

文章编号: 1000-4750(2015)05-0208-05

倾斜状态下静压油垫承载性能研究

刘志峰¹, 湛承鹏¹, 赵永胜¹, 李小燕¹, 夏龙飞¹, 马建川²

(1. 北京工业大学机械工程与应用电子技术学院, 北京 100124; 2. 北京北一机床股份有限公司重型事业部, 北京 101300)

摘要: 静压转台具有摩擦系数小、承载能力强、使用寿命长等优点。但其在使用过程中受载及变形情况复杂, 而转台倾斜或自身变形时油膜厚度的不均分布与油垫承载能力之间的规律依然不明晰。该文根据微元法建立一种引入转台倾角的雷诺方程, 用倾角 θ 描述转台导轨面与支承油垫封油边的不平行程度, 从而计算出油垫各处的油膜厚度, 然后基于有限差分法求解出不同倾角时定量式支承油垫的油膜压力分布状态以及油垫的油腔压力和承载能力, 并得到油腔工作参数及性能随倾角的变化规律, 为静压转台支承分析和油膜失效判定提供基础。

关键词: 静压转台; 油膜; 雷诺方程; 倾斜; 承载性能

中图分类号: TG659 文献标志码: A doi: 10.6052/j.issn.1000-4750.2013.11.1013

RESEARCH ON THE BEARING PERFORMANCE OF HYDROSTATIC OIL PAD UNDER TILTING CONDITION

LIU Zhi-feng¹, ZHAN Cheng-peng¹, ZHAO Yong-sheng¹, LI Xiao-yan¹, XIA Long-fei¹, MA Jian-chuan²

(1. College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100124, China;

2. Beijing NO.1 Machine Tool Co., Ltd, Beijing 101300, China)

Abstract: Hydrostatic turntable has many advantages, such as low coefficient of friction, high load capacity and long life. However, due to the complexity of load and deformation, the principle is still not clearly known between uneven distribution of film thickness and oil pad carrying capacity. Based on the infinitesimal method, a modified Reynolds equation which considers the tilting angle of turntable is derived. According to this formula, the interrelation of turntable guide surface and the sealing edge is described by an inclination angle. The equation is used to analyze the bearing performance of oil pad when the turntable is tilted. Applying the finite difference method to solve the modified Reynolds equation, the pressure distribution of oil film is obtained. Finally, the working parameters of the oil pad and the changing rule of bearing performance with the tilting angle are established. The research result presented in this paper provides theoretical basis for the bearing capacity analysis and the oil film failure determination.

Key words: hydrostatic turntable; oil film; Reynolds equation; tilt; bearing performance

静压转台是有有压力的流体使有相对运动的两个表面分开并借助流体静压来承载。由于运动副之间完全被油膜隔开, 所以运动副间的摩擦力大大减小, 同时其承载能力、运动精度与寿命却大大提

高。所以其在机床、航空航天、船舶、重型机械等领域得到了广泛的应用。

国内外学者对于静压支承进行了多方面的研究, 在弹性变形的影响方面; Manring 等^[1]应用雷

收稿日期: 2013-11-03; 修改日期: 2014-05-16

基金项目: 国家科技重大专项项目(2013ZX04013-011)

通讯作者: 刘志峰(1973—), 男, 黑龙江齐齐哈尔人, 教授, 博士, 从事数字化设计与制造及机械传动的研究(E-mail: lzf@bjut.edu.cn).

作者简介: 湛承鹏(1991—), 男, 陕西安康人, 硕士生, 从事数字化设计与制造及机械制造研究(E-mail: zcpeng91@126.com);

赵永胜(1975—), 男, 辽宁海城人, 副教授, 博士, 从事机床动力学及可靠性设计研究(E-mail: yszhao@bjut.edu.cn);

李小燕(1983—), 女, 河北邯郸人, 硕士生, 从事数字化设计与制造及机械制造研究(E-mail: lixiaoyan2012@emails.bjut.edu.cn);

夏龙飞(1990—), 男, 江西鹰潭人, 硕士生, 从事数字化设计与制造的研究(E-mail: rhyxlf1990@163.com);

马建川(1970—), 男, 四川自贡人, 高工, 学士, 从事重型机床设计与制造研究(E-mail: majianchuan@byjc.com.cn).

诺方程研究了液体静压止推轴承的线性变形对于静压止推轴承油膜压力分布和的刚度的影响。Kazuyuki 和 Joichi^[2]基于雷诺方程分析了弹性变形对于楔形滑块动压润滑的影响。在多油垫支承的设计计算方面；赵建华等^[3]借鉴“弹性体”的求解方法，列出油膜的变形协调方程，将超静定问题转化成静定问题，进而求解出多油垫支承时各油腔的承载力。高殿荣等^[4]对静压导轨面内油腔数目和导轨承载能力、静刚度进行了分析，结论表明在保证单个油腔长度在额定范围内的前提下油腔数目越少越好。赵永胜等^[5]将静压油膜等效为非线性弹簧建立静压转台支承系统的有限元模型，分析了转台受均载和偏载时各油垫的受力情况。在油垫流场特性方面；王新华等^[6-7]应用 CFD 技术对圆形静压油垫和新型结构静压油垫的流场特性、承载能力和刚度进行了定量分析和计算。Dennis 和 Douglas^[8]研究了油槽形状对于可倾瓦止推轴承性能的影响。在支承系统的动静态特性方面；Zuo 等^[9]建立了新型圆锥止推轴承的数学模型并用有限元法计算和分析了其动静态特性。Chen 等^[10]使用有限元软件 对于静压主轴系统的动静态特性进行了分析。文献 [11-14]对液体静压支承的动力学特性进行了研究。此外也有学者^[15-16]对于转台的优化设计方法及粗糙度对流体润滑影响进行了研究。

对于重型静压转台其尺寸和承载都比较大，自身的变形、加工误差及偏载作用时其倾斜与变形现象都比较明显，这些因素都会使每一个支承油垫的封油边与转台导轨面不平行，也就是支承油垫油膜的厚度分布不均匀，而学者在设计或计算转台承载量时大多忽略了这一效应的影响。本文基于微元法建立极坐标下圆形油垫的雷诺方程，用倾角 θ 角描述转台导轨面与支承油垫封油边的不平行程度，从而计算出油垫各处的油膜厚度，进而与雷诺方程联立并用有限差分法求解方程；分析定量供油的油垫油膜厚度分布不均匀时油腔压力、油垫承载能力及油膜压力分布的变化。

1 静压转台倾斜模型的建立

如图 1 所示，静压转台因为自身变形或加工误差或偏心载荷的影响使得油垫封油边与转台导轨面有一个倾角 θ ，所以在柱坐标系下油垫各处的油膜厚度为：

$$h = h_0 + r \tan \theta \cos \varphi \quad (1)$$

式中： h_0 为初始油膜厚度即在转台没有任何偏转时的油膜厚度； r 和 φ 是柱坐标系下的坐标参量。图中油垫为定量供油，供油量为 Q ，油腔压力为 p_s ，若不考虑沿程压力损失则供油压力也为 p_s ，经封油边流出的油液流量为 Q_1 。

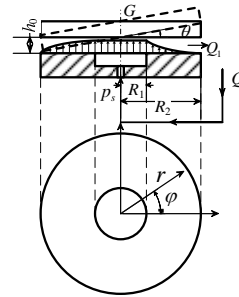


图 1 液体静压转台支撑油垫结构示意图
Fig.1 Configuration of hydrostatic oil pad

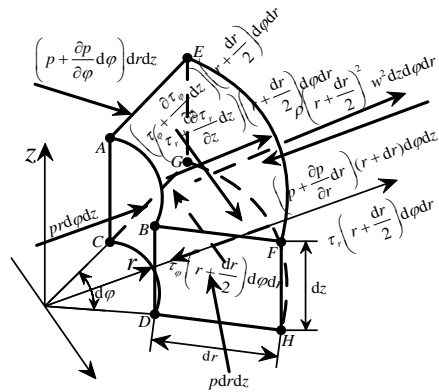


图 2 微元体上作用力的分布
Fig.2 Force distribution of infinitesimal body

在圆形油垫油膜上任取一微元，其受力如图 2 所示，微元在 r 方向受到正压力 $pr d\varphi dz$ 、

$\left(p + \frac{\partial p}{\partial r} dr\right) (r+dr) d\varphi dz$ 、离心力 $\rho \left(r + \frac{dr}{2}\right) \cdot w^2 dz d\varphi dr$ 和切应力 $\tau_r \left(r + \frac{dr}{2}\right) d\varphi dr$ 、 $\left(\tau_r + \frac{\partial \tau_r}{\partial z} dz\right) \left(r + \frac{dr}{2}\right) d\varphi dr$ 的作用在 φ 方向受到正压力 $p r dr dz$ 、 $\left(p + \frac{\partial p}{\partial \varphi} d\varphi\right) dr dz$ 和切向力 $\tau_\varphi \left(r + \frac{dr}{2}\right) d\varphi dr$ 、 $\left(\tau_\varphi + \frac{\partial \tau_\varphi}{\partial z} dz\right) \left(r + \frac{dr}{2}\right) d\varphi dr$ 的作用。 τ_φ 和 τ_r 分别指微元在 r 方向和 φ 方向的切应力，由牛顿内摩擦定律可知 $\tau_r = \eta \frac{\partial u_r}{\partial z}$ ， $\tau_\varphi = \eta \frac{\partial w}{\partial \varphi}$ 。根据微元的受力可列出微元体的力平衡方程为：

$$\left\{ \begin{array}{l} prd\varphi dz + \left(\tau_r + \frac{\partial \tau_r}{\partial z} dz \right) \left(r + \frac{dr}{2} \right) d\varphi dr + \\ \rho \left(r + \frac{dr}{2} \right)^2 w^2 dz d\varphi dr = \\ \left(p + \frac{\partial p}{\partial r} dr \right) (r + dr) d\varphi dz + \tau_r \left(r + \frac{dr}{2} \right) d\varphi dr \\ pdrdz + \left(\tau_\varphi + \frac{\partial \tau_\varphi}{\partial z} dz \right) \left(r + \frac{dr}{2} \right) d\varphi dr = \\ \left(p + \frac{\partial p}{\partial \varphi} d\varphi \right) drdz + \tau_\varphi \left(r + \frac{dr}{2} \right) d\varphi dr \end{array} \right.$$

化简并略去微小量可得到:

$$\left\{ \begin{array}{l} r \frac{\partial \tau_r}{\partial z} + \rho r^2 w^2 = \frac{\partial (pr)}{\partial r} \\ r \frac{\partial \tau_\varphi}{\partial z} = \frac{\partial p}{\partial \varphi} \end{array} \right.$$

将 $\tau_r = \eta \frac{\partial u_r}{\partial z}$, $\tau_\varphi = \eta \frac{\partial w}{\partial \varphi}$ 代入上式并认为油液不可压缩有:

$$\left\{ \begin{array}{l} r\eta \frac{\partial^2 u_r}{\partial z^2} + \rho r^2 w^2 = \frac{\partial (pr)}{\partial r} \\ r\eta \frac{\partial^2 w}{\partial z^2} = \frac{\partial p}{\partial \varphi} \end{array} \right. \quad (2)$$

由于油膜压力与 z 无关, 所以将式(2)分别对 z 进行二次积分并代入边界条件: $z=0$ 时 $w=0$ 、 $u=0$; $z=h$ 时 $w=0$ 、 $u=0$; 而后可以得到:

$$w = \frac{z^2 - zh}{2\eta r} \frac{\partial p}{\partial \varphi} \quad (3)$$

$$u_r = \frac{z^2 - zh}{2r\eta} \frac{\partial (pr)}{\partial r} - \frac{\rho r}{\eta} \left[\frac{2z^6 - 6z^5 h + 5z^4 h^2 - h^5 z}{240\eta^2 r^2} \left(\frac{\partial p}{\partial \varphi} \right)^2 \right] \quad (4)$$

由于认为油液不可压缩, 所以油膜在柱坐标系下的连续性方程为:

$$\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial (w)}{\partial \varphi} + \frac{\partial (u_z)}{\partial z} = 0 \quad (5)$$

将式(5)两边同时对 z 积分可得:

$$\int_0^h \left(\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial (w)}{\partial \varphi} + \frac{\partial (u_z)}{\partial z} \right) dz = 0$$

$$\Rightarrow \int_0^h \frac{1}{r} \frac{\partial (ru_r)}{\partial r} dz + \int_0^h \frac{1}{r} \frac{\partial (w)}{\partial \varphi} dz + \int_0^h \frac{\partial (u_z)}{\partial z} dz = 0$$

$$\Rightarrow \frac{1}{r} \frac{\partial}{\partial r} \int_0^h (ru_r) dz - u_{r,(z=h)} \frac{\partial h}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \varphi} \int_0^h (w) dz - w_{(z=h)} \frac{\partial h}{\partial \varphi} + u_{z,(z=h)} = 0 \quad (6)$$

由式(3)和式(4)有:

$$\int_0^h (ru_r) dz = \frac{-h^3}{12\eta} \frac{\partial (pr)}{\partial r} - \frac{\rho r^2}{\eta} \left[-\frac{h^7}{1120\eta^2 r^2} \left(\frac{\partial p}{\partial \varphi} \right)^2 \right] \quad (7)$$

$$u_{r,(z=h)} = 0 \quad (8)$$

$$\int_0^h (w) dz = \frac{-h^3}{12\eta r} \frac{\partial p}{\partial \varphi} \quad (9)$$

$$w_{(z=h)} = 0 \quad (10)$$

$$u_{z,(z=h)} = 0 \quad (11)$$

将式(7)~式(11)代入式(6), 则连续性方程变为:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\frac{-h^3}{12\eta} \frac{\partial (pr)}{\partial r} - \frac{\rho r^2}{\eta} \left[\frac{-h^7}{1120\eta^2 r^2} \left(\frac{\partial p}{\partial \varphi} \right)^2 \right] \right] + \frac{1}{r} \frac{\partial}{\partial \varphi} \left(\frac{-h^3}{12\eta r} \frac{\partial p}{\partial \varphi} \right) = 0 \quad (12)$$

由于油膜厚度 h 比较小, 所以我们可以略去高次 h 项, 于是得到极坐标下的雷诺方程为:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{-h^3}{12\eta} \frac{\partial (pr)}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left(\frac{-h^3}{12\eta r} \frac{\partial p}{\partial \varphi} \right) = 0 \quad (13)$$

令 $\bar{P} = \frac{p}{p_s}$ 联立雷诺方程式(13)和转台油膜厚度式(1)

有:

$$\left\{ \begin{array}{l} \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{h^3}{12\eta} \frac{\partial (r\bar{P})}{\partial r} \right) + \frac{\partial}{\partial \varphi} \left(\frac{h^3}{12\eta r^2} \frac{\partial \bar{P}}{\partial \varphi} \right) = 0 \\ h = h_0 + r \tan \theta \cos \varphi \end{array} \right. \quad (14)$$

方程的边界条件为^[17], 当 $r \in (0, R_1)$ 时 $\bar{P} = 1$, 当 $r = R_2$ 时 $\bar{P} = 0$ 。将其代入方程组式(14)中就可以解出无量纲的压力 \bar{P} , 由于转台从封油边流出的油液的流量为 $Q_1 = p_s \left(\int_0^{2\pi} \left(\frac{-h^3}{12\eta} \frac{\partial (r\bar{P})}{\partial r} \right) d\varphi \right)_{r=R_2}$ 且 $Q = Q_1$,

那么实际油腔的油压为:

$$p_s = \frac{Q_1}{\left(\int_0^{2\pi} \left(\frac{-h^3}{12\eta} \frac{\partial (r\bar{P})}{\partial r} \right) d\varphi \right)_{r=R_2}} \quad (15)$$

油垫的承载量为:

$$W = p_s \left(\pi R_1^2 + \int_0^{2\pi} \int_{R_1}^{R_2} \bar{P} r dr d\varphi \right) \quad (16)$$

2 数值仿真分析案例

如表 1 所示某静压转台支承油垫参数及油液参数。

表 1 油垫参数和油液参数取值

Table 1 The parameters of the oil and oil pad

油液密度 $\rho/(\text{kg}/\text{m}^3)$	油液粘度 $\eta/(\text{Pa} \cdot \text{s})$	封油边内径 R_1/mm	封油边外径 R_2/mm	油膜厚度 h_0/mm	定量泵流量 $Q/(\text{m}^3/\text{s})$
872	0.091	150	175	0.08	1×10^{-4}

用有限差分法求解雷诺方程式(14)和油腔压力方程式(15)要求油膜压力分布的整体误差和油腔压力的误差同时满足精度要求, 求解完成后得到油膜的压力分布和油腔压力, 再用式(16)求解油垫的承载能力, 之后改变倾角 θ 的值重复上述计算过程就可以得到不同倾角时油垫油膜压力分布、油腔压力和承载能力。具体计算流程如图 3 所示。

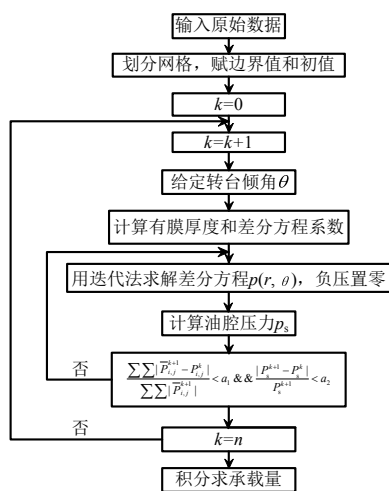


图 3 雷诺方程求解流程

Fig.3 The flow chart for the Reynolds equation solution

油垫在油膜失效前的最大倾角为 $h_0/R_2/\text{rad}$, 当转台倾角由 $0/\text{rad}$ 增加到 $0.6 \times h_0/R_2/\text{rad}$ 时油垫的承载能力和油腔压力变化如图 4 所示。

图 4 和图 5 是油垫承载能力和油腔压力随倾角的变化图, 当转台面与封油边平行时此时液阻最大封油能力最强承载能力最大, 而当转台与封油边有倾角时油垫的封油能力降低使得油腔压力降低, 进而使得油垫的承载能力有较大降低, 倾角为 $0.6 \times h_0/R_2/\text{rad}$ 时油垫的承载能力降低了 19%, 这说明倾斜对于油膜承载能力有较大影响。

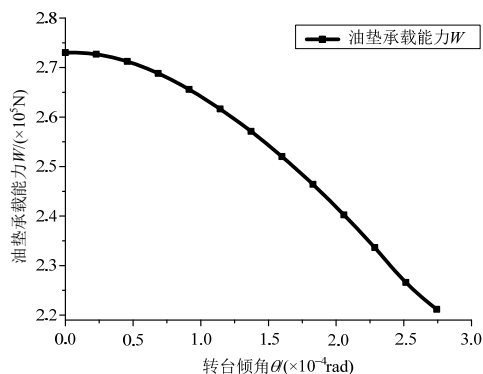


图 4 油垫承载能力 W 随转台倾角的变化曲线
Fig.4 The variation laws of oil pad load capacity

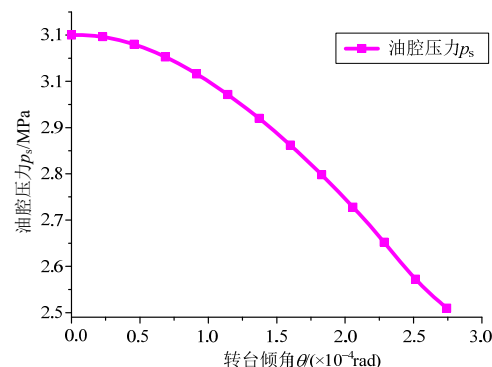


图 5 油腔压力 p_s 随转台倾角的变化曲线
Fig.5 The variation laws of oil pad cavity pressure

从图 6、图 7 可以看出当转台倾角变化时它影响最大的是油腔压力, 封油边的压力分布有变化但与油腔压力的变化相比还比较小。图 8 为转台倾角为 $0.6 \times h_0/R_2/\text{rad}$ 时封油边分别在 $\varphi = 0/\text{rad}$ 、 $\varphi = \pi/2/\text{rad}$ 和 $\varphi = \pi/\text{rad}$ 处的压力分布。从图中可以看出当转台倾斜时在转台下沉的一边油膜的压力增加, 而上升的一边油膜的压力有少量降低, 这说明转台倾斜对与油腔压力影响最大对与压力分布影响不太大。

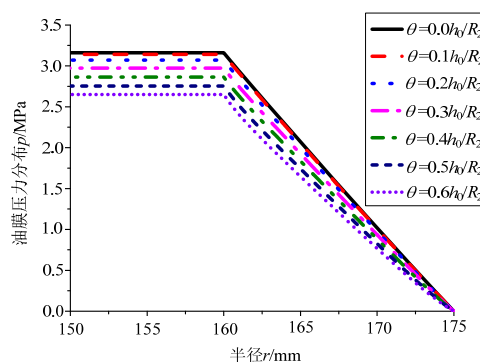


图 6 不同倾角时油膜在 $\varphi=0/\text{rad}$ 处的压力分布图
Fig.6 The pressure distribution under different tilting angles at $\varphi=0/\text{rad}$

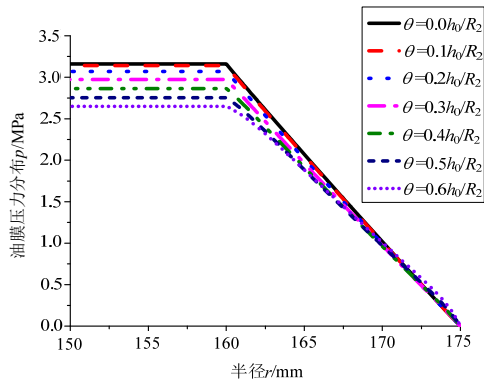


图7 不同倾角时油膜在 $\varphi = \pi/\text{rad}$ 处的压力分布图

Fig.7 The pressure distribution under different tilting angles at $\varphi = \pi/\text{rad}$

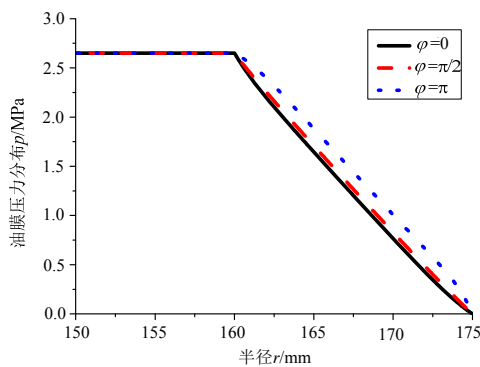


图8 倾角为 $0.6 \times h_0/R_2/\text{rad}$ 时封油边在不同位置时的压力分布图

Fig.8 The pressure distribution in different positions under the tilting angle of $0.6 \times h_0/R_2/\text{rad}$

3 结论

本文基于微元法和极坐标建立了一种倾斜状态下的雷诺方程,研究了转台导轨面与支承油垫封油边的不平行程度对油垫油膜厚度的影响关系,分析了转台上导轨面倾斜时油垫承载能力,油腔压力及油膜压力分布随倾角变化的变化规律。研究结果可知,油垫的承载能力和油腔压力随着转台倾角的增大而有较大减小约有19%,转台封油边的压力在转台倾斜下沉一侧有微量上升,转台上升一侧则有微量下降,并且压力的上升值大于下降值。由此可知,油垫的油膜厚度不均分布会降低油垫的承载能力,有必要在计算转台油垫的承载能力时考虑油膜厚度不均引起的承载能力降低的问题。

参考文献:

- [1] Manring N D, Johnson R E, Cherukuri H P. The Impact of Linear deformations on stationary hydrostatic thrust bearings [J]. Tribology, 2002, 124(4): 874—877.
- [2] Kazuyuki Y, Joichi S. Elastic deformation in thin film

hydrodynamic lubrication [J]. Tribology International, 2013, 59(1): 170—180.

- [3] 赵建华, 高殿荣, 张作超. 液体静压导轨定压供油式油腔承载力的超静定力学模型[J]. 机械工程学报, 2012, 48(22): 168—176.
Zhao Jianhua, Gao Dianrong, Zhang Zuochao. Indeterminate mechanics model of bearing capacity of constant pressure oil pockets in hydrostatic slide [J]. Journal of Mechanical Engineering, 2012, 48(22): 168—176. (in Chinese)
- [4] 高殿荣, 赵建华, 张作超. 液体静压导轨单一导轨面内油腔数目的分析[J]. 工程力学, 2013, 30(4): 423—441.
Gao Dianrong, Zhao Jianhua, Zhang Zuochao. Analysis of number of oil-pockets in one slide surface of liquid hydrostatic slide [J]. Engineering Mechanics, 2013, 30(4): 423—441. (in Chinese)
- [5] 赵永胜, 史霄, 董湘敏, 等. 重型数控机床静压转台承载能力分析[J]. 高技术通讯, 2013, 23(5): 540—545.
Zhao Yongsheng, Shi Xiao, Dong Xiangmin, et al. Load capacity analysis for the hydrostatic rotary table of heavy duty CNC machine tools [J]. Chinese High Technology Letters, 2013, 23(5): 540—545. (in Chinese)
- [6] 王新华, 郑秋月, 蔡力钢, 孙树文. 基于CFD的圆形静压油垫数值仿真技术研究[J]. 航空精密制造技术, 2010, 46(2): 17—20.
Wang Xinhua, Zheng Qiuyue, Cai Ligang, Sun Shuwen. Numerical simulation research of round hydrostatic oil pads based on CFD [J]. Aviation Precision Manufacturing Technology, 2010, 46(2): 17—20. (in Chinese)
- [7] 王新华, 郑秋月, 蔡力钢, 孙树文. 基于CFD的新型结构静压油垫数值仿真技术研究[J]. 北京工业大学学报, 2012, 38(1): 45—49.
Wang Xinhua, Zheng Qiuyue, Cai Ligang, Sun Shuwen. Numerical simulation of the hydrostatic oil pad with a new structure based on CFD [J]. Journal of Beijing University of Technology, 2012, 38(1): 45—49. (in Chinese)
- [8] Dennis V, Douglas J. An isoviscous, isothermal model investigating the influence of hydrostatic recesses on a spring-supported tilting pad thrust bearing [J]. Tribology International, 2012, 51(1): 25—35.
- [9] Zuo Xiaobo, Wang Jianmin, Yin Ziqiang, Li Shengyi. Performance analysis of multirecess angled-surface slot-compensated conical hydrostatic bearing [J]. Journal of Tribology, 2013, 135(4): 1701—1711.
- [10] Chen Dongju, Fana Jinwei. Dynamic and static characteristics of a hydrostatic spindle for machine tools [J]. Manufacturing Systems, 2012, 31(1): 26—33.
- [11] 赵建华, 高殿荣. 基于流量控制阀的液体静压导轨动态特性分析[J]. 中国机械工程, 2013, 24(4): 444—451.
Zhao Jianhua, Gao Dianrong. Dynamic characteristics analysis of liquid hydrostatic slide based on flow control valve [J]. China Mechanical Engineering, 2013, 24(4): 444—451. (in Chinese)

(参考文献[12]—[17]转第220页)

- 差的齿轮振动分析[J]. 机械传动, 2002, 26(1): 5—8.
Wang Yuxin, Yang Liu, Wang Yiming. Considering meshing time-varying stiffness and transmission error of gear vibration analysis [J]. Mechanical Transmission, 2002, 26(1): 5—8. (in Chinese)
- [9] 唐进元, 陈思雨, 钟掘. 一种改进的齿轮非线性动力学模型[J]. 工程力学, 2008, 25(1): 217—223.
Tang Jinyuan, Chen Siyu, Zhong Jue. A improved nonlinear model for a spur gear pair system [J]. Engineering Mechanics, 2008, 1(25): 217—223. (in Chinese)
- [10] 魏永祥, 陈建军, 马洪波. 随机参数齿轮系统的非线性动力响应分析[J]. 工程力学, 2012, 29(11): 319—324.
Wei Yongxiang, Chen Jianjun, Ma Hongbo, Analysis of nonlinear dynamic response of gear-rotor with random parametrics [J]. Engineering Mechanics, 2012, 29(11): 319—324. (in Chinese)
- [11] Zhang Wei, Ding Qian. Torsion vibration and parametric instability analysis of a spur gear system with time-varying and square nonlinearities [J]. International Journal of Applied Mechanics, 2014, 6(01): 45—67.
- [12] Kim W, Lee J Y, Chung J. Dynamic analysis for a planetary gear with time-varying pressure angles and contact ratios [J]. Journal of Sound and Vibration, 2012, 331(4): 883—901.
- [13] Chen Y C, Kang C H, Choi S T. Dynamic analysis of a geared rotor-bearing system with time-varying gear mesh stiffness and pressure angle [J]. Applied Mechanics and Materials, 2013, 284: 461—467.
- [14] 赵飞. 风力发电齿轮箱非线性耦合动态特性研究[D]. 大连: 大连理工大学, 2010.
Zhao Fei. Research on nonlinear coupling dynamic characteristic of the gear box for wind turbine [D]. Dalian University of Technology, 2010. (in Chinese)
- [15] 陆启韶, 彭临平, 杨卓琴. 常微分方程与动力系统[M]. 北京: 北京航空航天大学出版社, 2010: 29—57.
Lu Qishao, Peng Linping, Yang Zhuoqin. Ordinary differential equation and dynamical systems [M]. Beijing: Beijing University of Aeronautics and Astronautics Press, 2010: 29—57. (in Chinese)
- [16] 李万祥, 张永燕. 一类四自由度系统碰撞问题[J]. 工程力学, 2013, 30(9): 259—263.
Li Wanxiang, Zhang Yongyan, Collision problem of a four-degree-of-freedom vibro-impact system [J]. Engineering Mechanics, 2013, 30(9): 259—263. (in Chinese)
- [17] Errichello R. Encyclopedia of tribology [M]. Springer US, 2013: 1474—1478.
- [18] Nayfeh A H. Introduction to perturbation techniques [M]. New York: Wiley. 2011: 285—286.

(上接第 212 页)

- [12] 叶红玲, 郑小龙, 沈静娴, 刘赵森. 液体静压导轨转台轴向振动的动力学建模与分析[J]. 工程力学, 2012, 29(3): 218—225.
Ye Hongling, Zheng Xiaolong, Shen Jingxian, Liu Zhaomiao. Dynamic modeling and analysis of axial vibration of the hydrostatic slide turntable [J]. Engineering Mechanics, 2012, 29(3): 218—225. (in Chinese)
- [13] Kang Y, Chou H C, Wang Y P, et al. Dynamic behaviors of a circular worktable mountend on closed-type hydrostatic thrust bearing compensated by constat compensations [J]. Mechanics, 2013, 29(2): 297—308.
- [14] Garratt JE, Hibberd S, Cliffe KA, Power H. Centrifugal inertia effects in high-speed hydrostatic air thrust bearing [J]. Engineering Math, 2012, 76(1): 59—80.
- [15] 叶红玲, 文聘, 张颜明, 李耀明. 基于多层可行方向法液体静压转台优化设计[J]. 工程力学, 2014, 31(2): 249—256.
Ye Hongling, Wen Pin, Zhang Yanming, Li Yaoming. Design optimization of hydrostatic worktable based on multi-layer method of feasible direction [J]. Engineering Mechanics, 2014, 31(2): 249—256. (in Chinese)
- [16] 史宝军, 季家东, 杨廷毅. 粗糙度模式对硬盘气膜承载特性的影响[J]. 工程力学, 2012, 29(8): 313—318.
Shi Baojun, Ji Jiadong, Yang Tingyi. Effects of surface roughness modes on load carrying characteristics of air bearing films in hard disk drives [J]. Engineering Mechanics, 2012, 29(8): 313—318. (in Chinese)
- [17] 陈燕生. 液体静压支承原理和设计[M]. 北京: 国防工业出版社, 1980: 36—40.
Chen Yansheng. Theory and design of hydrostatic bearing [M]. Beijing: National Defence Industry Press, 1980: 36—40. (in Chinese)