Link-aware Precoding for Nonlinear Optical OFDM Transmission

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Abstract: We show that signal shaping schemes for the reduction of nonlinearities in optical OFDM systems need to take into account the signal evolution along the dispersive link. Rather than minimizing the transmit PAPR, a good precoding scheme should minimize the cumulative power variance.

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1. Introduction

Orthogonal frequency division multiplexing (OFDM) continues to receive considerable interest in optical communications, both for direct-detection [1] and coherent [2] systems. OFDM signals facilitate high spectral efficiencies and virtually unlimited compensation of linear channel effects (chromatic and polarization mode dispersion) using digital signal processing. In contrast, OFDM suffers from channel nonlinearities, which create interference between the signal's subcarriers [3]. It is well known that the distortions from Kerr nonlinearities, measured through the signal's Q factor, can be related qualitatively to the signal's peak-to-average power ratio (PAPR) [4]. The PAPR of uncoded OFDM signals is significantly higher than that of single-carrier signals. Therefore, a number of PAPR reduction schemes have been proposed for application in optical communication systems (see [5] and references therein). To measure the performance of PAPR reduction schemes, the gain between the uncoded and coded signal power's cumulative distribution function is traditionally used. This approach is sensible in wireless communications, where PAPR reduction is performed mainly to relax requirements on D/A converter and transmit amplifier. In fiber-optic communications, however, PAPR reduction aims at reducing fiber nonlinearities and therefore, the PAPR evolution along the link must be taken into account.

In this paper, we discuss the effect of waveform dispersion on the performance of PAPR reduction schemes. It is shown that the best performance is achieved by the scheme with the lowest *cumulative power variance* rather than by the one delivering the strongest PAPR reduction at the transmitter. We demonstrate this effect using Trellis shaping as a precoding method [6]. Section 2 reviews the principle of Trellis shaping with a focus on the coding metric. In Section 3, we present numerical results for a CO-OFDM 112 Gbit/s system and we give an interpretation that may help design optimum link-aware precoding schemes in the future.

2. PAPR reduction by trellis shaping

Trellis shaping is a coding method for altering signal properties e.g. to obtain a reduced average signal power or reduced PAPR [7]. The encoder and the decoder for trellis shaping are depicted in Fig. 1.

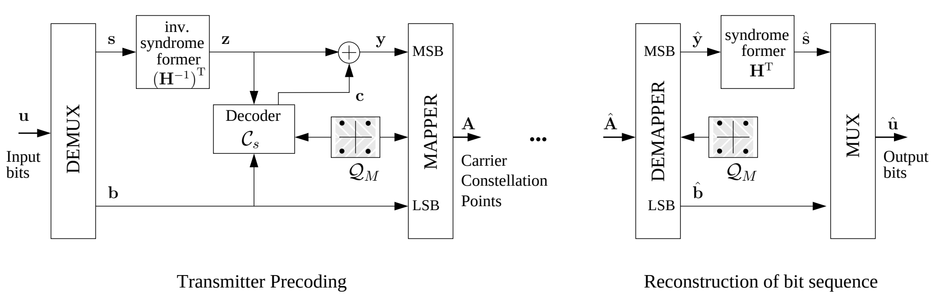

Fig. 1. Encoder (left) and decoder (right) for trellis shaping.

We assume that the prevailing OFDM system uses N_c carriers and that each carrier symbol $[A_0, \ldots, A_{N_c-1}]$ stems from a M-ary QAM-constellation $\mathcal{Q}_M = \{Q_0, \ldots, Q_{M-1}\}$. Hence, $\log_2 M$ bits are tranported in each subcarrier. The most significant bit (MSB) of each subcarrier will be used for shaping. The MSB of n_s neighbouring subcarriers form one shaping symbol, hence there are $\frac{N_c}{n_s}$ shaping symbols. The input-vector $\mathbf{s}$ of all MSBs consists of $\frac{n_c-1}{n_c}N_c$ bits, which are encoded to vector $\mathbf{z}$ by using an $(n_s-1)\times n_s$ inverse syndromeformer matrix $(\mathbf{H}^{-1})^{\mathrm{T}}$ of the convolutional shaping-code $\mathcal{C}_s$, i.e. $\mathbf{z}=\mathbf{s}(\mathbf{H}^{-1})^{\mathrm{T}}$. Consequently, the vector $\mathbf{z}$ consists of N_c bits, which can be used to select the MSB of the QAM constellation points. An arbitrary codeword taken from $\mathcal{C}_s$ can now be added to $\mathbf{z}$ to adjust signal properties. The original MSB vector $\mathbf{s}$ can be recovered in the receiver by using the syndromeformer $\mathbf{H}^{\mathrm{T}}$.

The codeword $\mathbf{c}$, which results in the lowest PAPR when added to $\mathbf{z}$ can be found using the Viterbi-Algorithm (VA) with a frequency-domain metric [7]. The derivation of this metric is based on the aperiodic autocorrelation function

$$R_m = \sum_{k=0}^{N_c-1-m} A_{k+m}A_k^*, \tag{1}$$

where A_k are the carrier symbols. For the i-th subcarrier the element z_k of the vector $\mathbf{z}$ and a $\log_2 M-1$ bit subvector $\mathbf{b}_k$ of the vector $\mathbf{b}$ are used to select the constellation-point $A_i = Q_{(z_i\oplus c_i,\mathbf{b}_i)_2}$. A reduction of the PAPR is achieved by selecting $\mathbf{c}$ such that the side-lobes of (1) are minimized. The optimization criterion can be expressed as

$$\mathbf{c} = \arg\min_{\mathbf{c}\in\mathcal{C}_s} \sum_{m=1}^{\Delta} |R_m|^2, \tag{2}$$

where $\Delta = N_c$. At the k-th trellis transition in the code trellis n_s code bits $\mathbf{c}_k = [c_{kn_s}\dots c_{(k+1)n_s-1}]$ are generated, which control the carrier symbols $\mathbf{A}_k = [A_{kn_s},\dots,A_{(k+1)n_s-1}]$. For the VA, the criterion (2) can be written as a recursive metric,

$$\mathbf{c}_k = \arg\min_{\mathbf{c}_k\in\mathcal{C}_s^k} \mu^{(k)}, \tag{3}$$

to determine the optimum shaping symbol $\mathbf{c}_k$ from the set of possible shaping symbols $\mathcal{C}_s^k$ at trellis transition k. The metric uses the sequence of carrier symbols $[A_0,\dots,A_{(k+1)n_s-1}]$ to compute $\mu^{(k)} = \sum_{m=1}^{\min\{k-1,\Delta\}} |R_m^{(k)}|^2$. The metric update is performed recursively:

$$\mu^{(k)} = \mu^{(k-1)} + \sum_{m=1}^{\min\{(k-1)n_s-1,\Delta\}} 2\mathrm{Re}\left\{R_m^{(k-1)}\left(\Phi_m^{((k-1)n_s)}\right)^*\right\} + \sum_{m=1}^{\min\{kn_s-1,\Delta\}} \left|\Phi_m^{((k-1)n_s)}\right|^2. \tag{4}$$

Recursive expressions for $R_m^{(k)}$ and $\Phi_m^{(k-1)n_s}$ are provided in [6].

The computational complexity of the scheme is dominated by the number of multiplications in the second term of (4) and can become prohibitive [7] for a large number of subcarriers. In order to reduce the complexity, the sidelobes can be truncated by using $\Delta < N_c$. The normalized truncation window is $\bar{\Delta} = \Delta/N_c$ and the complexity reduction amounts to a factor $\eta \approx (1-\bar{\Delta})^2$. The truncation of the sidelobes implies that the correlation properties between carriers spaced more than Δ apart are neglected in the metric. As a consequence, the selected codeword is not necessarily optimal with respect to PAPR reduction at the transmitter. However, an important feature of this modification is that the PAPR reduction is solely based on an interplay of subcarriers within range of the truncation window.

3. Numerical results and discussion

To demonstrate the effect of pre-coding using Trellis shaping with various window sizes Δ, we have performed numerical simulations of a coherent OFDM system with a gross data rate of 112 Gbit/s (56 Gbit/s PolMux). The system parameters are $N_c = 256$, 32 subcarriers cyclic prefix, 16-QAM, $n_s = 8$, simulation using 4-fold oversampling, ideal MZM, DAC, ADC and coherent receiver, EDFA NF $5\,\mathrm{dB}$, 20 spans of $80\,\mathrm{km}$ SSMF ($\alpha = 0.2\mathrm{dB/km}$, $D = 16\mathrm{ps/nm/km}$, $S = 0.057\mathrm{ps/nm^2/km}$, $\gamma = 1.3/\mathrm{W/km}$, no PMD).

Fig. 2 (left) depicts the mean PAPR as a function of the link distance. As expected, the lowest PAPR *at the transmitter* is achieved by the full-complexity metric, i.e. for $\bar{\Delta} = 1$ or $\Delta = N_c$. To see the mean PAPR rise as the waveforms are dispersed upon propagation along the uncompensated link is no surprise either. It is quite counter-intuitive, however, that the curves intersect at some point along the link, so that after 1600 km, their order is reversed and the lowest complexity metric ($\Delta = N_c/32$) delivers the best mean PAPR. Fig. 2 (right) depicts the normalized (i. e. divided by P^2) signal power variance. At $z = 0$, it can be seen that Trellis shaping achieves a reduction of the PAPR by decreasing the signal's variance. Since each time sample is a (weighted) sum of all subcarriers, the lowest variance (for a fixed redundancy) can be achieved by correlating all subcarriers in the OFDM spectrum jointly. However, as the signal propagates along the fiber, the decorrelation of two subcarriers scales with their spectral distance due to chromatic dispersion; with increasing distance, the joint optimization of remotely spaced subcarriers loses its effect.

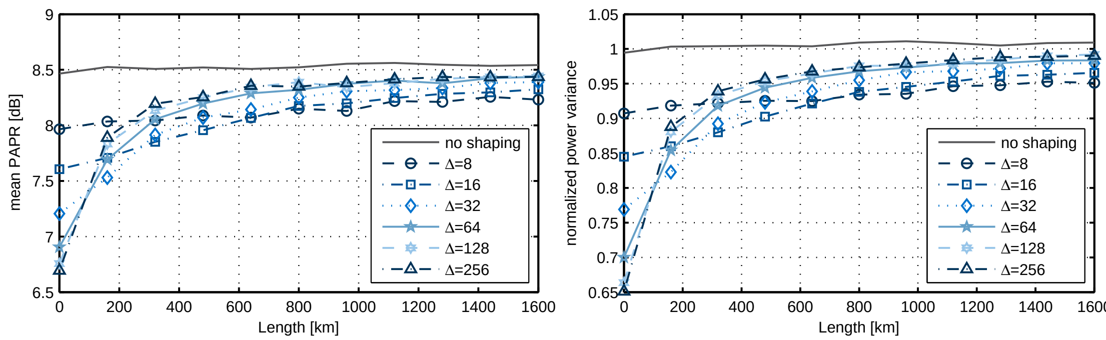

Fig. 2. Mean PAPR (left) and normalized variance of signal power (right) versus distance.

In contrast, pre-coding with low values of Δ optimizes subcarriers by taking only their closest neighbors into account. This will not deliver a globally optimum solution. However, as dispersion decorrelates all but the closest subcarriers, the "local" optimization with $\Delta < N_c$ outperforms the "global" solution using $\Delta = N_c$.

The optimum size of the truncation window Δ is determined from numerical simulations delivering an effective Q factor (calculated from the BER, which in turn is calculated from the measured OSNR). After transmission over 80 km, $\Delta = 64$ and $\Delta = 128$ already deliver a slightly higher Q factor than $\Delta = 256$ (not shown in Fig. 3 for clarity). Fig. 3 depicts the effective Q factors as a function of link length (left, $P = -4\mathrm{dBm}$ and $0\mathrm{dBm}$) and input power (right, $L = 800\mathrm{km}$). From Figs. 2 and 3, it is apparent that the nonlinear inpairments scale with the cumulated signal

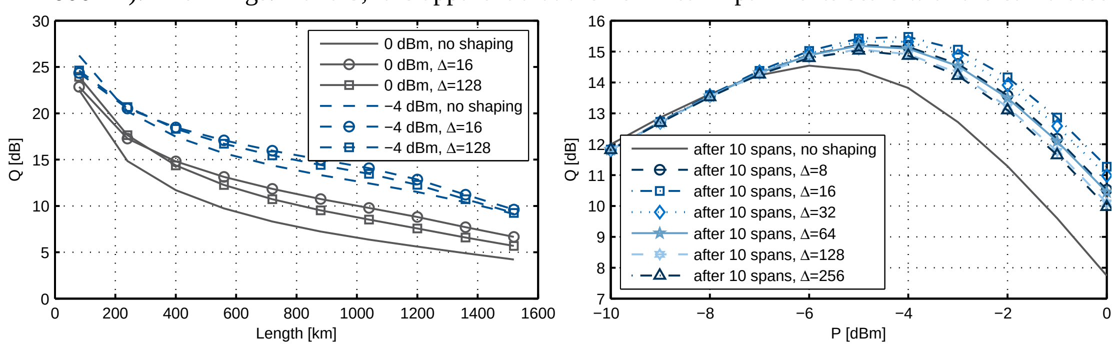

Fig. 3. Effective Q factor as a function of distance (left) and input power P (right).

power variance. After 7 spans, $\Delta = 16$ becomes the optimum window size. At this value, the pre-coding complexity is reduced by $\approx 88\%$. The gain achieved by Trellis shaping using $\Delta = 16$ over the uncoded signal amounts to $0.93\,\mathrm{dB}$; the optimum transmit power increases from $-6\,\mathrm{dBm}$ to $-4\,\mathrm{dBm}$ (cf. Fig. 3, right).

4. Summary

Pre-coding helps reducing nonlinearities in optical OFDM systems. Pre-coding schemes must not be evaluated by their performance at the transmitter, but in terms of the cumulative power variance. We found that using Trellis shaping, a truncation window size $\Delta = 16$ (corresponding to an 88% decrease in computational complexity) achieves the best performance at link lengths longer than $\approx$ 500 km.

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