Impact of Pump-Induced Nonlinear Phase Noise on Parametric Amplification and Wavelength Conversion of Phase-Modulated Signals

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Abstract *We calculate the signal SNR penalty resulting from pump-induced nonlinear phase noise in dual-pump fibre optical parametric amplifiers for various phase-shift keying formats.*

Introduction

Fibre optical parametric amplifiers (FOPAs) offer a variety of applications in future optical communication systems, e.g. wideband amplification at arbitrary centre wavelengths, regenerative properties and the possibility of wavelength conversion.[1] While FOPAs rely on ultrafast four-wave mixing (FWM), the underlying Kerr effect also mediates self- and cross-phase modulation (SPM and XPM) that can lead to signal degradation. In particular, the high power pumps can induce strong non-linear phase noise (NPN) on the amplified and the wavelength converted signals.[2,3] Here, we analyze the impact of pump-induced NPN for various phase-shift keying (PSK) formats. We derive an analytical formula for the NPN variance and use it to calculate signal-to-noise ratio (SNR) penalties for the amplified signal that also apply to the wavelength converted idler.

Theory

In a dual-pump FOPA[1], the input signal is amplified via FWM interaction with two high power pump signals in a dispersion-shifted highly non-linear fibre (DS-HNLF). At the same time, a wavelength-converted idler is generated. Although typically high power erbium-doped fibre amplifiers (EDFAs) are used to provide the pump signals in laboratory experiments, for real application they should be generated by two high power pump lasers. In this case, the unavoidable pump power fluctuations are dominated by the relative intensity noise (RIN) of the laser sources. Similar to the self-phase modulation of noisy signals leading to SPM-induced NPN, the pump power fluctuations modulate the signal phase via XPM leading to pump-induced NPN. Since the dispersion is low in the DS-HNLF, the pumps travel nearly at the same group velocity as the signal. Therefore, walk-off effects can be neglected and the probability density function (PDF) of the pump-induced NPN has a form that is similar to the PDF of SPM-induced NPN. When considering only pump-induced NPN that lies within the signal bandwidth, we can calculate the bit-error ratio (BER) using the PDF of the received phase that has been derived for phase modulated signals distorted by SPM-induced NPN.[4] We use here the Nicholson model[5] which takes into account the NPN via its variance. Although this is an approximation, it was shown to give error probabilities very close to the exact model.[5] For differentially detected signals, the PDF of the received phase is given by

$$\mathrm{p_D}(\theta) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{\mathrm{m}=1}^{\infty} \mathrm{c}_m^2 \exp\left(-\mathrm{m}^2\sigma_{\mathrm{NL}}^2\right)\cos(\mathrm{m}\theta)$$

while for synchronously detected signals it is given by

$$\mathrm{p_S}(\theta) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{\mathrm{m}=1}^{\infty} \mathrm{c}_m \exp\left(-\mathrm{m}^2\sigma_{\mathrm{NL}}^2/2\right)\cos(\mathrm{m}\theta)\,,$$

with σ_{NL}^2 the variance of the pump-induced NPN that will be derived below. The coefficients c_m are given by

$$\mathrm{c}_m = \frac{\sqrt{\pi\,\rho_s}}{2}\exp\left(-\frac{\rho_s}{2}\right)\left[\mathrm{I}_{\frac{m-1}{2}}\left(\frac{\rho_s}{2}\right) + \mathrm{I}_{\frac{m+1}{2}}\left(\frac{\rho_s}{2}\right)\right]$$

with the signal SNR ρ_s and the modified Bessel functions of first kind $\mathrm{I}_\nu(x)$. The BER is calculated using

$$\mathrm{BER} = \left[1 - \int_{-\theta_0}^{\theta_0} \mathrm{p}_{\{\mathrm{D,S}\}}(\theta)\,\mathrm{d}\theta\right]/\mathrm{m} \qquad (1)$$

with the integration limits $\pm\theta_0$ and the number of bits per symbol m taking into account Gray coding. Both depend on the modulation format: For 2-PSK and differential binary PSK (DBPSK), $\theta_0 = \pi/2,\ \mathrm{m} = 1$, for 4-PSK and differential quadrature PSK (DQPSK), $\theta_0 = \pi/4,\ \mathrm{m} = 2$, and for 8-PSK, $\theta_0 = \pi/8,\ \mathrm{m} = 3$.

For the derivation of the variance of the pump-induced NPN, we solve the coupled differential equations for non-degenerate four-wave mixing with neglected pump depletion[6] which is the normal case for linear amplification and wavelength conversion. The solutions show that the mean non-linear phase shift for the signal and the idler is $\Phi_{\mathrm{NL}} = \frac{3}{2}\gamma\,(P_1 + P_2)\,L$ with the pump powers P_1 and P_2, the fibre length L and the non-linear coefficient γ. Taking into account the pump power fluctuations it can be shown that the variance of the pump-induced NPN is given by

$$\sigma_{\mathrm{NL}}^2 = \left\langle \Delta\Phi_{\mathrm{NL}}^2 \right\rangle = 9\gamma^2 L^2 \frac{\left\langle \Delta P_p^2 \right\rangle}{2} = 9\gamma^2 L^2 \frac{P_p^2}{\mathrm{SNR}_p} \qquad (2)$$

where equal pump powers $P_1 = P_2 = P_p$ and uncorrelated pump power fluctuations with the same variance $\left\langle \Delta P_1^2 \right\rangle = \left\langle \Delta P_2^2 \right\rangle = \left\langle \Delta P_p^2 \right\rangle$ have been assumed. In Eq. (2), we have defined a pump SNR $\mathrm{SNR}_p = 2P_p^2/\left\langle \Delta P_p^2 \right\rangle$ that can be roughly related to the laser source RIN by $\mathrm{SNR}_p \approx (\mathrm{RIN} \times 10\ \mathrm{GHz})^{-1}$ assuming a constant RIN spectrum up to 10 GHz and

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a fast decrease beyond. Since RIN values down to -160 dB/Hz are reported[7], pump SNR values of 50-60 dB are used in the following. To verify Eq. (2), we conducted numerical simulations solving the full non-linear Schroedinger equation (NLSE). The DS-HNLF parameters were $L = 1$ km, dispersion slope $S = 0.02\ \mathrm{ps}^2/(\mathrm{nm\ km})$, $\gamma = 10/(\mathrm{W\ km})$, zero-dispersion wavelength $\lambda_0 = 1553\,\mathrm{nm}$. The pump SNR was set via Gaussian white noise with a 25 GHz bandwidth around the pump wavelengths $\lambda_{p1} = 1536$ nm and $\lambda_{p2} = 1568$ nm. The initially noise-free continuous-wave input signal was launched with a power $P_{sig} = -30\,\mathrm{dBm}$ at the wavelength $\lambda_{sig} = 1556\,\mathrm{nm}$. The phase standard deviations have been obtained over 32768 samples after filtering the signal and the idler at the DS-HNLF output with a 25 GHz 2nd order Gaussian bandpass filter. The results in Fig. 1 show a good agreement.

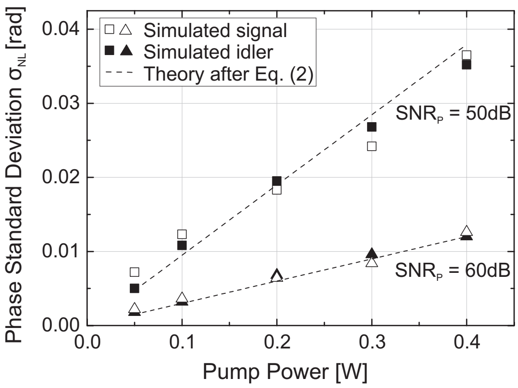

Figure 1: Comparison between the standard deviation of the signal and idler phase calculated with Eq. (2) and obtained by a numerical solution of the full NLSE.

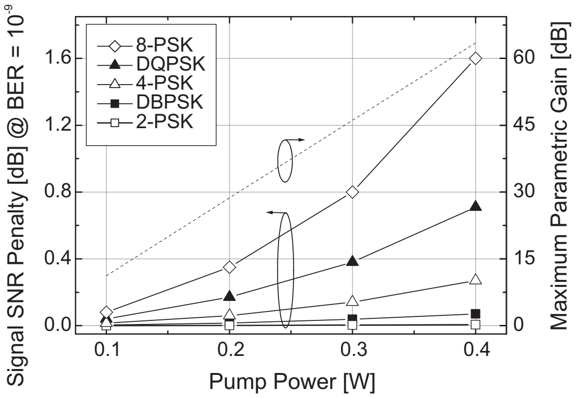

Figure 2: Left y-axis: Bitrate-independent signal SNR penalty @ $\mathrm{BER} = 10^{-9}$ for different modulation formats as a function of the pump power ($\mathrm{SNR}_p = 50$ dB, $L = 1$ km, $\gamma = 10/(\mathrm{W\ km})$). Right y-axis: Maximum parametric gain as a function of the pump power.

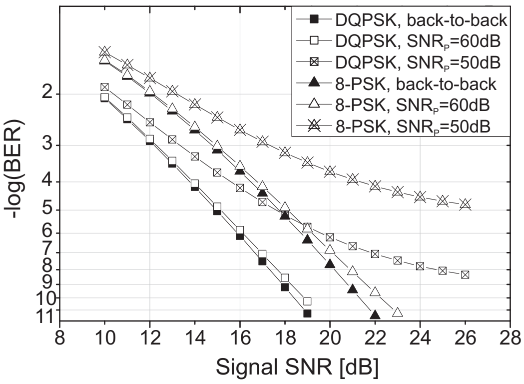

Figure 3: BER calculated with a NPN variance of 10 σ^2_{NL} resulting from 10 FOPA stages for DQPSK and 8-PSK for different pump SNR values ($P_p = 0.3$ W, $L = 1$ km, $\gamma = 10/(\mathrm{W\ km})$).

Discussion

In Fig. 2, we have plotted the bitrate-independent signal SNR penalty that occurs due to the pump-induced NPN. Note that it applies also to the idler. It is calculated with Eqs. (1) and (2) for a BER of 10^{-9}. The SNR penalty is shown for various PSK formats as a function of the pump power while the pump SNR was kept fix at 50 dB. As an orientation, the maximum parametric gain $\mathrm{G} = \sinh^2(2\gamma P_p L)$ is shown in the same figure. While the binary formats DBPSK and 2-PSK show negligible penalties, the higher order formats show measurable penalties for pump powers that are needed to generate parametric gain comparable to EDFA gain. The differentially detected modulation formats show generally higher penalties since two noisy bits are compared.

Typically, FOPAs are cascaded, e.g. when used as inline amplifiers or wavelength converters. Then, the NPN variances of each stage after Eq. (2) will add up since the noise contributions are uncorrelated. In Fig. 3, BER curves for DQPSK and 8-PSK are shown that are calculated with a NPN variance of 10 σ^2_{NL} resulting from 10 FOPA stages with $P_p = 0.3$ W. A pump SNR of 60 dB, i.e. approximately a laser RIN of -160 dB/Hz, is needed to prevent high signal SNR penalties even at a BER of 10^{-4}.

Conclusions

The results show that pump-induced non-linear phase noise is a critical issue for dual-pump FOPAs when dealing with phase-modulated signals. For 10 cascaded FOPAs with gain comparable to EDFA gain, the required pump SNR is 60 dB to avoid degradations for DQPSK and 8-PSK signals. Since the corresponding high power laser RIN of -160 dB/Hz can be realized[7], the use of FOPAs for cascaded amplification and wavelength conversion of higher order phase-modulated signals seems feasible. Note that because the pump-induced NPN is not correlated to the received power, a receiver-based compensation for synchronously detected signals[8] is not possible.

References

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