept has many similarities with both Satellite Based Augmentation Systems and the Galileo Safety-of-life integrity concept [2], [3], and is compatible with ARAIM. For this reason, we will first review the ARAIM algorithm and its availability. Secondly, we will show how the measurements made by the ground monitor van be integrated in the ARAIM solution. Then we will show the availability figures for this concept as a function of the assumptions on the possible failures, the size and location of the ground reference network, and the constellation size. In the last section, we will go back to the ARAIM assumptions and stress the role of offline monitoring.

ARAIM

A complete description of Absolute RAIM can be found in [1], [4], and [5]. In ARAIM, for each possible fault i, the receiver computes a position solution that is not affected by the fault –this is done simply by not including the measurement in the least squares solution - , as well as a standard deviation and maximum bias for that position solution. The standard deviation and the bias are computed using the nominal error model - an overbound of the error in nominal conditions [1]. Then for each possible fault i, a partial Vertical Protection Level VPL_i is computed. The partial VPL_i includes the error bound around the subset solution and the solution separation term. The solution separation term is the actual solution separation between the all-in-view solution and the subset solution in real time VPL, and an upper bound on the separation in the predicted VPL. The final VPL is computed by taking the maximum among the partial VPL_i:

$$VPL = \max_{0 \le i \le n} \left(VPL_i \right) \tag{1}$$

In this equation, the index zero corresponds to the all-in-view solution, n is the number of satellites, and i corresponds to each possible failed satellites. Although in this work we focus on single failures, one can easily include the multiple failure case by just computing the corresponding partial *VPL* [4]. The details of the calculation can be found in [1], [4] or [5]. The final *VPL* is dependent on the integrity budget (the Probability of Hazardously Misleading Information), the continuity budget, the nominal error model (which includes a Gaussian overbound and a maximum bias), the a priori probability of failure for each satellite (or set of satellites), and the geometry. Also, each partial VPL_i depends on the integrity and continuity allocation made to the fault mode *i*.

ARAIM AVAILABILITY

Here we provide results for North America (a study of worldwide ARAIM performance can be found in [1]).

Error Model

The error model for each error source used here was determined by the GPS Evolutionary Architecture study, and can be found in [1]. It includes the tropospheric error bound, the receiver noise, and the multipath. The User Range Accuracy (URA), which includes the clock and ephemeris errors, was taken to be .75 m. A bias of .5 m is also assumed (b_{nom}). It is supposed to cover deviations from the Gaussian distribution, possible correlations among measurement errors, and the nominal signal deformation [6].

Requirements

The integrity requirement (Probability of Hazardously Misleading Information) is 10^{-7} per approach, and the continuity requirement (Probability that the VPL exceeds the predicted VPL once the approach has started) is 4.10^{-6}. These requirements are very similar to the SBAS requirements.

Simulation conditions

We used the Service Volume analysis tool MAAST [7] to evaluate the performance of the concept investigated here. The VPL was computed for North America by simulating a user every 2 degrees between15 and 75 degrees in latitude and -170 and -50 degrees in longitude, for a period of 24 hours every 5 minutes.

Results

Figure 1 is a map of the resulting 99.5%quantile VPL at each location for a 24 satellite constellation. At each location, the VPL exceeded the indicated value only .5% of the time. One can see that the performance is insufficient. This is due to the very weak subset solutions when computing the partial VPLs.

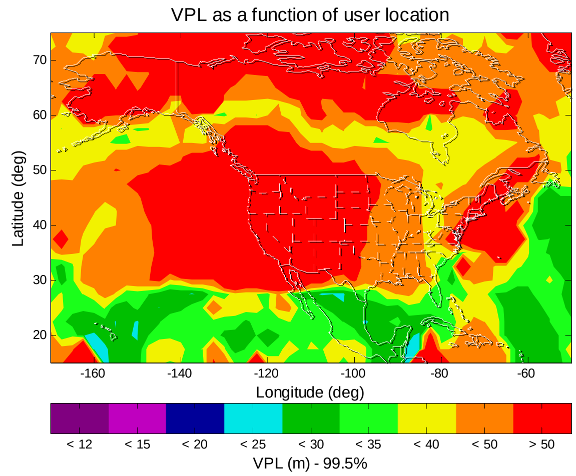

Figure 1. *99.5% Percentile of the VPL over the course of a day for a 24 satellite constellation.*

REAL TIME GROUND MONITORING

List of threats

The threats considered here include clock and ephemeris errors (clock run-offs, bad ephemeris uploads, unannounced maneuvers). A real time monitoring should also monitor the signal deformation and code carrier coherence as it is done in WAAS and LAAS. This will not be covered here, but it should be understood that such a monitor would be needed in the concept proposed here, and a flag should be raised if the signal deformation exceeds the assumed nominal bias for the worst case user. This being said, we will limit our description to the clock and ephemeris errors.

Limiting the magnitude of the failure mode

As was described earlier, the ARAIM user protects against possible failures by computing several possible solutions and their associated error bound. In each solution, the possibly failed satellite is excluded completely. With real time monitoring, the situation is different: even if there is a failure, the ground can estimate the offset between the broadcast position and the position estimated by the ground.

Joint estimation of satellite positions and receiver clock offset

At each epoch (each second for example) the ground computes a snapshot position solution of each satellite, and its difference with the broadcast position. Because the receiver clocks and inter-frequency biases are unknown, they are estimated jointly with the satellite positions. Let X be the set of unknowns. The measurement model is given by:

$$Y = HX + z \quad (2)$$

The matrix H includes all the lines of sight (including the clock) from each satellite to each reference station and the receiver clocks. One of the reference receivers was used as time reference (otherwise H is ill conditioned) – in a real system, the reference should be either one satellite or the average of the satellites in view, because no clock corrections are broadcast. The measurements Y are iono-free and carrier smoothed. The ground errors z are characterized by a Gaussian overbound and a maximum bias (for this work, we took the current WAAS ground measurement overbound [8]). We will label W_{ref} the inverse of the covariance matrix describing z.

Factoring in the prior probability of failure

Just like the ARAIM user computes a position solution for each possible threat, the ground estimates the unknowns for each possible threat. Here, each threat corresponds to the user range error not being overbounded by the URA. This means that the system above is solved as many times as there are failure modes. Each failure mode corresponds to a different prior on the satellite position.

Prior covariance in the nominal case

The prior covariance in the nominal case was chosen such that the standard deviation of the induced pseudorange error is σ_{URA}. It was chosen to be:

$$P_{sat,nom}^{-1} = \frac{\sigma_{URA}^2}{2} I_4 \tag{3}$$

The matrix I_4 is the 4 by 4 identity matrix.

Prior covariance in the faulted case

The prior covariance in the faulted case assumes a possibly unbounded clock error and a large but bounded prior on the ephemeris.

$$P_{sat,fault}^{-1} = \begin{bmatrix} \sigma_{eph,fault}^2 I_3 & \underline{0} \\ \underline{0} & +\infty \end{bmatrix} \tag{4}$$

Prior for the set of unknowns

Here we define the prior covariance for each fault mode. There are as many fault modes as there are satellites in view of the reference network (N_{sat}). The prior covariance corresponding to the first satellite being faulted would be:

$$P_{allsat,1} = \begin{bmatrix} P_{sat,fault} & 0 & \cdots & 0 \\ 0 & P_{sat,nom} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & P_{sat,nom} \end{bmatrix} \tag{5}$$

The total prior covariance for this failure mode is given by:

$$P_{total,1} = \begin{bmatrix} P_{allsat,1} & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

The zero in the lower right corner of the matrix is a square matrix of zeros corresponding to the inverse of the a priori covariance of the receiver clocks (they are assumed to be completely unknown at every epoch).

Covariance of the estimate for each failure mode

Now that we have defined the a priori covariance and the observation matrix, we can compute the covariance of the estimate after the measurements. It is given by:

$$C_{all,1} = \left(H^T W_{ref} H + P_{total,1}\right)^{-1} \tag{7}$$

Similarly, the ground computes $C_{all,i}$ for all possible satellite failures.

Estimated offset between broadcast position and true position

It is assumed here that the measurements Y are linearized with respect to the broadcast satellite positions. For a given fault mode, the estimated satellite offset and receiver clocks is computed using:

$$X_{est,i} = \left(H^T W_{ref} H + P_{total,i}\right)^{-1} H^T W Y \tag{8}$$

This estimate includes all satellites and all reference receiver clocks.

User error mitigation

Ideally, users would receive all the measurements and apply the RAIM algorithm to all the available measurements. However, this is not possible due to the bandwidth limitations and the obvious complexity that such a scheme would require. Instead, the ground only sends information about the position of the satellite as measured by the ground, in the case the a priori covariance is not trusted. The covariance corresponding to the failed satellite i is extracted from the matrix $C_{all,i}$ and the position is extracted from $X_{est,i}$. They will be labeled C_i and X_i respectively.

Because the user will only use the information gathered at the ground to limit the size of the failure mode, it is not necessary that the actual position offset be sent. Instead, a bound on the error can be computed as a function of the covariance C_i. The difference between the broadcast position and the estimated position is expected to be bounded by the multivariate Gaussian with covariance:

$$P_{sat,nom}^{-1} + C_i \quad (9)$$

The ground checks that:

$$X_i^T \left(P_{sat,nom}^{-1} + C_i\right)^{-1} X_i \le T \quad (10)$$

Here T is related to the probability of false alarm. Assuming that X_i is a multivariate Gaussian, the above statistic is chi-square distributed. In this work, a probability of false alarm of 10^{-6} was chosen. T is therefore determined by:

$$T = \left(\chi_4^2\right)^{-1}\left(1-10^{-6}\right) = 5.78^2 \quad (11)$$

A user knowing the nominal URA and C_i can calculate an upper bound on the difference between the broadcast position and the estimated position:

$$\begin{aligned}&\left|u^T\left(X_i - X_{i,broadcast}\right)\right| \le \sqrt{Tu^T\left(P_{sat,nom}^{-1} + C_i\right)u}\\ &= \sqrt{T}\sqrt{\sigma_{URA}^2 + u^T C_i u}\end{aligned} \quad (12)$$

In this equation, the vector u contains both the line of sight and the clock. In addition to the bound on the maximum deviation, the user computes the standard deviation on the uncertainty around the ground estimate, which is given by:

$$\sigma_{i,sat,mon}^2 = u^T C_i u \quad (13)$$

Effect of measurement biases

The biases on the ground (due to antenna biases), are relatively small (tens of centimeters), but need to be taken into account. The effect on a line of sight u – corresponding to the satellite one, for example- is given by:

$$\begin{aligned}\underline{u} &= \begin{bmatrix}u^T & 0 & \cdots & 0\end{bmatrix}^T\\ s &= \underline{u}^T\left(H^T W_{ref} H + P_{total,i}\right)^{-1} H^T W\\ b_1 &= \sum_{k=1}^{N_{ref}}\left|s_k b_{ref,i}\right|\end{aligned} \quad (14)$$

where b_{ref} is a vector of biases (which as stated earlier is mostly due to ground antenna biases). In this study, the bias on one line of sight was assumed to be bounded by a multiple of the standard deviation on the uncertainty of the ground estimate. For the line of sight i:

$$b_{sat,i} \le \alpha\sigma_{i,sat,mon} \quad (15)$$

An offline analysis showed that for the values of the biases assumed in this paper [8], a conservative value of one for α was conservative.

User Messages

In the concept proposed here, users would receive a flag stating whether the satellite is monitored and can be used, or is monitored and cannot be used – for example because the estimate is not within the bounds determined by the continuity constraint-, or is not monitored (in which case the satellite can be used in the ARAIM sense). In addition, a covariance C_i is sent per satellite –like Message Type 28 in WAAS [9]-, which allows the user to compute a bound on the maximum deviation between the broadcast position solution and the ground estimated position using equation (12), the standard deviation around this estimate using equation (13), and a bias using equation (15) (with α=1). To summarize, the user can assume that in the failure mode, the pseudorange error is overbounded by a biased distribution with standard deviation $\sigma_{i,sat,mon}$ and maximum bias:

$$b_i = \sqrt{T}\sqrt{\sigma_{URA}^2 + u^T C_i u} + b_{sat,i} \quad (16)$$

The monitored standard deviation is similar to the Signal in Space Monitored Accuracy (SISMA) in the Galileo Safety-of-Life concept, and the URA corresponds to the Signal in Space Accuracy (SISA).

Message rate

In this paper, we did not evaluate the necessary bandwidth and the best way to send the covariance matrix. It is expected that the covariance per satellite could be sent less than every two minutes (like Message Type 28). Also, there are ways to further compress the covariance matrix. One way is by choosing a basis where the covariance is better conditioned than in MT 28.

Error correlation

As said earlier, for each possible threat (each possible satellite failure) the ground monitor solves a joint system where all satellites in view are used. As a consequence, for each possible threat, the estimate on the possibly failed satellite is correlated with the errors in the remaining satellites. Fortunately, the correlation is very weak and can be conservatively taken into account by slightly inflating the covariance.

USER ALGORITHM

The user algorithm is a very simple modification of the ARAIM algorithm briefly described earlier. As said earlier, the ARAIM user computes a position solution for each possible satellite fault (or group of satellites), by excluding the satellite that could be faulted. With ground monitoring, the user receives a bound on the clock and ephemeris error (signal-in-space) that is valid even in the faulted case. As a consequence, instead of excluding completely the satellite from the sub-solution, it can be included assuming that the clock and ephemeris error is overbounded by the standard deviation $\sigma_{i,sat,mon}$, and the maximum bias b_i (instead of σ_{URA} and $b_{nom,i}$). The solution separation has the same expression as in the ARAIM equations (see [1], [4]), [5].

Sub-solution coefficients
As pointed out in [10], the optimal solution in the presence of biases is computationally intensive. One way to adjust the coefficients is by using modified weights in the least square solution. For this work, we used the weighting matrix whose diagonal terms are given by:

$$w_i^{-1} = \sigma_{i,DF\_air}^2 + \sigma_{i,tropo}^2 + \sigma_{i,sat,mon}^2 + \left(\frac{b_i}{K}\right)^2$$

$$K = -Q^{-1}\left(10^{-2}\right) = 2.33$$

Please refer to [1] for the definition of the two first terms in w_i. This determines the projection matrix from the measurements onto the position solution S. Once the S matrix is defined, the partial VPL can be computed [4], [5].

AVAILABILITY

The availability and coverage evaluation were done in the conditions described in the section on ARAIM availability. Two parameters were studied: the a priori ephemeris assumptions in the case of a failure, and the effect of the ground network size. The baseline reference station network is the current WAAS 38 station network [11].

Effect of a priori ephemeris
Table 1 shows the results with three possible values of $\sigma_{eph,fault}$: 10 m, 1000 m and 1000 km. The first row shows the percentage of the region shown in Figure 1 that has a 99.5% availability. The second row shows the average availability. The third row shows the average 99.5% VPL (the average over the values plotted in a map like Figure 1). There is a significant improvement from 1000 km to 10 m, since the size of the region with insufficient coverage is more than halved. Given the size of the observed ephemeris faults [12], an a priori of 1000 m seems appropriate and conservative.

	10 m	1000 m	1000 km
99.5% cov.	99.31%	98.93%	98.22%
Availability	99.99%	99.98%	99.97%
Av. 99.5 VPL	21.5 m	22.5 m	22.7 m

Table 1. *Effect of a priori ephemeris covariance*

An effect that cannot be appreciated in this chart was observed on the maps: in some regions: decreasing the a priori from 1000 km to 1000 m would decrease the VPL from 30 m to 20 m. This shows that the choice on the a priori should not be neglected.

Effect of ground network
Three situations are compared: no real time ground monitoring, a small reference stations network of 8 reference stations (see Appendix), and the baseline 38 stations network. The constellations are the same that were used in [1]. The results are shown in Table 2.

		24-1	24	27-1	27	30-1	30
No Real Time Mon.	99.5% cov	**3.65%**	**27.47%**	**9.56%**	**87.90%**	**79.8%**	**99.59%**
	Avail.	94.49%	98.24%	97.22%	99.85%	99.75%	99.99%
	Av. 99.5% VPL	∞	54.0 m	104.2 m	27.3 m	30.8 m	20.0 m
8 stat.	99.5% cov	**50.80%**	**88.26%**	**71.47%**	**96.70%**	**98.68%**	**100%**
	Avail.	98.91%	99.85%	99.60%	99.96%	99.97%	100%
	Av. 99.5% VPL	51.8 m	26.7 m	32.5 m	21.7 m	22.6 m	18.2 m
38 stat.	99.5% cov	**71.15%**	**98.93%**	**90.03%**	**100%**	**99.89%**	**100%**
	Avail.	99.48%	99.87%	99.87%	100%	99.89%	100%
		37.7 m	22.5 m	27.5 m	19.4 m	20.24 m	17.0 m

Table 2. *Effect of ground network size and location*

For 24 satellites, the coverage increases dramatically with only 8 reference stations. With 38 stations, the coverage is almost as good as with 30 satellites and no real time ground monitoring. Table 2 allows us to compare the value of the reference station to the value of redundant satellites at a fundamental level, since the underlying assumptions are the same for all the entries. It is interesting to notice that with more than 30 satellites, real time ground monitoring brings a very modest improvement.

DIFFERENCES AND SIMILARITIES WITH SBAS DUAL FREQUENCY AND GALILEO INTEGRITY CONCEPT

In this concept, real time monitors would not send a correction, only a bound on the difference between the broadcast position and the ground estimated satellite position. This is similar to the Galileo integrity. The advantage of this approach over SBAS is the simplicity of both ground monitors processing and messaging. In Satellite Based Augmentation Systems where the ground improves the accuracy of the position solution by sending corrections based on an orbit estimator. Because the orbit estimator is typically distrusted, a complex chain of monitors needs to ensure that all the outputs are correct and consistent. Also, the proof of safety is arduous to establish, as the ground processing adds several new threats. In the concept presented here, the fault tree is much simpler than in SBAS, as it is much closer to the ARAIM fault tree.

The main difference with both SBAS and the Galileo integrity concept is the fact that it is designed to be compatible with Absolute RAIM: if a satellite is not being monitored by the real time ground network, then it can be used in the ARAIM sense (that is, it can only be trusted up to the specified a priori probability of failure). This feature would allow the ground network to adapt its size to the available constellation. For example, if two constellations are operational and trusted in the ARAIM sense, it would not be necessary to send real time error bounds. However, if there is only one small constellation (GPS with 24 satellites for example), it would be necessary to send the error bounds.

Another difference with both SBAS and Galileo is the scalability of this concept. It could be used both as a local solution, with only one reference station (similar to Local Augmentation System), or as a global solution with a worldwide network. The equations shown in this paper would work for any size of reference network.

As opposed to SBAS, there are no ionospheric corrections for single frequency users, which would limit the reversionary modes to non-precision approach in the case of the loss of one frequency. Also, this concept does not provide a hedge against the possibility of a worse than expected dual frequency performance. This might be the larger difference with SBAS: like ARAIM, it relies heavily on a very accurate GNSS constellation with very few failures.

OFF-LINE MONITORING-ARAIM ASSUMPTIONS

The Absolute RAIM assumptions used in [1] were obtained by using the historic probability of satellite failure in GPS and the expected error in the clock and ephemeris for the dual frequency satellites. As said earlier, the clock, ephemeris and signal deformation are assumed to be well bounded by a standard deviation σ_{URA} of .75 m and a maximum bias b_{nom} of .5 m in the nominal case, and the prior probability of a failure is than 10^{-5}. The ARAIM algorithm interprets this the following way: the expected error is overbounded by σ_{URA} and b_{nom} in the paired bounding sense (for example) [13] up to the 10^{-5} probability, that is there can be tails that are not within the nominal distribution but their weight is less than 10^{-5}.

Because in ARAIM the errors are not monitored in real time, we need to be convinced that the assumptions will hold in the near future. The question is therefore how to convince ourselves that they will. One way to proceed is by examining how it has been done for the current use of RAIM. The current RAIM assumptions (for horizontal guidance) are based on a very conservative overbound on the accuracy and a probability of failure that is linked to the historical performance of GPS satellites [RAIM ref]. This approach can be adapted to dual frequency RAIM by defining a length of time (for example, a year) and making sure that for any period of this length, the empirical distribution of the errors and their convolution are overbounded by the nominal model and its convolutions. However, the final overbound should also account for the fact that the expected errors increase with the age of data, and that some satellites might have a known risk of failure (because they are older, or have a known defect, for example). It will also be necessary to clearly specify what the errors include, in particular regarding receiver specific errors induced by signal characteristics (like Signal Deformation).

CONCLUSION

The dual frequency GNSS integrity monitoring concept presented here is compatible with ARAIM, and may be compatible with Galileo Safety-of-life. Since the ground monitors are very simple and do not rely on an orbit estimation filter (not even for the receiver clocks), the proof of safety could be greatly simplified –compared to SBAS. This concept complements the three architectures that have been studied in the GPS Evolutionary Architecture Study (GIC, Relative RAIM, ARAIM) by adding another midpoint between the GIC architecture and the ARAIM architecture. At a fundamental level, it can be used to evaluate the value of redundant satellites versus ground monitoring. Finally, both ARAIM and the concept proposed here rely heavily on a very accurate GNSS constellation with very few failures. It will therefore be imperative that offline monitoring be in place. However, before that, it will be necessary to define on which conditions a signal is deemed trustworthy in the ARAIM sense.

ACKNOWLEDGEMENTS

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APPENDIX

The reduced network of reference stations is given in the following table:

	Lat.	Lon
Los_Angeles	34.604	-118.084
Atlanta	33.380	-84.297
Cold Bay	55.200	-162.718
Honolulu	21.313	-157.921
Kansas City	38.880	-94.791
San Juan	18.431	-65.994
Gander	48.94	-54.569
Mexico City	19.439	-99.067

Table A. *Reduced Network*

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