Sensitivity of Code Tracking Methods to PRN Differences

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BIOGRAPHY

R. Eric Phelts is a Research Associate in the Department of Aeronautics and Astronautics at Stanford University. He received his B.S. in Mechanical Engineering from Georgia Institute of Technology in 1995, and his M.S. and Ph.D. in Mechanical Engineering from Stanford University in 1997 and 2001, respectively. His research involves signal deformation analysis and monitoring for WAAS and LAAS.

ABSTRACT

Many of the future GNSS ranging codes produce correlation functions that have two or more prominent sidelobes immediately adjacent to the primary peak. These more complex code modulations have inspired numerous inventive techniques for reliable code tracking. Most of these techniques have been conceived principally to prevent the accidental tracking of the sidelobes. They have also been evaluated for their noise and multipath mitigation performance. Few if any, however, have been evaluated for their sensitivity to range biases due to differences in the PRN codes themselves.

This paper analyzes the biases that may be present when ranging on the Galileo OS E1b and E1c codes using four proposed methods. In contrast to previous, more-simplistic analyses, it models SAW filters used in actual GNSS receivers to evaluate the relative sensitivity of each to PRN differences. The results suggest that some receiver code tracking techniques may provide far superior robustness to PRN-to-PRN variations than others. It is suggested that this type of analysis should be considered when evaluating potential tracking loop implementations for receivers to be used in high-accuracy and high-integrity applications.

BACKGROUND

In a single GNSS receiver, common-mode errors are those that are the same from signal to signal. They are estimated as a part of the clock term in the navigation solution. They have no effect on user position accuracy. Conversely, errors that are not common-mode may cause range errors and decrease the accuracy of the final position solution.

One potential error source that is typically considered common-mode is the user receiver processing. More specifically, the receiver filter channel and tracking loop or discriminator implementation, are often assumed to have no effect on range (and, hence, position) accuracy. If all incoming signals of a given receiver were themselves identical, receiver processing would introduce only common-mode errors. However, the received signals are never completely identical. Because receiver hardware is also imperfect, errors can result if this is not taken into account.

Unmodeled Biases

Signal deformations such as those nominally present on navigation satellites [1] are caused by imperfections or faults in the signal generating hardware on the satellites. They can lead to range errors by causing one or more signals to differ from the others. Receiver filtering and discriminator implementations that then modify the incoming signal structure may sometimes worsen these biases by effectively increasing the differences between each signal.

Timing biases, as discussed in [2] between different codes centered on the same frequency may result from the use of different receiver types and delay-lock loop (DLL) configurations. These differences are known to result from (filtered) correlation peak asymmetry; however, this asymmetry is still presumed to be constant from PRN-to-PRN within any given receiver. This implies that within a given receiver, by incorporating a sufficient number of signals such timing biases may be estimated and removed by the addition of a state to the navigation equation.

Interchannel biases are presumed to be those biases introduced by the receiver electronics alone, but some have observed a correlation between observed inter-channel biases with receiver filtering and PRN codes in 2001 [3]. Still, few currently analyze the potential for such variation in modern receiver implementations. This is likely because common analog filter prototype topologies typically used in such analyses, such as Butterworth, Chebychev, Elliptical, etc. are perfectly symmetric and have smooth, magnitude and group delay responses [4].

A 6th-order Butterworth filter was used in [2]. Similar topologies have been used in signal deformation analyses for WAAS and LAAS. [5] These kinds of filters generally cause larger correlation peak distortion so they tend to be conservative for the types of analyses where the errors of interest depend most on the correlator spacing differences along a single correlation peak.

SAW filters

Many, perhaps most, modern GNSS receivers use Surface Acoustic Wave (SAW) type pre-correlation filters. They provide a wealth of advantages for communications applications including small size, stability, high phase linearity [6]. Because of the latter in particular, they tend to cause less correlation peak distortion in GNSS receivers. They are frequently modeled as Finite Impulse Response (FIR) filters, which have perfectly linear phase, or constant group delay [7]. The group delay responses of actual SAW filters, however, have important characteristics that are not as easily modeled.

The group delay (and magnitude) responses of actual SAW filters are not perfectly smooth. They have ripples that vary as a function of frequency. This is a result of the so-called triple-transit response, reflection of the desired signal occurs to varying degrees in all SAW filters. It is discussed in [8] and [9]. Across the filter passband, these tend to average out; so the mean group delay is nearly constant. However, the group delay ripples (or, alternatively, the phase ripple) are usually somewhat irregular functions of frequency. (See Figure 1.)

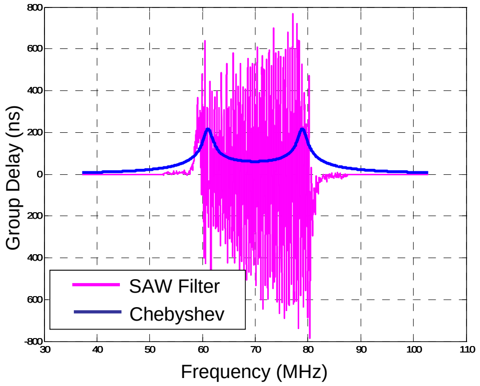

Figure 1. Comparison of the (20MHz) SAW filter group delay response of an actual GNSS precorrelation filter to an analytical (20MHz) Chebychev group delay response.

The SAW filter triple-transit responses and their potential effects on GNSS ranging are discussed in detail in [9]. There, the author also provides model equations for the response. The paper stops short of analyzing the effects of actual SAW filters—with their irregular group delay responses—on the correlation functions different PRNs, and (probably as a result) concludes that the effects are generally negligible.

This more thorough analysis is important, however. Precise positioning requires ranging with different PRN codes. And future GNSS receivers target ever more accurate and more reliable position solutions. Group delay ripples in SAW filters can potentially lead to position errors by introducing irregular distortions onto each code.

BOC(*n*,*m*) Discriminator Implementations

Generally speaking, a traditional "Early-Minus-Late" (EML) delay-lock loop (DLL) is the de-facto standard method of estimating the time of arrival of GNSS signals in receivers. However, the desire for increased multipath performance has led to the creation of many others. (The double-delta technique, for example, is one such commonly-accepted technique for tracking GPS signals [5].) For new signal structures like BOC(*n*,*m*) codes (where n>0), the increased potential for tracking correlation peak sidelobes has led to the creation of even more innovative code-tracking techniques.

Many of these newer sidelobe cancellation methods (SCMs) propose to modify the correlation peak shape in various ways. They have been analyzed for their robustess to false-peak tracking, multipath, and thermal noise. [10] However, to date none have been analyzed for their sensitivity to PRN differences. Implicitly, they presume the filters can introduce no such variation. However, SAW filters, in combination with these techniques may lead to unexpected range biases.

Calibration of these biases is limited by filter tolerances and drift. Even Doppler uncertainty can make it difficult to calibrate a receiver to mitigate them for some applications. This is particularly true if the accuracy required is on the order of only centimeters (or less). Dynamic estimation may be limited by the redundancy that can be leveraged, as each PRN will have its own bias. Alternatively, it may be impractical to attempt estimation of numerous additional states over long periods of time. Careful design and selection of the tracking technique may therefore be critical for high-accuracy and high integrity applications.

The remaining sections of this paper analyze several SCMs applied to the Galileo BOC(1,1) signals filtered using typical SAW filters for their sensitivity to PRN-to-PRN biases.

ANALYSIS

Code correlation models

The analysis approach is nearly identical to the one discussed in [4]. In short, the autocorrelation functions of each signal were modeled according to

$$R(\tau) = R_{USER}(\tau) = \int_{-\infty}^{\infty} H_{USER}(f)H(f)C(f)C_R^*(f)e^{j2\pi f}df \qquad (1)$$

where $H_{USER}(f)$ is the user receiver filter transfer function, $H(f)$ is the transfer function of the satellite signal filter. $C(f)$ and $C_R^*(f)$ represent the power spectra of the of the incoming code and the replica, respectively. ,

In this paper, the actual E1b and E1c memory codes obtained from the Galileo OS SIS ICD (Draft 0) were used for the Galileo codes. GPS PRNs 1 through 32 were analyzed as well. The satellite signal filter transfer function, $H(f)$, was assumed to be a single ideal, rectangular filter with a 40MHz bandwidth centered at L1.

Tracking error models

Four proposed tracking techniques were modeled. The first is the "bump and jump" discussed in [10] and [11]. In steady-state, while the DLL is centered on the main peak, this "bump-jumping" technique is equivalent to a traditional EML. It does not attempt to modify the correlation peak and instead uses additional correlators to simply determine whether the loop is centered on the main peak or not.

The three others selected for comparison are methods proposed by Ward, Julien, et al (ASPeCT), and Schmid, et al (Differential Correlation (DC)). For the remainder of this paper these will be referred to as the Quadra-BOC (Q-BOC), Autocorrelation Side-peak Cancellation Technique (ASPeCT), and Direct Correlation (DC) methods, respectively. A detailed description of each of these methods is beyond the scope of this paper; however they are each described in detail in [12], [13], [10], and [14].

Figure 2 plots a comparison of the unfiltered correlation peaks for each of these methods.

(Also plotted for reference are the peaks for an L1 C/A code PRN and an infinite bandwidth L5 code.)

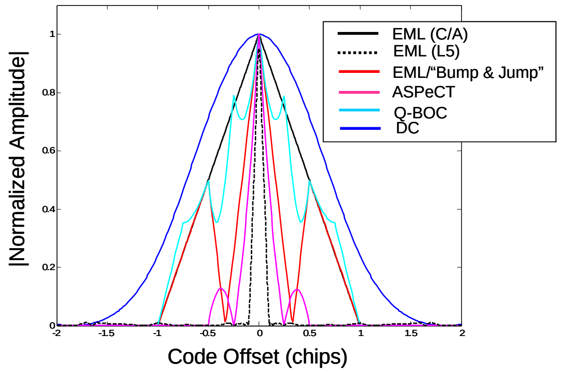

Figure 2. Comparison of ideal, infinite bandwidth correlation peaks subjected to traditional EML discriminators and three sidelobe cancellation methods (SCM).

Assuming coherent tracking and negligible phase error, the steady-state tracking error for a standard dot-product early-minus-late discriminator about the equilibrium point is given by

$$\tau = \arg\left\{ R_{d(\bullet)}(\tau)\left[R_{d(\bullet)}\left(\tau - \frac{d}{2}\right) - R_{d(\bullet)}\left(\tau + \frac{d}{2}\right)\right] = 0\right\} \qquad (2)$$

The above equation is also assumed for the bump-jumping DLL.

For the Q-BOC SCM, the tracking loop equation is

$$\tau = \arg\left\{ \frac{1}{2}\left[\frac{R_{d(\bullet)}\left(\tau - \frac{d}{2}\right) - R_{d(\bullet)}\left(\tau + \frac{d}{2}\right)}{R_{d(\bullet)}\left(\tau - \frac{d}{2}\right) + R_{d(\bullet)}\left(\tau + \frac{d}{2}\right)} \right] = 0 \right\} \qquad (3)$$

The equation for ASPeCT tracking errors is

$$\tau = \arg\left\{ \frac{R(\tau)\left(R\left(\tau - \frac{d}{2}\right) - R\left(\tau + \frac{d}{2}\right)\right) - R_{B/P}(\tau)\left(R_{B/P}\left(\tau - \frac{d}{2}\right) - R_{B/P}\left(\tau + \frac{d}{2}\right)\right)}{(6+d)R^2(\tau)} = 0 \right\} \qquad (4)$$

where $R_{B/P}(\tau)$ is the correlation of the BOC(1,1) code with the PRN code, not modulated by the associated square wave.

The Differential Correlation SCM discriminator is defined by

$$\tau = \arg\left\{ R_{DC}(\tau)\left(R_{DC}\left(\tau - \frac{d}{2}\right) - R_{DC}\left(\tau + \frac{d}{2}\right)\right) = 0\right\} \qquad (5)$$

where

$$R_{DC}(\tau)\left|\sum_{\mu=1}^{M} R_{\mu}(\tau) R_{\mu}^{*}(\tau)\right|^{2} \qquad (6)$$

and M is the total number of samples μ in a single correlation function period of $R_{\mu}(\tau)$.

Example: GIOVE A codes

Using the GIOVE A code generators derived in [15], the 20MHz bandwidth SAW filter of Figure 1 and procedure discussed in [4], 50 GIOVE-A, filtered correlation functions were computed. Figure 3 compares the relative tracking errors for each of the four proposed BOC(1,1) tracking methods described. The GPS C/A code (BOC(0,1)) PRN sensitivity is shown the figure for comparison.

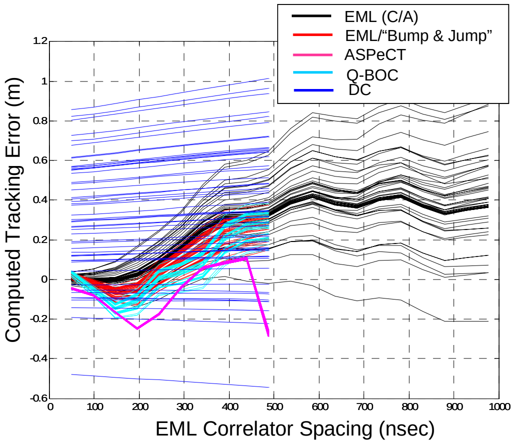

Figure 3. Relative tracking errors induced by EML, ASPecT, Q-BOC, and DC discriminators. (EML tracking for the C/A code PRNs is also plotted for comparison.)

The Differential Correlation SCM code clearly results in the largest sensitivity to the GIOVE A PRN differences. The bump jumping and Q-BOC techniques display significantly smaller variations that are comparable to each other. The ASPeCT SCM results in the smallest sensitivity to PRN differences as the traces corresponding to each PRN are nearly coincident.

Peak asymmetry, measured as the maximum variation of a given PRN trace over all correlator spacings, is of less importance in this paper, but this too can be observed from the figure. All but one of the methods create similar degrees of peak asymmetry to the Q-BOC and conventional dot-product EML tracking (including "bump jumping") methods. The tracking error curves for the DC method remain relatively constant as a function of correlator spacing.

RESULTS

The E1b and E1c memory codes each have 50 possible PRNs. (Since E1c is modulated with a secondary code, making it effectively 25 times longer than E1b, it can be determined whether code length has an effect on PRN sensitivity.) After all 100 correlation functions were computed, they were filtered using each of three 20MHz SAW filters used in modern GNSS receivers with the following approximate group delay response characteristics:

- Test Filter 1 had an approximate maximum group delay ripple amplitude of 600ns and a ripple frequency of 2 cycles per MHz. (This is the same one used in the GIOVE-A code example and in [4])
- Test Filter 2 had an approximate maximum group delay ripple amplitude of 15ns and a ripple frequency of 2.5 cycles per MHz.
- Test Filter 3 had an approximate maximum group delay ripple amplitude of 25ns (peak-to-peak) and a ripple frequency of 1.5 cycles per MHz.

Each of the four proposed tracking methods were then implemented on all filtered Elb and E1c PRN correlation functions and the relative tracking errors were computed.

The maximum PRN variation for any given correlator spacing, *d* and PRN *j* is given by the following equation

$$\text{Maximum PRN-to-PRN Variation} = \max_{j}\left\{\tau_d^j\right\} - \min_{j}\left\{\tau_d^j\right\} \tag{7}$$

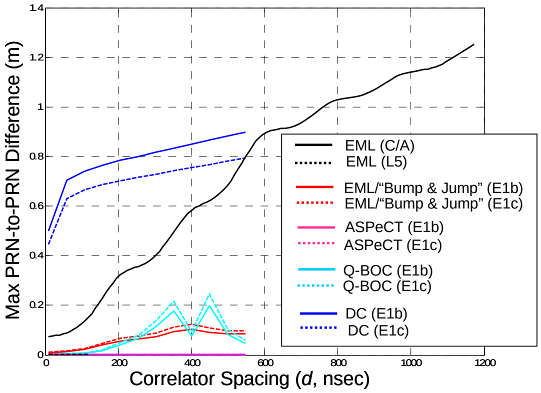

Figure 4. Maximum PRN variations for SAW filter Case 1 (20MHz, 600ns ripple amplitude at 2 cycles/MHz)

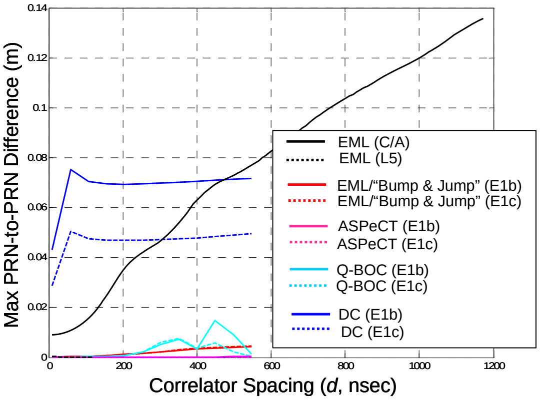

Figure 5. Maximum PRN variations for SAW filter Case 2 (20MHz, 15ns ripple amplitude at 2.5 cycles/MHz)

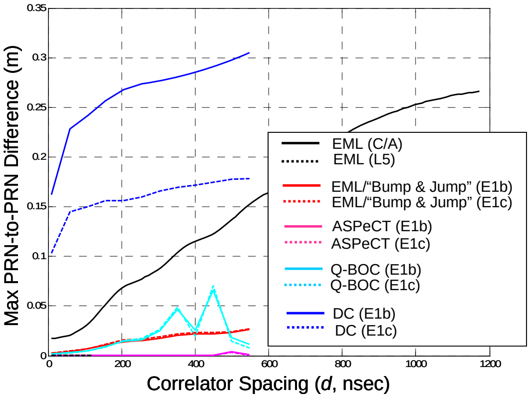

Figure 5. Maximum PRN variations for SAW filter Case 3 (20MHz, 25ns ripple amplitude at 1.5 cycles/MHz)

The above equation is plotted as a function of correlator spacing for each of the three test filters in Figures 4 through 6. For comparison, the results for 32 C/A code and 32 L5 (I) codes are plotted as well for the traditional dot-product EML discriminator.

Again, the Differential Correlation SCM code results in the largest biases from PRN-to PRN. Also as with the GIOVE A codes, the bump jumping and Q-BOC techniques result in smaller, more-comparable variations. In all cases the ASPeCT SCM, at the narrowest correlator spacings, results in the smallest sensitivity to PRN differences of all the codes and implementations shown. The errors are frequently sub-millimeter. This is true even in case 1, where the SAW filter group delay ripples are largest.

The likely reason the ASPeCT method is so robust against these biases can be see from Equation 4. In that equation, a related PRN correlation function is essentially subtracted from itself. This cancels many differences that would normally exist between the functions of different PRNs. In short, it makes the signals appear to be more identical so the filters—even the irregular SAW filters—have essentially the same effect on each of them.

It should be noted that, in addition to PRN variations, satellite hardware imperfections (e.g., filter variations and code modulator tolerances from satellite-to-satellite) also contribute to range error. They are also affected by DLL implantation; however, these error sources have not been analyzed in this paper. It is unlikely these can be mitigated by any of the tracking techniques discussed herein.

CONCLUSIONS

From the analysis and results presented in this paper, the following may be concluded:

- In modern GNSS receivers, SAW filters can induce distortion on each received signal that is a function of the PRN codes.
- SAW filters with larger group delay ripples tend to yield larger PRN variations, regardless of DLL implementation.
- The longer E1c codes do not in general do not display more robustness against PRN variations than E1b.
- The Differential Correlation method is significantly more sensitive to PRN variations than the other techniques.
- The ASPeCT correlation method seems to be more resistant to PRN variations by design.
- None of the proposed tracking methods are effective against nominal signal deformations that may result from differences or imperfections in satellite hardware. (These effects have not been modelled here.)

It is recommended that future proposed tracking loop implementations include similar analyses prior to selection for use in high-accuracy and high-integrity applications.

ACKNOWLEDGEMENTS

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