Use of the Zero Forcing Method for Compensation of Polarization Dependent Loss in Coherent Fiber-Optic Links

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Abstract *We show that a zero forcing receiver can be used to achieve nearly optimal performance in coherent polarization multiplexed systems. The advantage is in the receiver simplicity relative to the joint detection scheme for which the ultimate performance can be achieved.*

Introduction

One of the most attractive features of coherent optical communications is that manipulation of the entire optical signal (phase and amplitude) can be performed in the electronic domain. Through the use of electronic signal processing, elimination of distortions induced by unitary optical effects can be achieved, as has been shown experimentally[1]. Polarization dependent loss (PDL), on the other hand, is a non-unitary phenomenon and thus it cannot be eliminated by signal processing, even in principle. Since compensation for loss can only be achieved through gain, and since gain involves noise, a penalty from PDL is unavoidable. It is therefore a fundamental limitation in systems. The presence of PDL is particularly significant in polarization multiplexed (PMUX) systems, where in addition to the distortion of the signal to noise ratio, the orthogonality between the two launched optical signals is compromised.

In a recent publication[2], the ultimate (i.e. smallest possible) PDL induced reduction in the raw BER of coherent PMUX systems was calculated. The ultimate performance is achieved with a maximum likelihood receiver performing joint detection of signals in both polarizations. Yet, the implementation of such, joint detection receivers is quite challenging. In the case of M-ary modulation on each of the two polarizations, the overall number of constellation points is M^2 and they reside in a 4 dimensional space (2 quadratures and 2 polarizations). The optimal receiver checks the Euclidean distance between the M^2 constellation points and the received sample, in order to decide upon the symbol that is most likely to have been transmitted. With the high symbol rates in optical communications, the computational complexity at the receiver becomes prohibitive even with moderate values of M. More practical schemes for the detection of PDL distorted symbols are therefore required.

In this work we consider the performance of a simple sub-optimal scheme that cancels the loss of orthogonality between the two channels, and detects them separately, while ignoring the statistical dependence between their samples. The practical implementation of this scheme is identical in principle, to the implementation of PMD compensation in coherent links, which has been demonstrated experimentally[1]. Mathematically, it begins with the inversion of the link Jones matrix $\mathbf{T}_0(\omega)$ that represents the relation between the link input and output[3], and follows with the independent detection of the two polarization components. Unlike in the case of PMD, this scheme is sub-optimal for PDL, as it ignores the statistical dependence of the noise in the two polarizations, which is caused by the fact that $\mathbf{T}_0$ is not unitary. In the theory of wireless MIMO communications channels, the method described above is referred to as the zero forcing (ZF) method.

Our goal in this paper is to evaluate the inferiority of the ZF scheme with respect to the ultimate receiver. We show that for the relevant range of link PDL values, the sub-optimality of this scheme remains practically unnoticed. At the same time, the advantage with respect to brute force detection (i.e. one in which the presence of PDL is ignored) is shown to be very significant. The performance of the ZF method can be shown to match that of detection using a splitter with two separate polarizers[4,5], but it has the advantage of simultaneous compensation for PDL and for dispersive effects.

Analysis

A linear, PMUX link can be represented mathematically by the relation[2]

$$\underline{r} = \mathbf{T}_0(\omega)\underline{a} + \mathbf{\Lambda}^{1/2}\underline{n}\,, \qquad (1)$$

where underlines denote a 2 component column vector and where boldface and Greek letters represent matrices. Thus, the components of $\underline{a}$ are the launched signals, the components of $\underline{r}$ are the received waveforms and $\underline{n}$ represents normalized white Gaussian noise. The coherency matrix of the noise is $\Lambda = N_0 g'(1+\vec{\Gamma}'\cdot\vec{\sigma})$, where N_0 is the noise power density in the absence of PDL, g' is the polarization averaged noise enhancement, $\vec{\Gamma}'$ is the Stokes vector of the noise (its modulus is the degree of polarization) and $\vec{\sigma}$ is the vector of Pauli matrices. Since Λ is Hermitian, the matrix $\Lambda^{1/2}$ is uniquely defined. In the presence of PDL $\vec{\Gamma}' \neq 0$ and thus the components of the noise are statistically dependent. For ZF detection, one defines a new vector $\underline{r}' = \mathbf{T}_0^{-1}\underline{r} = \underline{a} + \underline{n}'$, where $\underline{n}' = \mathbf{T}_0^{-1}\Lambda^{1/2}\underline{n}$. The coherency matrix of $\underline{n}'$ is $\mathbf{T}_0^{-1}\Lambda\left(\mathbf{T}_0^{-1}\right)^{\dagger}$, and thus its components are statistically dependent. In spite of this dependence, the elements of $\underline{r}'$ are processed separately in order to recover the transmitted information. Since in all cases, the BER is dominated by minimally distant constellation points in each polarization, the Q factor of the *j*'th channel (*j*=1,2) is proportional to $|\Delta a_{\min}|^2 / N_j$, where N_j is the power spectrum associated with the *j*'th component of $\underline{n}'$. Notice that in the absence of PDL, the Q factor is proportional to $|\Delta a_{\min}|^2 / N_0$, implying that the effect of PDL is equivalent to an OSNR reduction by a factor of N_0 / N_j. We define the equivalent OSNR penalty as $\eta_{ZF} = 10\log_{10}(N_0 / \max\{N_1, N_2\})$. After significant algebraic manipulation, this quantity can be expressed explicitly, without approximation in terms of the link PDL parameters as,

$$\eta_{ZF} = \min_{j=1,2}\left\{\log_{10}\left(\frac{1-|\vec{\Gamma}|^2}{g'\left(1+\vec{\Gamma}\cdot\hat{e}_j^{(in)}\right)\left(1-\vec{\Gamma}\cdot\hat{e}_j^{(out)}\right)}\right)\right\}, \quad (2)$$

where $\vec{\Gamma}$ is the PDL vector of the entire link[2], $\hat{e}_j^{(in)}$ is the unit Stokes vector corresponding to the launch polarization of the *j*'th channel and $\hat{e}_j^{(out)}$ is the polarization of that channel at the output. Equation 2 includes no assumptions as to the magnitude of PDL in the link and it can be used as a basis for the numerical assessment of the penalty. When approximated to low PDL values, it can be shown that

$$\eta_{ZF} \approx \eta_{Opt} - \gamma\left[\left(\vec{\Gamma}'-\vec{\Gamma}\right)\times\hat{e}_1^{(in)}\right]^2, \quad (3)$$

where η_{Opt} is the equivalent OSNR penalty in the case of the optimal receiver[2], and $\gamma = 10/\ln(10)$. Equation (3) indicates that the inferiority of the ZF receiver with respect to the optimal receiver is only of second order in the PDL parameters. It is important to point out that as in the case of the optimal receiver[2], the PDL induced penalty does not depend on the modulation format.

Results

We consider a system consisting of 10 amplified spans. Each is represented by a non-unitary transfer Jones matrix that is constructed from the local PDL vectors following the conventions presented in ref. 2. The matrix $\mathbf{T}_0$ is given by the product of all those matrices. The coherency matrix of the noise is obtained from the individual matrices via the procedure described in ref. 2. Throughout our numerical procedure, the distribution of the local PDL vectors from which the matrices are constructed, is assumed to be Gaussian, consistent with previous literature. We perform a large set of Monte-Carlo simulations and asses the equivalent OSNR penalty for random fibre realizations. One million random fibre realizations were done for each value of the mean PDL. In addition to η_{ZF} we also calculate the penalty in two cases of brute-force (BF) detection, where ther is no compensation for PDL. We perform the simulations for BPSK and QPSK modulation. While in the case of ZF, there is no dependence on the format, as explained earlier. Some dependence exists in the BF case[4]. By evaluating the cumulated probabilities of the penalty in each type of receiver and modulation format, we find the tolerable amount of mean link PDL for each scenario. The tolerable PDL is defined as the PDL value for which the probability that the PDL penalty exceeds the allocated margin, is smaller than the outage probability, which is typically quoted as $\Pr_{out} = 4\times10^{-5}$. The tolerable link PDL versus allocated system margin described in Fig. 1.

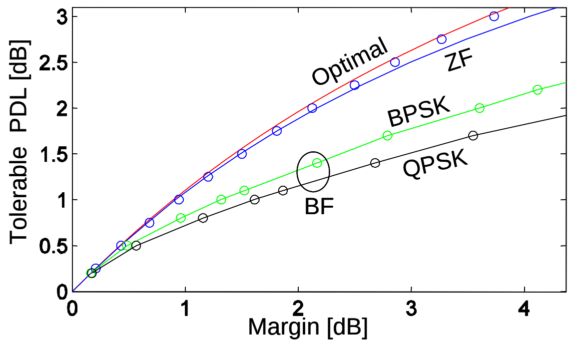

Fig. 1: Tolerable mean link PDL for given allocated system margin.

As predicted earlier, the difference between the results for ZF and for optimal detection is very small. Whereas, with the BF receiver, the tolerable amount of PDL is significantly lower. The solid curve in the optimal case shows an analytical solution taken from ref. 2. In the case of the ZF receiver the solid curve is an analytical approximation, whose derivation will be provided separately. In the BF curves the solid curves were added for illustration purposes only. Simulation results are represented by circular marks.

Conclusions

We have studied the viability of the ZF receiver to coherent PMUX systems in the presence of PDL. It has been shown analytically that the inferiority with respect to the much more complex optimal detection is only of second order in the PDL parameters. A numerical study also indicates that the difference with respect to optimal detection is negligible in the relevant range of mean PDL values. The advantage over the BF receiver that ignores compensation for PDL is considerable.

References

1 J. Renaudier et al., JLT, vol. 26, p. 36 (2008).
2 M.Shtaif, Opt. Express. Vol. 16, p. 13918 (2008)
3 S. J. Savory, Opt.Express. vol. 16 p. 804 (2008)
4 A.Andrusier and M.Shtaif, "Disjoint detection in polarization multiplexed communication," to be published, Opt. Express, 2009.
5. Z. Wang et al., paper We.3.E.2, ECOC (2008).