Concatenated QC-LDPC and SPC Codes for 100 Gbps Ultra Long-Haul Optical Transmission Systems

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Abstract: We propose and investigate a novel FEC coding scheme based on a concatenation of QC-LDPC with single-parity-check codes. High performance of a Q-limit of 5.8 dB with 20.5% overhead has been achieved with FPGA-based simulations.

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1. Introduction

Improving system margin with forward error correction (FEC) has become increasingly important for ultra long-haul (ULH) DWDM transmission system in order to meet continuous demand for higher bit-rates over single wavelength. In current systems, FEC with net coding gain (NCG) in the range of 6.0 dB to 8.5 dB with 7 % overhead (OH) is commonly used [1]. For 100-Gbps long-haul transmission system, as recently discussed in OIF [2], digital coherent receiver with DP-QPSK modulation is regarded as one of the promising technologies. Digital coherent receiver inherently employs high speed ADC, which will enable use of *soft-decision decoding* techniques for FEC. FEC coding with soft-decision decoding offers potential of much higher NCG than that of current systems.

Several soft-decision FEC (SD-FEC) coding schemes have been recently presented [3–6] for 100-Gbps coherent transmission systems, where high-coding-rate low-density parity-check (LDPC) codes [7] of short-to-moderate lengths have been employed. It is known [9] that carefully designed longer LDPC codes can provide better error performance very close to the Shannon limit. However, they generally increase encoder/decoder complexity and will not be suitable for high-speed transmissions. In general, error performance and implementation simplicity exist in a trade-off relationship.

In this paper, we propose a novel class of long quasi-cyclic (QC) LDPC codes to overcome this trade off. Each code in this class is formed by concatenating single-parity check (SPC) codes and QC-LDPC codes of shorter lengths, and we refer to the codes as concatenated QC-LDPC codes. Further, we propose a code for OTU-4 signals. The 7 % FEC OH in the standard OTU-4 frame is replaced with 20.5 % OH, composed of a concatenated QC-LDPC code as inner code and a Reed-Solomon (RS) code as outer code. The FEC OH due to the outer RS code is 2.1 %, and it plays a role in cleaning up the residual errors after decoding of the inner code. FPGA simulated bit-error-rate (BER) performance shows that our coding scheme can achieve a Q-limit of 5.8 dB and an NCG of 10.4 dB at a BER of 10^{-12}. At a BER of 10^{-15}, the NCG will be 11.3 dB, which is sufficiently high compared with previously reported LDPC codes for 100-Gbps coherent transmission systems and approximately 5 dB better than that of the standard RS $(255, 239)$ code employed in OTU-4.

2. Concatenation of QC-LDPC Codes with Single-Parity Check Codes

We introduce long high-rate QC-LDPC codes which can be viewed as a concatenation of QC-LDPC codes and SPC codes of shorter lengths. We first briefly introduce the structure of the component QC-LDPC codes, followed by that of the overall codes, and then present an example which will be suitable to apply to OTU-4.

We denote by $\boldsymbol{P}$ an $m \times m$ circulant matrix whose first row is $(0, 1, 0, \cdots, 0)$. Then, for $0 \leqslant i < m$, $\boldsymbol{P}^i$ is a circulant matrix whose first row has only one non-zero entry, equal to 1, at the i-th position. We denote by $\boldsymbol{H}$ an $r \times n$ block matrix of the form $\boldsymbol{H} = (\boldsymbol{P}^{\mu(i,j)})_{0 \leqslant i < r,\, 0 \leqslant j < n}$, where $\mu(i, j) \in \{0, 1, \cdots, m-1\} \cup \{-\infty\}$ and $\boldsymbol{P}^{-\infty} \triangleq \mathbf{0}$ by convention. A binary code C with parity-check matrix $\boldsymbol{H}$ has length mn and rate at least $1 - (r/n)$. In order for C to perform well with such iterative decoding algorithms as sum-product and min-sum algorithms (cf. [8, 9]), we assume that each $\mu(i, j)$ is set so that the Tanner graph of $\boldsymbol{H}$ is free of four-cycles [10, 11]. Then C is referred to as a QC-LDPC code. Several methods for choosing $\mu(i, j)$'s to generate good QC-LDPC codes have been recently presented. These methods can be grouped under two main classes: random or pseudo-random methods [11], and algebraic methods [3, 4, 10–13]. QC-LDPC codes constructed using algebraic methods will be amenable to low-complexity implementation. For high coding rates and moderate code lengths, it can also outperform randomly constructed codes in terms of error-performance. Thus, we use algebraic QC-LDPC codes as component codes. Next we propose codes whose parity-check matrix can be written as follows:

$$\boldsymbol{H}^{(l)} \triangleq \begin{pmatrix} \boldsymbol{H} & & & \\ & \boldsymbol{H} & & \\ & & \ddots & \\ & & & \boldsymbol{H} \\ \boldsymbol{T}_{l-1} & \boldsymbol{T}_{l-2} & \cdots & \boldsymbol{T}_0 \end{pmatrix}, \quad (1)$$

where l is an integer in the range $1 < l \leqslant n$ and where, for each $0 \leqslant k < l$, $\boldsymbol{T}_k$ is an $n \times n$ block matrix of the form $\boldsymbol{T}_k = (\boldsymbol{P}^{\gamma_k(i,j)})_{0 \leqslant i,j < n}$ that satisfies $\gamma_k(i,j) = -\infty$ if $j - i \not\equiv k \bmod n$. The matrix $\boldsymbol{T}_k$ is an $mn \times mn$ sub-permutation matrix, i.e., there is at most a single non-zero entry, equal to 1, in each row and in each column of $\boldsymbol{T}_k$. We denote by $C^{(l)}$ the null space of $\boldsymbol{H}^{(l)}$ over GF(2). Clearly, the Tanner graph of $\boldsymbol{H}^{(l)}$ is free of four-cycles, and $C^{(l)}$ is a binary QC-LDPC code of length lmn and rate at least $1 - (r/n) - (1/l)$. We refer to $C^{(l)}$ as a concatenated QC-LDPC code.

We note that by (1), a codeword $\boldsymbol{c}$ of $C^{(l)}$ is an l-tuple of codewords of C and that a tuple of SPC codewords can be obtained from $\boldsymbol{c}$ by permuting its bit-order in accordance with sub-permutation matrices $\boldsymbol{T}_0, \boldsymbol{T}_1, \cdots, \boldsymbol{T}_{l-1}$. Thus $C^{(l)}$ can be regarded as a concatenation of C and SPC codes. We also note that no two $n \times n$ block matrices $\boldsymbol{T}_k, \boldsymbol{T}_h$, $k \neq h$, can have a position where they both have non-zero blocks. These properties of the matrix $\boldsymbol{H}^{(l)}$ can be used to facilitate implementation of an efficient decoder. It can be shown that a layered decoder architecture (cf. [14]) for $C^{(l)}$ can be directly derived from those for its component QC-LDPC code C simply by expanding memory capacity.

Example ([12, 13]) Here we show an example of concatenated QC-LDPC codes, which will be used in the next section for constructing an FEC scheme for OTU-4. Let $m = 63$, $r = 5$, $n = 36$, $l = 33$, and let $\boldsymbol{H}$ be a parity-check matrix of a $(2268, 1986)$ QC-LDPC code C. Note that $\boldsymbol{H}$ contains 33 redundant row vectors and that such a matrix can be obtained by using the algebraic method presented in [12, 13]. For each $0 \leqslant k < 33$, we set $\gamma_k(i,j) = -\infty$ if $j - i \not\equiv k \bmod 36$ or $i \geqslant 31$; otherwise, we set $\gamma_k(i,j)$ to an integer chosen from the set $\{0, 1, \cdots, 62\}$. Then, the concatenated QC-LDPC code $C^{(33)}$ has length 74844 bits. A codeword of $C^{(33)}$ can be thought of as a 33-tuple of codewords of C, and each of the first 32 codewords contains 1986 information bits. The total of $63552 (= 32 \times 1986)$ information bits can be easily encoded by using the systematic encoding algorithm [13] for the component code C. Simulation results show that the code with a practical min-sum decoder performs within 1.0 dB from the Shannon limit at a BER of 10^{-10} (see Fig. 2).

3. FEC Coding Scheme for OTU-4

Fig. 1 illustrates an FEC structure applied to OTU-4. The original $7\ \%$ FEC OH is replaced with $20.5\ \%$ OH, i.e., the overall coding rate is about 0.83.

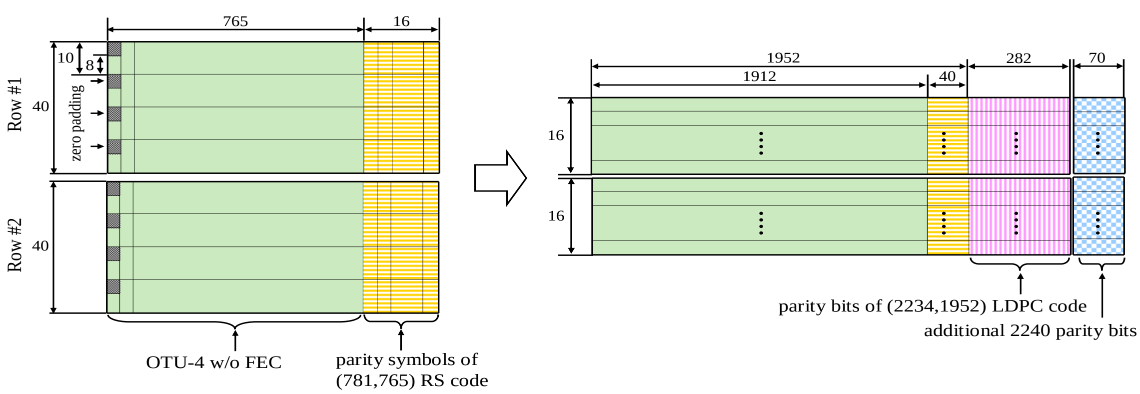

Fig. 1: Proposed FEC structure applied to OTU-4

The OTU payload can be naturally segmented into four data sequences, Row$\#1, \cdots,$ Row$\#4$, each of which consists of 3824 bytes [5]. Each Row is further divided into four sequences, and they are then encoded with four RS $(781, 765)$

encoders over GF(2^{10}). We note here that since 3824 bytes $= 4 \times 7648$ bits, two zeros are prepended before being fed into each encoder. The total number of parity bits appended to each Row is 4×160 bits (see left side of Fig. 1). Each of these RS encoded Row data is then divided into sixteen sequences, and they are encoded with sixteen QC-LDPC $(2234, 1952)$ encoders. Here, the $(2234, 1952)$ code is a shortened code of the $(2268, 1986)$ code described in the previous section. As illustrated in right-hand side of Fig. 1, additional 2240 parity bits are appended to two consecutive Rows. These 2240 parity bits are computed from the 32-tuple of codewords corresponding to the two Rows. These parity bits together with the 32 codewords form a codeword of a concatenated QC-LDPC $(73728, 62464)$ code which is obtained by shortening and puncturing the $(74844, 63552)$ code $C^{(33)}$ explained in the previous section.

4. Simulation Results

Fig. 2 shows FPGA simulated BER performance over an AWGN channel of the coding scheme applied to OTU-4 described in the previous section. For the decoding of inner concatenated QC-LDPC code, we adopted a layered min-sum algorithm. Here, the input log-likelihood ratios (LLRs) from the channel is quantized to 4 bits, and the maximum number of iterations, $I_{\max}$, is set to 5, 10, and 15. Results of 5 and 15 iterations differ by only about 0.2 dB, showing that the decoding of this code with a layered min-sum algorithm converges very fast. It can also be observed that the outer RS $(781, 765)$ code works effectively to correct residual errors after iterative decoding of the $(73728, 62464)$ code eliminating error-floor below BER of 10^{-9}. Our coding scheme can achieve a Q-limit of 5.8 dB and thus an NCG of 10.4 dB at a BER of 10^{-12}, where the number of quantization levels of the LLRs is set to 4 bits and $I_{\max} = 15$. At a BER of 10^{-15} the NCG will be 11.3 dB, which outperform conventional RS $(255, 239)$ code by about 5 dB. When the number of quantization levels is reduced to 3 bits (resp., to 2 bits), it incurs approximately 0.1 dB (resp., 0.4 dB) degradation in the coding gain.

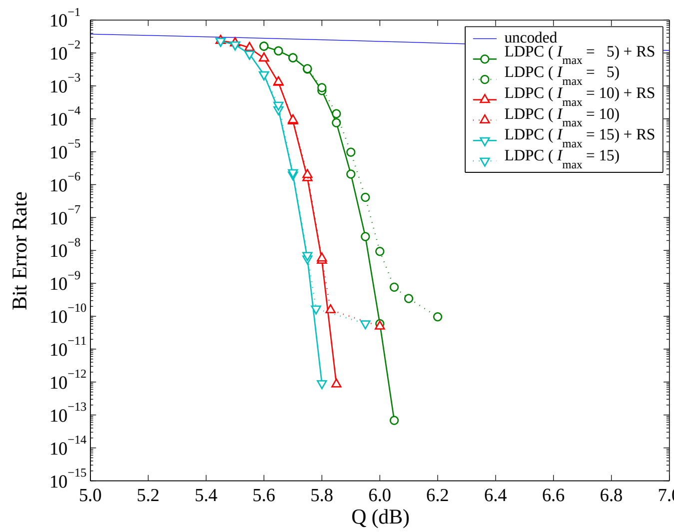

Fig. 2: Simulation Results

5. Conclusions

We have proposed a concatenated QC-LDPC code for OTU-4 replacing the 7 % FEC OH and investigated its performance. Q-limit of 5.8 dB and an NCG of 10.4 dB at a BER of 10^{-12} was achieved with 20.5 % OH. At a BER of 10^{-15}, the NCG is 11.3 dB. This is approximately 5 dB better than that of standard RS $(255, 239)$ 7 % OH FEC. Moreover, this proposed LDPC code, since it is highly structured, is expected to inherently enable efficient high-speed implementation. Our approach thus provides a promising solution for 100-Gbps systems and beyond.

References

[1] ITU-T G. 975. 1, *Forward Error Correction for high bit-rate DWDM Submarine System*, Feb. 2004.
[2] Optical Internetworking Forum (OIF), *100 G Ultra Long Haul DWDM Framework Document*, http://www.oiforum.com.
[3] M. Arabaci, et al., Proc. OFC/NFOEC 2009, **JThA38**, San Diego, CA, March 2009.
[4] I. B. Djordjevic, et al., Proc. OFC/NFOEC 2008, **JWA53**, San Diego, CA, March 2008.
[5] Y. Miyata, et al., Proc. OFC/NFOEC 2009, **NThB2**, San Diego, CA, March 2009.
[6] T. Mizuochi and Y. Miyata, Proc. 2008 IEEE/LEOS Summer Topical Meetings, pp.217–218, Acapulco, Mexico, July 2008.
[7] R. G. Gallager, *Low Density Parity Check Codes*, Cambridge, MA:MIT Press, 1963.
[8] D. J. C. MacKay, *Information Theory, Inference, and Learning Algorithms*, Cambridge University Press, 2003.
[9] T. J. Richardson and R. Urbanke, *Modern Coding Theory*, Cambridge University Press, 2008.
[10] J. L. Fan, Proc. 2nd Int. Symp. Turbo Codes and Related Topics, Brest, France, pp.545–546, September 2000.
[11] M. P. C. Fossorier, IEEE Trans. Inform. Theory, vol.50, no.8, pp.1788–1793, April 2004.
[12] N. Kamiya, IEEE Trans. Inform. Theory, vol.53, no.4, pp.1444–1459, April 2007.
[13] N. Kamiya and E. Sasaki, IEEE J. Select. Areas Commun., vol.27, no.6, pp.846–854, August 2009.
[14] M. Mansour and N. Shanbhag, IEEE Trans. Very Large Scale Integr. (VLSI) Syst., vol.11, no.6, pp.976–996, December 2003.