

Liquidity as Social Expertise

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Abstract

This paper proposes a theory of liquidity dynamics. Illiquidity results from asymmetric information. Observing the historical track record teaches agents how to interpret public information and helps overcome information asymmetry. There can be an illiquidity trap: too much asymmetric information leads to the breakdown of trade, which interrupts learning and perpetuates illiquidity. Liquidity falls in response to unexpected events that lead agents to question their valuation models, especially in newer markets, may be slow to recover after a crisis and is higher in periods of stability.

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1 Introduction

This paper proposes a theory of what determines market liquidity and how liquidity evolves over time. The theory is based on the interplay between information asymmetry and social learning.

Liquidity is an elusive concept and the literature on it is plagued by challenges of both definition and measurement.¹ The theory in this paper refers to one particular notion of liquidity: the liquidity of an asset class is the fraction of the potential gains from trade in that asset class that are realized in equilibrium. If the potential gains from trade are large, it is therefore tautologically true that asset liquidity has important consequences for social welfare.

The theory is based on a minor modification of an otherwise standard model of trade under asymmetric information in the spirit of Akerlof (1970). The assumption is that, rather than being purely uninformed, asset buyers have access to some information, but their ability to make use of that information depends on their collective experience. The model abstracts from the specific features of the assets that are traded. However, to fix ideas it is useful to think of it as applying to markets like IPOs, the primary market for asset backed securities or the primary market for sovereign bonds.

The model works as follows. Every period, “investors” can create assets of heterogeneous quality; these are more productive if held by “managers” rather than investors, so there are gains from trade. Investors know the asset qualities but managers do not observe them, so a classic lemons problem ensues.

Managers can alleviate their informational disadvantage by observing public, asset-specific signals. However, for these signals to be useful, managers need to know how they correlate with asset qualities. The key assumption in the model is that the joint distribution of signals and asset qualities, summarized by the single parameter μ , is not known exactly. Agents can learn about μ from commonly-observed past data. This data is based on a sample of past assets and for each of them contains the signal it produced when it was created and an indicator of how it turned out. The idea, then, is that agents turn to the historical track record to guide them in making sense of the information available on any given asset, which is standard practice among financial analysts. Damodaran (2008) offers a guide on the use of “comparables” to determine which pieces of information about a particular asset one should focus on and how to use them in valuation. One of the key challenges in practice is finding

¹See Brunnermeier and Pedersen (2009) for a discussion of the conceptual issues and Goyenko et al. (2009) for a discussion of measurement.

a sufficiently large and sufficiently similar sample of historical precedents. In terms of the model, the key question is at what rate observations are added to the historical sample. This rate is assumed to depend positively on the level of investment and trade, a form of learning-by-doing.

The model delivers several predictions about the relationship between investment, information, and liquidity. The first result is that liquidity is increasing in the precision of agents' estimates of μ . Knowing μ makes traders better able to extract information from signals, reducing the degree of information asymmetry and increasing liquidity. Hence, liquidity is a function of traders' expertise. Asset prices, the level of investment and the volume of trade are increasing in liquidity.

Depending on parameters, the model may feature an "illiquidity trap". If at any point in time, estimates of μ are sufficiently imprecise, then assets will be completely illiquid and trade will break down. Depending on the parameters of the learning process and the investment technology, this may imply that markets generate no data for agents to learn from, which perpetuates the illiquidity. Whether or not the economy falls into an illiquidity trap depends on the sample realizations during the first few periods. If the first observations lead to precise and correct estimates of μ , then this will increase liquidity and reinforce the learning process, which becomes self-sustaining. If instead they lead to imprecise estimates of μ , because they conflict with each other or with agents' prior, then signals will become uninformative, leading to the illiquidity trap. Even under parameters such that there is no permanent illiquidity trap, markets liquidity can be slow to recover after a disruption.

The model also predicts that markets will tend to become more liquid over time, as traders accumulate more observations with which to estimate μ . Several studies have documented this feature of market maturation. Buckley (1997a,b) describes how this took place in emerging market debt markets; Anderson and Gascon (2009) do so for the commercial paper market.

In the model, unexpected events disrupt liquidity in the short term because they increase buyers' uncertainty as to whether they are using the correct model (i.e. the correct value of μ) to evaluate assets. This increases informational asymmetry and lowers liquidity. Adrian and Shin (2009) and Brunnermeier (2008) document how liquidity in the market for asset-backed securities fell sharply after an unexpected increase in mortgage defaults in 2007. Furthermore, according to the model, unexpected events will be more disruptive in newer markets, where traders have not had time to accumulate a long track record of observations and thus revise their beliefs strongly in light of new information. Caballero and Krishnamurthy (2008)

emphasize that market reactions to surprises tend to be more extreme in newer markets. If the underlying structure of the economy can change over time, unexpected events will be more common in times of structural change and therefore liquidity will be higher when the underlying economy is more stable.

The learning-by-doing mechanism at the heart of the model is similar to that assumed by Veldkamp (2005), van Nieuwerburgh and Veldkamp (2006) and Fajgelbaum et al. (2014), all of whom assume that there is an association between the level of economic activity and the rate at which agents learn. The main difference with this literature lies in what is the object that agents are trying to learn about. Here agents are not learning about the level of aggregate productivity but about the right way to evaluate assets. The mechanics of illiquidity persistence in the model are also similar to the informational cascades described by Banerjee (1992), though the logic is somewhat different. Here the learning process may be interrupted not because agents stop paying attention to information but because trade and investment collapse and no further information is generated.

The rest of the paper is organized as follows. Section 2 describes the environment; section 3 characterizes the static outcomes of the model; section 4 contains the main results about liquidity dynamics and section 5 offers some final remarks.

2 The Model

Agents, endowments and technology There are two types of agents, investors and managers. Each lives for only one period; once I introduce dynamics I study the continuous time limit where the period is short. Both types of agents have large endowments of the consumption good and are risk neutral.

Investors are ex-ante identical and have access to a technology that lets them convert $c(K)$ consumption goods into K units of capital. The function c is increasing and convex. As a side-product, the technology also produces λK “lemons”, which look like units of capital but are useless. λ is an exogenous parameter. I refer to capital and lemons collectively as “assets”. An investor may (within the period) use a unit of capital to produce ZK consumption goods. Z is a nonnegative random variable, distributed *iid* across investors according to a continuous distribution F . It is realized after K is chosen. The law of large numbers applies so F also represents the distribution of Z among investors. After production, capital depreciates completely.

Managers are not able to convert consumption goods into capital but have access to a

technology that produces θK consumption goods with one unit of capital.

Assumption 1.

1. $F(\theta) = 1$
2. $\frac{F(p)}{F(p)+\lambda}\theta < p$ for all $p > 0$

Part 1 of Assumption 1 says that investors are always less productive than managers. A social planner would want all capital to be operated by managers. Part 2 of Assumption 1 ensures that no trade would take place under pure asymmetric information. It will hold if λ is sufficiently high (i.e. there is a high proportion of lemons) and the left tail of $F(\cdot)$ is sufficiently thin (i.e. there are few investors with extremely low productivity).

Information Each investor can tell apart the units of capital from the lemons among the assets that he produced, but neither managers nor other investors can do so.

However, each asset emits a publicly observable signal s , which takes two possible values, A and B , according to the following conditional probabilities:

$$\begin{aligned} \Pr [s = A|\text{Capital}] &= \Pr [s = B|\text{Lemon}] = \mu \\ \Pr [s = B|\text{Capital}] &= \Pr [s = A|\text{Lemon}] = 1 - \mu \end{aligned} \tag{1}$$

where $\mu \in (0, 1)$. Assets are divisible and the law of large numbers applies so the probabilities (1) also represent proportions in any investor's portfolio.²

The value of μ is itself a random variable, realized at the beginning of time but initially unknown to the agents, drawn from a distribution G_0 . This is meant to capture the idea that market participants might not know what is the right model that applies to a particular asset class.

Occasionally, after its payoff is realized, an asset it will emit an additional "ex-post" public signal $z \in \{H, L\}$ with the following conditional probabilities:

$$\begin{aligned} \Pr [z = H|\text{Capital}] &= \Pr [z = L|\text{Lemon}] = \gamma \\ \Pr [z = L|\text{Capital}] &= \Pr [z = H|\text{Lemon}] = 1 - \gamma \end{aligned} \tag{2}$$

where $\gamma > 0.5$ is a *known* parameter (if $\gamma = 1$ this means agents find out for sure whether a particular asset was capital or a lemon). When an ex-post signal z is published, agents can

²Due to risk neutrality, this assumption is inconsequential.

contrast it to the original signal s to update their beliefs about μ . Each cohort can observe the entire history of past $\{s, z\}$ pairs.

The two signals s and z play different roles in the model. The original signal s is a composite of hard-to-interpret public information about an asset that is available at the time it trades. In the context of an IPO, s represents the content of the company's prospectus, analyst reports about its industry, the biography of its founders, etc. This information is used to value the company; uncertainty about μ represents uncertainty about how to translate information into valuation.

The ex-post signal z represents the information that is publicly available after the asset trades. In the context of an IPO, z may represent the company's financial statements for the first few years after the IPO. While these might also not be definitive about whether the company turned out to be a good investment (in terms of the model, γ is less than 1) they have the advantage that there is less ambiguity about how to interpret them. Since it's too late to use this information for trading the asset that generated it, agents only use it to update their valuation models for the next asset that comes along.³ For instance, after observing IPOs of several packaged foods companies, they might observe that the profit margins of individual companies in the industry are extremely stable but sales growth is hard to predict. The next time they observe a packaged foods company with high sales growth but low margins, they will conclude that it's likely to be a lemon in a way that would not have been possible with less experience.

The publication of ex-post signals takes place randomly over time at an aggregate Poisson rate ϕ_t . ϕ_t is endogenous; its determinants are discussed below and capture a form of learning by doing.

Markets There is no market for management services. In order for an agent to operate a unit of capital he must either build it himself (which is only possible for investors) or buy it in the market for capital. The market for capital is competitive but subject to information asymmetries.

³Of course, this information will be reflected in secondary markets for the asset, but the focus here is on the primary market.

3 Static Equilibrium

Equilibrium definition Since all agents are short-lived, it is possible to define an equilibrium of the static economy that takes place each period. The equilibrium will depend on the beliefs about μ that agents have upon entering the period. As is well known (Wilson 1980, Hellwig 1987), there is more than one way to formulate a competitive equilibrium in environments with asymmetric information, depending on whether the sellers have ways of signaling their type, for instance by committing to retain a fraction of the asset. The definition here implicitly assumes they cannot do so.

An investor must decide how much to invest and, conditional on the realization of Z , whether to sell each kind of asset in his portfolio. An investor that invests K will have a portfolio made up of

μK	units of capital with $s = A$
$(1 - \mu) K$	units of capital with $s = B$
$\lambda(1 - \mu) K$	lemons with $s = A$
$\lambda\mu K$	lemons with $s = B$

Denote by $\sigma_{Xs}(Z) \in [0, 1]$ the fraction of units of type $X \in \{\text{Capital, Lemon}\}$ with signal $s \in \{A, B\}$ that an investor chooses to sell if his productivity is Z . The investor's problem is:

$$\max_{K, \sigma_{Xs}(Z)} \mathbb{E} \left[\left(\begin{array}{c} [Z + \mu\sigma_{CA}(Z)(p_A - Z) + (1 - \mu)\sigma_{CB}(Z)(p_B - Z)] \\ + \lambda[(1 - \mu)\sigma_{LA}(Z)p_A + \mu\sigma_{LB}(Z)p_B] \\ - c(K) \end{array} \right) K \right] \quad (3)$$

The expectation operator is needed because both the realization of Z and the true value of μ are uncertain.⁴

Equilibrium prices must satisfy a zero-profit condition for managers, given by

$$p_A = \frac{\mathbb{E}[\mu\sigma_{CA}(Z)]}{\mathbb{E}[\mu\sigma_{CA}(Z) + \lambda(1 - \mu)\sigma_{LA}(Z)]} \theta \quad (4)$$

$$p_B = \frac{\mathbb{E}[(1 - \mu)\sigma_{CB}(Z)]}{\mathbb{E}[(1 - \mu)\sigma_{CB}(Z) + \lambda\mu\sigma_{LB}(Z)]} \theta \quad (5)$$

In equations (4) and (5), the numerator is the expected number of s -labeled units of capital sold by investors and the denominator is the expected total number of s -labeled assets

⁴The investor could infer μ from observing the proportions of each signal in his own portfolio but by then the investment decision is already sunk, and knowing μ does not change the selling decision.

sold. Equations (4) and (5) could have multiple solutions. Equilibrium requires that prices correspond to the highest solution.⁵

Definition 1. An equilibrium consists of

1. A level of investment K and selling decisions $\sigma_{X_s}(Z)$.
2. Prices p_s .

such that

1. K and $\sigma_{X_s}(Z)$ solve the investor's problem (3) taking p_s as given.
2. Prices are equal to the highest solution of equations (4) and (5) respectively.

Notice that the quantities of A -labeled and B -labeled asset that are traded will depend on the realized value of μ , since that determines how many units of capital and lemons end up carrying each label. The definition of equilibrium implicitly assumes that managers stand ready to absorb however many units end up being put up for sale at the equilibrium price.

Equilibrium characterization It is immediate from problem (3) that $\sigma_{L_s}(Z) = 1$ whenever $p_s > 0$ and that $\sigma_{C_s}(Z) = 1$ if and only if $p_s \geq Z$. In other words, all investors sell their lemons if they can get a positive price for them, but only sell their capital if the price is greater than their own productivity. Replacing this in (4) and (5) results in

$$p_A = \frac{\hat{\mu}F(p_A)}{\hat{\mu}F(p_A) + \lambda(1 - \hat{\mu})}\theta \quad (6)$$

$$p_B = \frac{(1 - \hat{\mu})F(p_B)}{(1 - \hat{\mu})F(p_B) + \lambda\hat{\mu}}\theta \quad (7)$$

where $\hat{\mu} \equiv \mathbb{E}(\mu)$. Notice that beliefs about μ enter the pricing condition only through their mean $\hat{\mu}$. The binary distribution has the property that that $\hat{\mu}$ is a sufficient statistic for the problem of inferring whether an asset is a lemon. In other words, agents interpret signals as though they knew that the value of μ was equal to $\hat{\mu}$.

Definition 2. The *informativeness* of signals given beliefs about μ is $\tau \equiv 2|\hat{\mu} - 0.5|$.

⁵This requirement can be derived formally by assuming that there are markets at every possible price and agents select at what prices to trade, as in Kurlat (2012). Generically, if there are prices p and $p' > p$ that satisfy the zero-profit condition, there must be a price $p'' \in (p, p')$ such that both buyers and seller prefer to trade at p'' rather than at p .

When $\hat{\mu} = 0.5$, then $\tau = 0$ and the signals convey no information, since each of them is believed to be equally likely to come from a unit of capital or from a lemon. When τ approaches 1, then signals perfectly reveal the type of asset that generates them.

Lemma 1. *Let $p(\tau) \equiv \max\{p_A, p_B\}$.*

1. $p(\tau)$ is equal to the highest solution to

$$p = \frac{(1 + \tau) F(p)}{(1 + \tau) F(p) + \lambda(1 - \tau)} \theta \quad (8)$$

2. There exists a cutoff $\tau^* > 0$ such that for $\tau \leq \tau^*$, $p(\tau) = 0$ and for $\tau > \tau^*$, $p(\tau)$ is strictly increasing in τ .

3. For any τ , $\min\{p_A, p_B\} = 0$.

Lemma 1 says that, under Assumption 1, when signals are sufficiently uninformative, prices collapse to zero, which implies no capital is traded. Trade can only take place if signals are sufficiently informative. When this is the case, there will be no trade conditional on a bad signal and positive trade conditional on a good signal, with prices increasing in the signal's informativeness. Which of the two signals is considered good depends on whether $\hat{\mu} > 0.5$, but the entire model is symmetric around $\hat{\mu} = 0.5$ and allocations depend only on τ .

Turn now to the investment decision. Assume without loss of generality that $\hat{\mu} \geq 0.5$ so $p_A \geq 0$ and $p_B = 0$. The ex-post value of an asset for an investor depends on whether it is a unit of capital or a lemon, what signal it emits and the productivity of the investor, according to:

Case	Value	
Lemon, $s = A$	p_A	always sold
Lemon, $s = B$	0	useless and $p_B = 0$
Capital, $s = A$	$\max\{p_A, Z\}$	sold if $Z < p_A$
Capital, $s = B$	Z	never sold since $p_B = 0$

Therefore the ex-ante value is

$$\begin{aligned} V &= \mathbb{E}[\lambda(1 - \mu)p_A + \mu \max\{p_A, Z\} + (1 - \mu)Z] \\ &= \lambda(1 - \hat{\mu})p_A + \hat{\mu} \left[F(p_A)p_A + \int_{Z \geq p_A} Z dF(Z) \right] + (1 - \hat{\mu})\mathbb{E}(Z) \end{aligned}$$

The case where $\hat{\mu} \leq 0.5$ is symmetric so V can be expressed simply as a function of τ :⁶

$$V(\tau) = \frac{1}{2} \left((1 - \tau) (\lambda p(\tau) + \mathbb{E}(Z)) + (1 + \tau) \left[F(p(\tau)) p(\tau) + \int_{Z \geq p(\tau)} Z dF(Z) \right] \right) \quad (9)$$

Lemma 2. V is increasing in τ

From expression (9), it is immediate that $V(0) = \mathbb{E}(Z)$ and $V(1) = \theta$. When signals are uninformative, no gains from trade are realized so the value for investors is just the expected marginal product under their own technology. When information is perfect, investors can capture the entire value that is generated when capital is operated by managers at their higher productivity θ . Lemma 2 says that, between these two extremes, the value for investors increases monotonically with the informativeness of signals. It is worth noting that this is not a general feature of all models of trade under asymmetric information. Levin (2001) shows that there may be cases where V can be a nonmonotonic function of buyers' information. Here, Assumption 1 ensures that the information asymmetry is sufficiently severe that improving buyers' information is always helpful.

Definition 3. The level of *liquidity* is $\frac{V(\tau) - \mathbb{E}(Z)}{\theta - \mathbb{E}(Z)}$.

I define the liquidity of assets as the fraction of the potential gains from trade that are realized. Its complement is the loss of value that results from the fact that they can only be traded in a market where there is asymmetric information. In this definition, liquidity is not purely an attribute of the assets themselves but of the entire equilibrium. In particular, it is immediate from Lemma 2 and the definition of liquidity that liquidity is increasing in the informativeness of signals.

⁶By assumption, the technology for producing capital also produces lemons in a fixed proportion λ . One could instead imagine that there might be separate technologies for producing capital and for producing lemons. In that case, it's immediate that the ex-ante value of a unit of capital is

$$V^C(\tau) = \frac{1}{2} \left((1 - \tau) \mathbb{E}(Z) + (1 + \tau) \left[F(p(\tau)) p(\tau) + \int_{Z \geq p(\tau)} Z dF(Z) \right] \right)$$

and the value of a lemon is

$$V^L(\tau) = \frac{1}{2} (1 - \tau) p(\tau)$$

so that $V(\tau) = V^C(\tau) + \lambda V^L(\tau)$. $V^C(\tau)$ is monotonically increasing in τ while $V^L(\tau)$ is nonmonotonic. For $\tau < \tau^*$, then $V^L(\tau) = 0$ because trade breaks down so lemons cannot be sold; on the other hand, $\lim_{\tau \rightarrow 1} V^L(\tau) = 0$ since all lemons are detected and cannot be sold either. Hence the incentive to produce lemons would be maximal for intermediate values of τ .

Under this definition of liquidity, it's straightforward to show that both investment and trade are positively associated with liquidity. From program (3), the first order condition for investment reduces to:

$$c'(K) = V(\tau) \quad (10)$$

The expected fraction of capital that will be traded is

$$x(\tau) = F(p(\tau)) \frac{1+\tau}{2} \quad (11)$$

Proposition 1. *The level of investment $K(\tau)$ and the expected fraction of capital traded $x(\tau)$ are increasing in the informativeness of signals τ .*

Proposition 1 establishes a link between liquidity, investment and trade. Liquidity is valued by investors because it measures the extent of gains from trade that are realized and (given that managers make zero profits) captured by investors. Investment will be higher when signals are more informative because this makes assets more liquid. This in turn results in more trade.

4 Dynamics

The only link between different periods in the economy comes from the evolution of beliefs about μ . These will depend on the initial distribution G_0 from which μ is drawn and on the $\{s, z\}$ pairs that are publicly observed thereafter.

Assumption 2. *μ is drawn from the following distribution:*

$$\mu = \begin{cases} \bar{\mu} & \text{with probability } g_0 \\ 1 - \bar{\mu} & \text{with probability } 1 - g_0 \end{cases} \quad (12)$$

where $\bar{\mu} > 0.5$.

If $\mu = \bar{\mu}$, then equation (1) implies that signal A is more likely to come from a unit of capital and signal B is more likely to come from a lemon, but it's the other way around if $\mu = 1 - \bar{\mu}$. At any given point in time, beliefs about μ can be summarized by $g_t = \Pr_t(\mu = \bar{\mu})$ and the resulting informativeness of signals is

$$\tau_t = 4 |g_t - 0.5| (\bar{\mu} - 0.5)$$

If the value of μ is known ($g_t = 1$ or $g_t = 0$), signals are equally informative whether the true value is $\bar{\mu}$ or $1 - \bar{\mu}$. In this case, the informativeness is

$$\bar{\tau} \equiv 2(\bar{\mu} - 0.5)$$

As g_t becomes close to 0.5, informativeness is lower, and becomes zero when $g_t = 0.5$. In other words, the symmetric formulation (12) is a way to formalize the assumption that the maximum possible informativeness of signals $\bar{\tau}$ does not depend on the true value of μ ; the only determinant of their actual informativeness is the extent to which agents know the correct way to make use of the information.

Assumption 3. $p(\bar{\tau}) > 0$

Assumption 3 says that $\bar{\tau}$ is sufficiently informative that if agents knew the value of μ this would result in trade at positive prices. In the absence of this assumption assets would always be completely illiquid.

The evolution of g_t is easy to characterize. g_t is revised whenever an ex-post signal z is observed from some asset and remains constant in the intervals in between. Whenever an ex-post signal is observed, agents will compare it with the original signal s . Letting m be the indicator of the event $\{s, z\} \in \{\{A, H\} \cup \{B, L\}\}$, its conditional probability is $\Pr[m|\mu] = \omega(\mu) \equiv \mu\gamma + (1 - \mu)(1 - \gamma)$. Letting $\bar{\omega} \equiv \omega(\bar{\mu})$, $\bar{\omega} - 0.5$ measures the informational content of ex-post signals for inferring μ . It is increasing in γ because more accurate ex-post signals provide more feedback about the correct interpretation of the original signals. By Bayes' Rule, the posterior upon observing an ex-post signal is:

$$g'_t = \frac{[\bar{\omega}^m (1 - \bar{\omega})^{1-m}] g_t}{[\bar{\omega}^m (1 - \bar{\omega})^{1-m}] g_t + [\bar{\omega}^{1-m} (1 - \bar{\omega})^m] (1 - g_t)} \quad (13)$$

I take the continuous time limit where each period is short and model the arrival of ex-post signals as a Poisson process with intensity ϕ_t . The key driver of the model's dynamics is the relationship between ϕ_t and the static equilibrium objects.

Assumption 4. ϕ is an increasing function $\phi(K, x)$ of investment K and the fraction of capital traded x

Assumption 4 says there is learning-by-doing. The amount of feedback that agents obtain depends on the volume of economic activity: both investment and trade result in learning. I shall leave unspecified the exact channels by which economic activity determines the number

of signals; more than one mechanism is possible. For instance, it could be that a fraction of all assets gets examined by analysts, who then report both the original signal s and an indicator z of how the asset eventually turned out; in this example, information flow would be proportional to investment K . Alternatively, one could imagine that the original investors prepare an IPO prospectus (which contains s) only for assets that they sell and once the asset is publicly traded its ex-post payoffs z are publicly available; in this case, information flow would be proportional to xK .

A full model of these mechanisms would have to take into account incentives to collect information, for instance through experimentation, and also specify the channels through which this information diffuses. Here, because agents are short-lived, learning is a pure externality: the experience of one cohort teaches the following ones.

Since both investment and trading volume depend on informativeness τ , it is possible to express the arrival rate of signals simply as a function of τ :

$$\phi(\tau) \equiv \phi(K(\tau), x(\tau))$$

Since $\phi(K, x)$ is increasing in both arguments and both $K(\tau)$ and $x(\tau)$ are increasing, then $\phi(\tau)$ is increasing as well.

An illiquidity trap

Depending on parameters, the model may have the feature that when τ is sufficiently low no learning will take place, which implies that τ will remain low. I refer to this outcome as an illiquidity trap.

Definition 4. There is an *illiquidity trap* if there exists a cutoff τ^{IT} such that $\phi(\tau) = 0$ for all $\tau \leq \tau^{IT}$.

The following result characterizes the conditions on the primitives of the model for this condition to be met.

Lemma 3.

1. *If either*

(a) $\phi(K, x) = 0$ whenever $x = 0$ (trade is essential for learning) or

(b) $\phi(K, x) = 0$ whenever $K = 0$ and $c'(0) > \mathbb{E}(Z)$ (investment is essential for learning and positive investment requires gains from trade)

there is an illiquidity trap, with $\tau^{IT} \geq \tau^*$.

2. If either

(a) $\phi(K, x) > 0$ for all K, x or

(b) $\phi(K, x) > 0$ whenever $K > 0$ and $c'(0) < \mathbb{E}(Z)$

there is no illiquidity trap.

Two conditions make an illiquidity trap possible. The first possibility is that agents are unable to learn from experience if no trade takes place. In this scenario, if τ falls below τ^* and trade breaks down, the learning process stops. The second is that investors' own technology is not sufficiently productive to justify positive investment unless assets are sufficiently liquid. In this case, if τ falls below a threshold τ^{IT} , there will be no investment and the learning process stops; the threshold might be $\tau^{IT} = \tau^*$ or, if higher liquidity than τ^* is required to justify investment, it might be even higher. If neither of these two conditions hold, then learning will always take place, so there is no illiquidity trap.

Short-run dynamics

Turn now to the short-run dynamics of the model, assuming there is no illiquidity trap. The first result says that on average informativeness increases over time.

Proposition 2. *Suppose there is no illiquidity trap. Then for any $t' > t$, $\mathbb{E}(\tau_{t'}|\tau_t) > \tau_t$*

Proposition 2 provides a characterization of how the learning dynamics evolve in the short run. The law of iterated expectations implies that g_t must be a martingale; since τ is a convex function of g_t then in expectation it increases. On average, new observations push beliefs towards either $g_t = 0$ or $g_t = 1$, increasing agents' confidence in their estimates of μ and increasing liquidity. This provides an account of how markets become more mature. As market participants' experience increases, they become better able to use the available information, which makes valuations more accurate, alleviates information asymmetry and increases liquidity.

Other things being equal, the upward trend in liquidity will be stronger in more productive economies.

Proposition 3. *Suppose there are two otherwise identical economies without an illiquidity trap that differ in the cost of investment function $c(\cdot)$ such that $c'(K)$ is lower in one of them for all K . Then $\mathbb{E}(\tau_t)$ is higher in the economy with lower $c'(\cdot)$, for all t .*

Proposition 3 says that more productive economies will be more active and therefore more liquid. A lower cost of investment results in higher levels of investment, which speeds up the process of learning-by-doing, resulting in more precise estimates of μ , less information asymmetry and more liquidity.

The relationship between measures of market volume and liquidity has been the subject of a large empirical literature (Easley et al. 1996, Chordia et al. 2001, Amihud 2002). The positive association is often attributed to some form of thick market effect through easier search or increased competition. Here the effects are through information asymmetry and are both static and dynamic. The static effect is standard: information asymmetry interferes with trade, lowering volume and liquidity. Glosten and Milgrom (1985), Kyle (1985) and indeed Akerlof (1970) examine aspects of this effect. The dynamic effect is specific to this model. Higher volume leads to more learning, which leads to lower informational asymmetry and higher liquidity in future periods.

Unlikely events

Whenever $g \neq 0.5$, one of the two possible values of $\bar{\mu}$ is considered more likely; hence some realizations of $\{s, z\}$ are more likely than others.

Definition 5. An observation $\{s, z\}$ is *unlikely* if either (i) $g > 0.5$ and $\{s, z\} \in \{\{A, H\} \cup \{B, L\}\}$ or (ii) $g < 0.5$ and $\{s, z\} \in \{\{A, L\} \cup \{B, H\}\}$

Liquidity will react differently to likely and unlikely events, especially if the level of informativeness required for trade is large.

Definition 6. τ^* is *large* if

$$\tau^* > \bar{\tau} \frac{\left(\frac{\bar{\omega}}{1-\bar{\omega}}\right)^{0.5} - 1}{\left(\frac{\bar{\omega}}{1-\bar{\omega}}\right)^{0.5} + 1} \quad (14)$$

Condition (14) relates the cutoff level of informativeness required for positive liquidity τ^* to the informational content of any given observation $\bar{\omega}$. If it holds, then no single observation carries sufficient information to overturn agents' beliefs to the point that markets remain liquid while reversing which of the two signals is considered good. The model is very stark in that there are only two possible models of the world, one where $\mu = \bar{\mu}$ and one where $\mu = 1 - \bar{\mu}$. Condition (14) guarantees that questioning one model cannot lead to immediately adopting the other with a sufficiently high degree of confidence that markets remain liquid. In a limit where each single piece of information is small (with $\bar{\omega}$ close to 0.5) then condition

(14) always holds: evidence against the preferred model leads to uncertainty before it leads to adoption of the alternative model, which implies that τ^* is large. τ^* can fail to be large only if information comes in sufficiently discrete pieces.

Proposition 4. *If τ^* is large, then*

1. *If liquidity is positive, an unlikely observation leads to lower liquidity.*
2. *Starting from any beliefs, there is a finite n such that a sequence of n unlikely observations will make assets completely illiquid.*
3. *The number of unlikely observations needed to make assets completely illiquid is increasing in τ_t .*

Assets are more liquid the more agents understand (or think they understand) the information structure of the economy. When unlikely events take place, they cast doubt on whether the agents are using the right valuation model to guide their actions. The increased uncertainty lowers liquidity by worsening the information asymmetry between investors and managers.⁷ Proposition 4 shows that this effect makes asset liquidity fragile: a sufficiently unlikely sequence of observations will push beliefs from levels that support positive liquidity to levels where assets are completely illiquid. The more confidence that agents have to begin with, the less their beliefs will be swayed by contradictory information and therefore the more unlikely events are necessary to render markets illiquid. Given that, by Proposition 2, τ_t tends to increase over time, this predicts that market liquidity tends to become less fragile when markets are more mature.

The result relies on τ^* being large, but only in a technical sense. If τ^* is not large, then there are values of τ such that any observation, even an unlikely one, increases liquidity, but only if liquidity was very low to begin with.

Proposition 4 establishes that, fixing the values of parameters, if assets are more liquid, this liquidity will be less fragile in response to unlikely events. Comparing across different parameter values, the same level of liquidity can be more or less fragile depending on the exact circumstances that give rise to it.

Proposition 5. *Suppose there are two otherwise identical economies that differ in $\bar{\mu}$ and g_t so that informativeness $\tau_t = 4|g_t - 0.5|(\bar{\mu} - 0.5)$ is the same in both. An unlikely observation will make liquidity fall more in the economy with higher $\bar{\mu}$ and lower $|g_t - 0.5|$.*

⁷Routledge and Zin (2009) propose a complementary mechanism based on uncertainty aversion rather than asymmetric information.

Informativeness results from the combination of how informative signals would be if agents knew the information structure (as measured by $\bar{\tau} = 2(\bar{\mu} - 0.5)$) and how confident agents are that they know the true value of μ (as measured by $|g_t - 0.5|$). Proposition 5 shows that liquidity is more resilient to unlikely events when it results from a high degree of certainty about the information structure, even if the signals themselves are less informative. In contrast, a market where the signals, properly interpreted, carry a lot of information but where the right value of μ is uncertain is more subject to illiquidity-inducing surprises. This implies that, even controlling for how liquid they are at a given point in time, liquidity in more mature markets will be less fragile.

The link between surprising events in relatively new markets and the onset of financial crisis has been emphasized by Caballero and Krishnamurthy (2008) and Caballero and Kurlat (2009). One historical example of this dynamic at play is the bankruptcy of Penn Central in 1970. Penn Central was a large issuer of commercial paper, which was a relatively new asset class, total outstanding issues having risen from about \$10 billion to about \$40 billion in four years (Schadrack and Breimyer 1970). According to Schadrack and Breimyer (1970), investors mistakenly believed that paper issued by large corporations was safe, so when Penn Central filed for bankruptcy in June 1970 this came as a large surprise. According to the account by Calomiris (1993), it then became “necessary for the market to reevaluate its methods for pricing paper generally in light of this surprising event”. The resulting uncertainty led to a freeze in new issues of commercial paper, so that the outstanding stock fell by about 10% in the first month after Penn Central’s bankruptcy.

A more recent example is the reaction of the asset backed securities market to the downturn in the housing market. Foote et al. (2012) cite financial industry reports around 2005 that discuss different scenarios for the performance of the various tranches of mortgage-backed securities. The most pessimistic scenarios in these reports (labeled “meltdown” in one of them) far underestimate the losses that materialized in the following years. In other words, the housing crisis was considered an unlikely event. As documented by Adrian and Shin (2009) and Brunnermeier (2008), issuance of all kinds of asset-backed securities fell almost to zero when this unlikely event materialized in 2008.⁸

Through the lens of the model, these episodes can be interpreted as follows. Before the onset of each crisis, investors believed they had reliable (high $\bar{\mu}$) indicators of asset quality (the size of the corporation issuing the commercial paper, credit ratings for mortgage-backed

⁸Caballero and Krishnamurthy (2008) describe similar pattern following the stock market crash of 1987, the LTCM crisis in 1998 and the terrorist attacks of September 11, 2001.

securities). Therefore asset buyers were confident that they were not at a large informational disadvantage and markets were liquid, i.e. most of the potential gains from trade were realized. Then the unexpected events took place. Since the markets were relatively new (in model terms, g_t had not had time to converge all the way to 0 or 1 yet), the unexpected events could not be dismissed as outliers and led investors to question their valuation models. Without confidence in a valuation model, the logic of asymmetric information took over and markets became illiquid, at least until confidence in a revised valuation model was built up.

Long-run outcomes

The long-run properties of the model depend on whether there is an illiquidity trap and, if so, on initial conditions and how large the trap is.

Proposition 6.

1. *If there is no illiquidity trap, then $\tau_t \rightarrow_{a.s.} \bar{\tau}$ and $K_t \rightarrow_{a.s.} \bar{K} \equiv (c')^{-1}(V(\bar{\tau}))$ for any initial beliefs g_0 .*
2. *If there is an illiquidity trap and $\tau_0 \leq \tau^{IT}$, then $K_t = K(\tau_0)$ and $\tau_t = \tau_0$ for all t .*
3. *If there is a large illiquidity trap (i.e. τ^{IT} satisfies condition (14)) and $\tau_0 > \tau^{IT}$, then for any $\epsilon > 0$,*

$$\lim_{t \rightarrow \infty} \Pr [\tau_t \geq \bar{\tau} - \epsilon] \in \left[\frac{\tau_0 - \tau^*}{\bar{\tau} - \tau^*}, \frac{\tau_0 + \tau^*}{\bar{\tau} - \epsilon + \tau^*} \right] \quad (15)$$

and $\lim_{t \rightarrow \infty} \Pr [\tau_t \leq \tau^{IT}] = 1 - \lim_{t \rightarrow \infty} \Pr [\tau_t \geq (1 - 2\epsilon) \bar{\tau}]$.

4. *If there is an illiquidity trap but it is not large, then there are values of $\tau_0 > \tau^{IT}$ such that $\lim_{t \rightarrow \infty} \Pr [\tau_t \leq \tau^{IT}] > 0$ and values of $\tau_0 > \tau^{IT}$ such that $\Pr [\tau_t \leq \tau^{IT}] = 0 \quad \forall t$.*

Proposition 6 establishes under what conditions assets will eventually either become as liquid as possible or fall into the illiquidity trap. Part 1 says that if parameters are such that there is no illiquidity trap, then learning always takes place. Eventually large samples will accumulate and agents will learn the value of μ perfectly. Informativeness will converge to $\bar{\tau}$ and liquidity and investment will converge to their maximum possible levels. Part 2 says that if there is an illiquidity trap and the economy begins in it, then there will never be learning and assets will remain illiquid. The economy will repeat the initial period static equilibrium forever.

If there is an illiquidity trap but the economy begins outside it, some learning will take place at first. Since the sequence of realizations that agents learn from is random, there is a positive probability that it leads g_t towards 0.5. Parts 3 and 4 distinguish between two possibilities. Part 3 says that if the illiquidity trap is large, then as g_t approaches 0.5 it will fall into the trap, so the probability of falling into it is positive no matter what the initial beliefs were. Part 4 says that if the trap is not large, a single observation may lead beliefs to jump across the illiquidity trap; in this case there are initial beliefs such that no sequence of observations leads into the trap. Conversely, if initial observations lead g_t away from 0.5, liquidity increases, and there is a positive probability that it converges to $\bar{\tau}$. The probabilities that τ_t falls into the trap or converges to $\bar{\tau}$ respectively can be bounded. Condition (15) says that the closer the economy begins to the edge of the illiquidity trap, the higher the probability that it falls in it; the further away, it begins, the more likely it is to avoid it.⁹

The reason for the possibility of two long-run outcomes is that the model features a form of dynamic strategic complementarities in investment and trading decisions. Higher investment and trading at date t results in a higher rate of learning which, on average, leads over time to more precise estimates of μ and therefore more informativeness. This informativeness in turn sustains higher levels of learning and investment. Due to this complementarity, there is path dependence: randomness in the early stages of the learning process may have long-term consequences for how financial markets develop, a feature of many models with complementarities. This suggests a possible role for government intervention to get the learning cycle started (or re-started if it starts but then enters the illiquidity trap), for instance by subsidizing the early stages of the financial industry. This is not unlike other “big-push” policies based on a learning-by-doing-arguments (see Easterly (2006) for a sceptical look at this type of argument).

Figure 1 shows an example where the economy may or may not fall into the illiquidity trap depending on the initial observations. For this example, investment is always positive and the frequency of observations is proportional to the total volume of trade, so that if trade breaks down then learning breaks down too. Hence $\tau^{IT} = \tau^*$. Initial beliefs imply $\tau_0 > \tau^*$ so the economy begins outside the illiquidity trap. The figure shows two possible realizations. In both of them, beliefs oscillate in response to the first few observations, which include instances of both $\{s, z\} \in \{\{A, H\} \cup \{B, L\}\}$ (which raises g) and $\{s, z\} \in \{\{A, L\} \cup \{B, H\}\}$ (which lowers g). Since the true value of μ is $\bar{\mu}$, the first type of observation is more common so

⁹This probability can also be computed exactly, but the answer depends on $\bar{\omega}$. The bounds in (15) hold for any $\bar{\omega}$.

over time g drifts upwards towards $g = 1$, which leads to the maximum possible level of informativeness, asset prices and investment. However, in one of the realizations the early observations include many instances of $\{s, z\} \in \{\{A, L\} \cup \{B, H\}\}$, upon which which g falls and τ falls below τ^* . After that, the economy is in the illiquidity trap: there is no trade and therefore no learning. Note that even before actually entering the trap, whenever τ is low then the frequency of observations ϕ is low so the learning process slows down before stopping entirely.

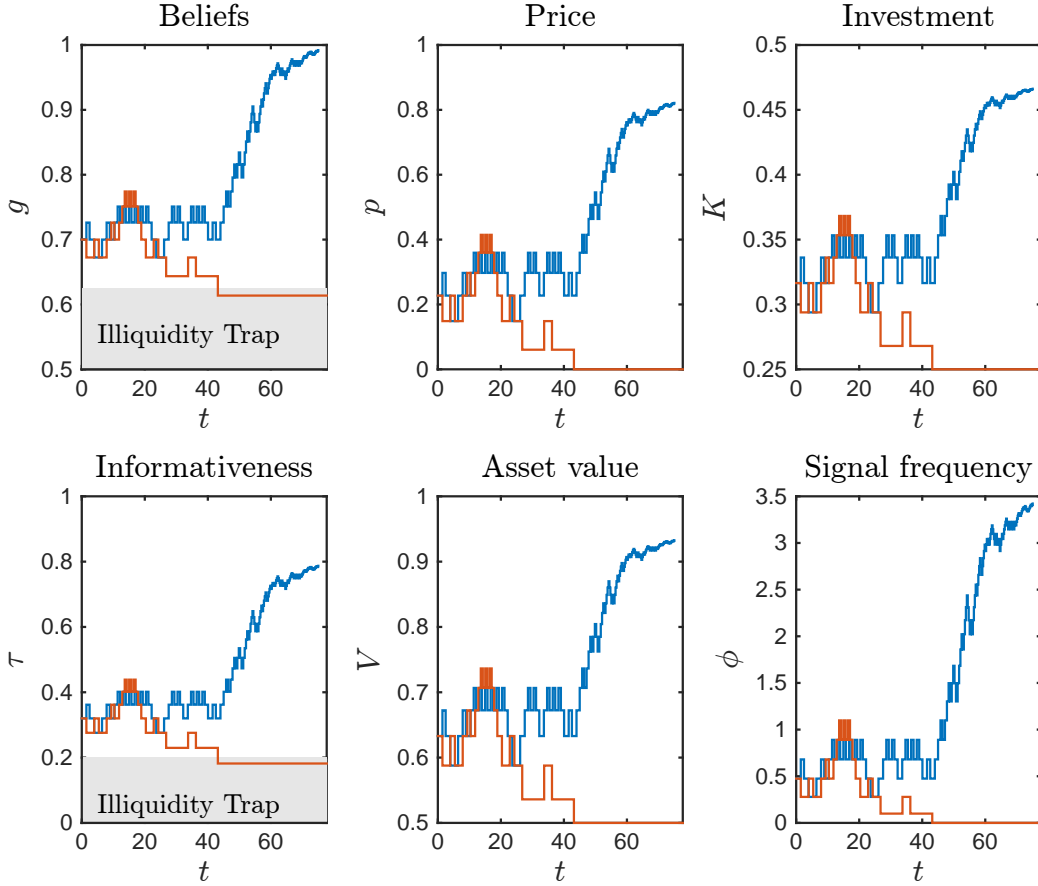


Figure 1: Two possible long run outcomes. The example uses $\mu = \bar{\mu} = 0.9$, $g_0 = 0.7$, $\lambda = 1.5$, $c(K) = K^2$, $Z \sim U[0, 1]$, $\theta = 1$, $\gamma = 0.54$ and $\phi = 10Kx$

Changing μ

Suppose that instead of being fixed forever, μ switches randomly between its two possible values at a Poisson rate δ . This is meant to capture the idea that the structure of the economy,

which determines the correct way to use information, may change over time. Agents do not observe the changes in μ directly, so they face a filtering problem: keeping track of the changing value of μ on the basis of their observations. The solution of this problem is as follows. During any interval where no ex-post signals are observed, g_t evolves according to:

$$\frac{dg_t}{dt} = \delta (1 - 2g_t) \quad (16)$$

and any time there is a new observation, g_t jumps according to (13). Equation (16) says that during periods with no new observations, g_t mean-reverts towards 0.5. This implies that experience depreciates: unless new observations are added, agents are rationally aware that the true model may have changed so their beliefs are based on possibly outdated data, and they become less confident in them. This lowers the informativeness of signals and therefore liquidity falls.

Lemma 4. *If $\delta > 0$ and there is an illiquidity trap then $\tau_t \rightarrow_{a.s.} 0$.*

Lemma 4 says that if there is an illiquidity trap, the economy will eventually fall into it and never emerge. This is in contrast with Proposition 6, which says that with a constant μ the economy can, with positive probability, avoid the illiquidity trap in the long run. The difference is that with constant μ experience does not depreciate, so as beliefs converge to either $g = 0$ or $g = 1$ the sequence of unlikely events that is needed for the economy to fall back into the illiquidity trap gets longer and less probable. Instead, when μ can change, unlikely events are not the only way to drive the economy into the illiquidity trap: a prolonged period of no new observations will suffice, and this is always possible.

If there is no illiquidity trap, standard arguments imply that informativeness will converge to an invariant distribution. In the long run, informativeness will fluctuate. Sometimes there will be many recent observations that all point in the same direction, giving agents high confidence in their estimates of μ , leading to high informativeness and liquidity; at other times there will be few or contradictory observations, pushing g towards 0.5 and lowering liquidity. Prices, the level of investment and learning rates will therefore also fluctuate.

One of the sources of these fluctuations will be actual changes in the true value of μ . In expectation, these changes lower informativeness.

Proposition 7. *Starting from any beliefs $g_t \neq 0.5$, let $t + T$ be the first time an ex-post signal is observed after t . $\mathbb{E}(\tau'_{t+T} | \mu_{t+T} = \mu_t) > \mathbb{E}(\tau'_{t+T} | \mu_{t+T} \neq \mu_t)$*

Proposition 7 establishes that informativeness, and therefore liquidity, is higher when the economy remains stable than when it suffers shocks. If the true value of μ has changed since

the last observation, it is more likely that the next observation will conflict with the agents' beliefs, which will make them uncertain about the true value of μ , lowering liquidity. In periods of stability, it is more likely that new observations will reaffirm agent's prior, which shifts beliefs away from $g_t = 0.5$, increasing the informativeness of signals and increasing liquidity.

Figure 2 shows a simulated path for the same economy as in Figure 1 with two minor differences. First, the arrival rate of ex-post signals is $\phi = [10x + 0.15(1 - x)]K$, which means it's positive even when $x = 0$ (i.e. when trade breaks down). This implies there is no illiquidity trap. Second, the value of μ changes with a Poisson intensity $\delta = 0.0001$, so expertise depreciates at a small positive rate.

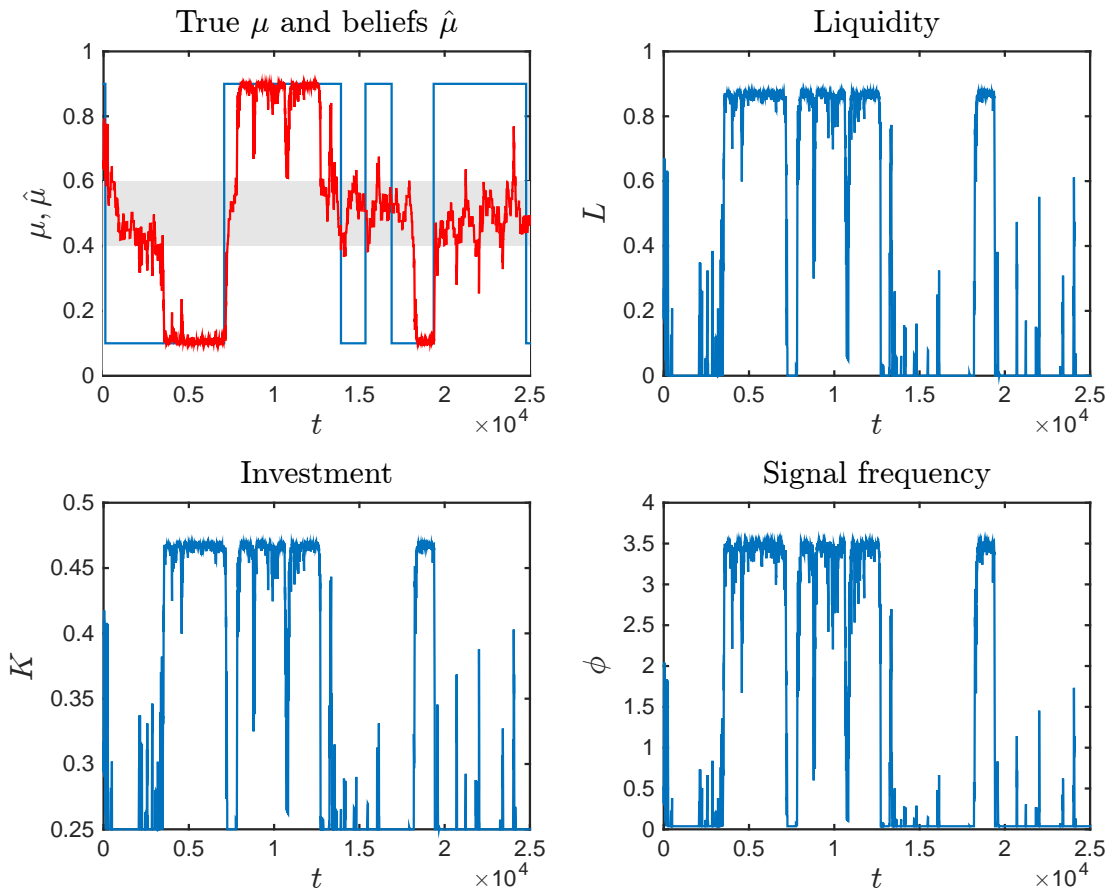


Figure 2: Simulated path. The example uses $\mu = \bar{\mu} = 0.9$, $g_0 = 0.7$, $\lambda = 1.5$, $c(K) = K^2$, $Z \sim U[0, 1]$, $\theta = 1$, $\gamma = 0.54$ and $\phi = [10x + 0.15(1 - x)]K$ and $\delta = 0.0001$. The grey area in the first panel shows the region of $\hat{\mu}$ where trade breaks down.

In the example, the signals at first move the economy towards the region of beliefs where

assets are completely illiquid; this slows down the learning process so the economy remains there for a long time. Eventually, a string of signals that all indicate that $\mu = 1 - \bar{\mu}$ arrive close to each other and the economy escapes the illiquid region. The learning rate accelerates, so agents receive a lot of signals that on average confirm that μ has not changed and liquidity remains high for many periods. Eventually, a string of signals arrives that points (correctly, as it turns out) to μ having changed sign and the economy becomes illiquid again, and the cycle repeats itself, with relatively long periods of sustained liquidity and sustained illiquidity. Liquidity, investment and the arrival rate of signals all co-move with beliefs.

Figure 3 shows the long-term distribution of informativeness. Since both the learning rate when there is no trade and the depreciation rate of expertise are very low, the economy spends most of the time in states where informativeness is either close to its maximum $\bar{\tau}$ or at levels that are insufficient to sustain trade. This shows that the model's predictions are continuous with respect to whether or not there is an illiquidity trap. If lack of trade leads to a large slowdown in the learning rate but not quite a complete shutdown, then liquidity will eventually recover, but will spend a large fraction of time in an illiquid state.

Figure 4 shows an impulse response of the economy if, starting from the median level of liquidity, it receives a series of signals that push beliefs into the region where assets are illiquid. In the example, illiquidity makes the learning process very slow. Therefore it takes (on average) a long time for liquidity to recover after the shock. The example of course is meant as an illustration and not as a quantitative statement. The point is that if $\phi(\tau)$ is very low when $\tau < \tau^*$, then recovery after assets become illiquid may take a long time. This mechanism could be part of the explanation of why recoveries after financial crises tend to be slow (Cerra and Saxena 2008, Reinhart and Rogoff 2009): unexpected shocks makes financial expertise outdated, liquidity falls and drags investment with it, and recovery requires rebuilding the stock of expertise.

In this example, shocks affect information directly, but the learning dynamics could also be a transmission mechanism for shocks that originate elsewhere. For instance, shocks to productivity can lower the level of investment, which lowers the rate of learning and leads to lower liquidity and investment in the future. Kurlat (2010) shows how this effect can create persistence in a business cycle model.

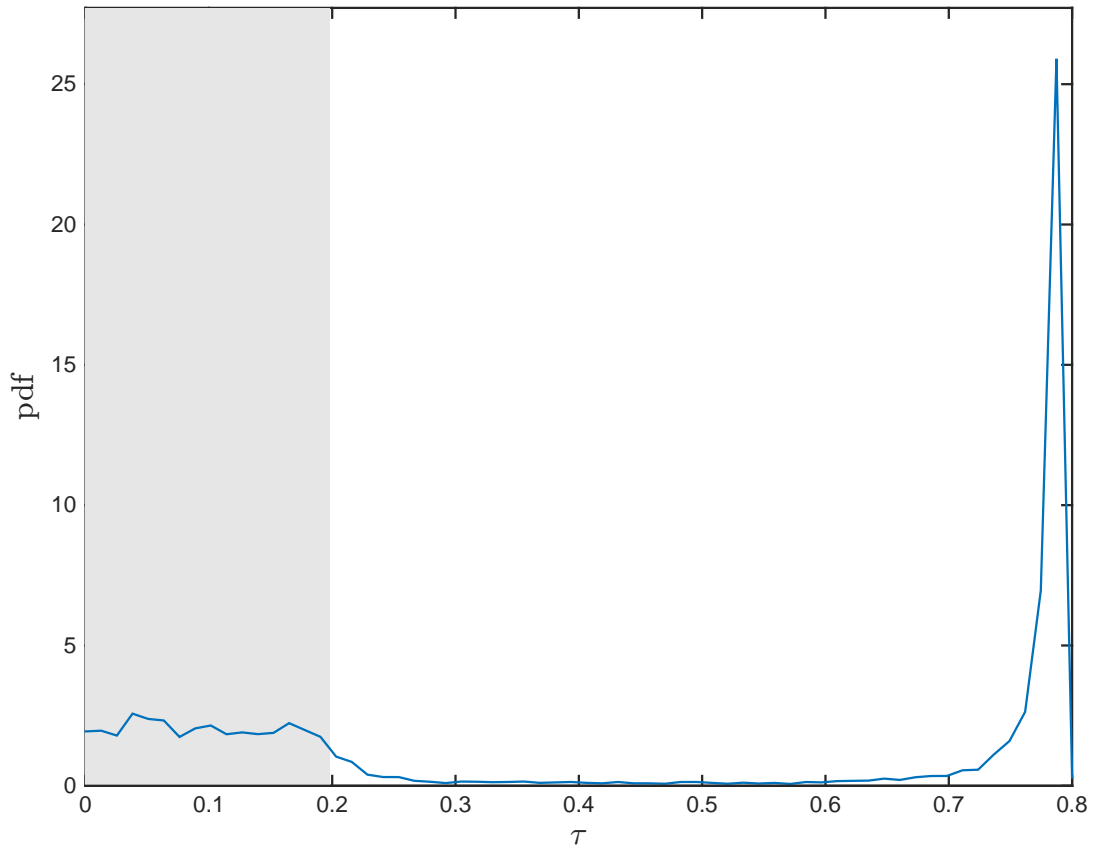


Figure 3: Long-term distribution of informativeness. This is computed by sampling 10,000 equally-spaced observations over a simulation of 400,000 periods in calendar time and approximating the density with a Gaussian kernel. The grey area shows the region of τ where trade breaks down.

5 Conclusion

Introducing social learning dynamics in a standard model of trade under asymmetric information yields a rich set of implications. It provides a dynamic link between liquidity and trading volume; a theory of why liquidity increases over time, why it can be volatile, why it falls after unexpected events and why illiquidity can become long-lasting.

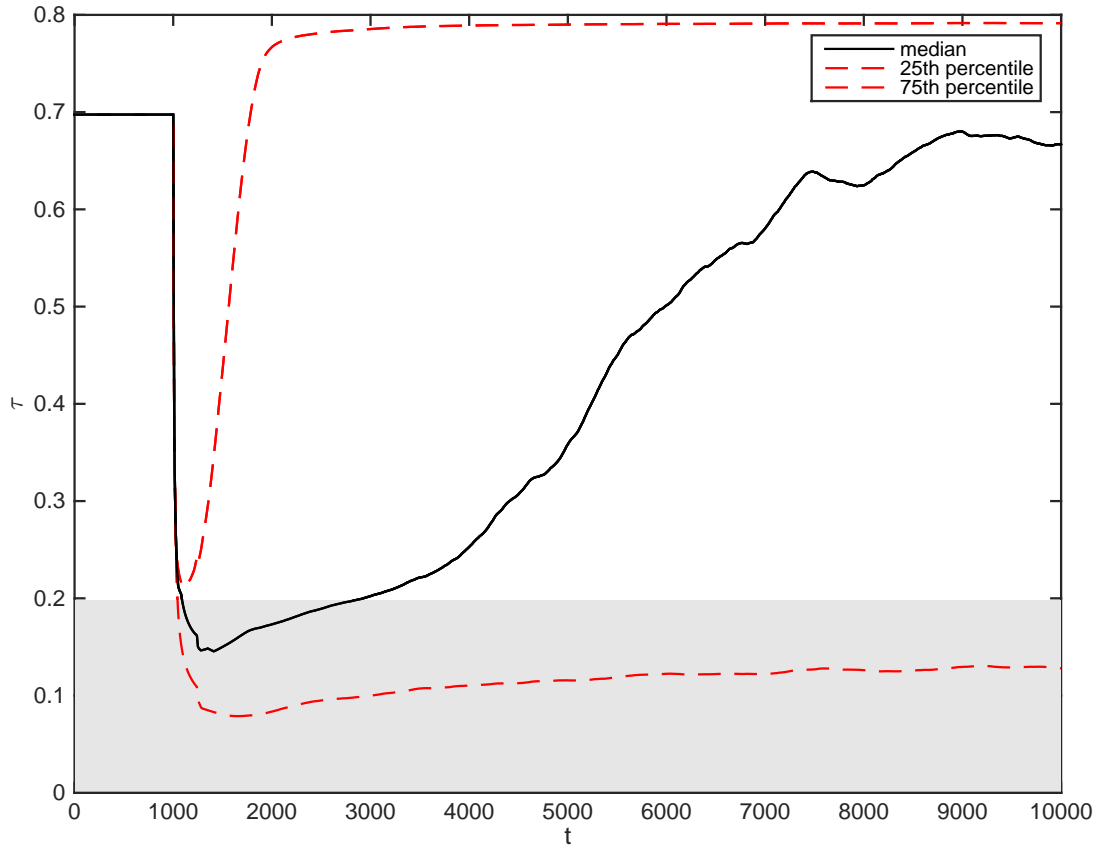


Figure 4: Impulse response after in period 1,000 a series of consecutive unlikely observations shift the economy into the illiquid region. The quantiles are computed over 5,000 simulations, each 10,000 periods long in calendar time.

Appendix

Proof of Lemma 1

1. From equations (6) and (7) and the definition of τ , the two prices must be the highest solutions to (8) and

$$p = \frac{(1 - \tau) F(p)}{(1 - \tau) F(p) + \lambda(1 + \tau)} \theta \quad (17)$$

respectively. The result then follows because

$$\frac{(1 + \tau) F(p)}{(1 + \tau) F(p) + \lambda(1 - \tau)} \geq \frac{(1 - \tau) F(p)}{(1 - \tau) F(p) + \lambda(1 + \tau)} \quad \forall p$$

so the solution to (8) defines $\max \{p_A, p_B\}$ and the solution to (17) defines $\min \{p_A, p_B\}$.

2. When $\tau = 0$, (8) reduces to

$$p = \frac{F(p)}{F(p) + \lambda} \theta$$

By Assumption 1, there is no positive solution, so the only solution is $p = 0$. Since Assumption 1 is a strict inequality, by continuity there is a neighborhood around $\tau = 0$ where (8) has no positive solution. The fact that $p(\tau)$ is strictly increasing whenever it is positive results from the fact that the right hand side of equation (8) is increasing in τ . This in turn implies that the cutoff τ^* is unique.

3. The right hand side of equation (17) is decreasing in τ and lies below the 45 degree line for $\tau = 0$. Therefore it must lie below the 45 degree line for any τ , so the only solution of equation (17) is $p = 0$.

Proof of Lemma 2

Using (8), rewrite (9) as

$$\begin{aligned} V(\tau) &= \frac{1}{2} \left[((1 + \tau) F(p(\tau)) + \lambda(1 - \tau)) p(\tau) + (1 - \tau) \mathbb{E}(Z) + (1 + \tau) \int_{Z \geq p(\tau)} Z dF(Z) \right] \\ &= \frac{1}{2} \left[(1 + \tau) F(p(\tau)) \theta + (1 - \tau) \mathbb{E}(Z) + (1 + \tau) \int_{Z \geq p(\tau)} Z dF(Z) \right] \end{aligned}$$

so

$$\begin{aligned} \frac{dV}{d\tau} &= \frac{1}{2} \left[F(p(\tau)) \theta - \mathbb{E}(Z) + \int_{Z \geq p(\tau)} Z dF(Z) + (1 + \tau) (\theta - p(\tau)) f(p(\tau)) \frac{dp(\tau)}{d\tau} \right] \\ &= \frac{1}{2} \left[\int_{Z \leq p(\tau)} (\theta - Z) dF(Z) + (1 + \tau) (\theta - p(\tau)) f(p(\tau)) \frac{dp(\tau)}{d\tau} \right] \geq 0 \end{aligned}$$

where the last inequality follows from the fact that $p(\tau) < \theta$ and $\frac{dp(\tau)}{d\tau} \geq 0$ by Lemma 1.

Proof of Proposition 1

For $K(\tau)$, the result follows from Lemma 2 and condition (10) because c is convex. For $x(\tau)$, the result follows because by Lemma 1 $p(\tau)$ is increasing in τ .

Proof of Lemma 3

1. Suppose first that $\phi(K, x) = 0$ whenever $x = 0$. Then if $\tau \leq \tau^*$ then $p(\tau) = 0$ which by (11) implies $x = 0$ and therefore $\phi = 0$. Now suppose $\phi(K, x) = 0$ whenever $K = 0$ and $c'(0) > \mathbb{E}(Z)$. Then (9) and (10) imply that for $\tau \leq \tau^*$ the equilibrium will have $K = 0$, which implies $\phi = 0$, which means τ will remain constant. The threshold of the illiquidity trap could be $\tau^{IT} = \tau^*$ or, in case $c'(0) > V(\tau^*)$, $\tau^{IT} > \tau^*$.
2. If $\phi(K, x) > 0$ for all K, x , then by assumption $\phi > 0$. If $c'(0) < \mathbb{E}(Z)$, then equation (10) implies that $K(\tau) > 0$ for any τ and then if $\phi(K, x) > 0$ whenever $K > 0$, this also implies $\phi > 0$. Therefore ex-post signals will always arrive at a positive rate.

Proof of Proposition 2

Assume w.l.o.g. that $g_t > 0.5$ so $\tau_t = 4(g_t - 0.5)(\bar{\mu} - 0.5)$. Let n be the number of ex-post signals that agents observe between t and t' . n is a random variable with a Poisson distribution with parameter $\bar{\phi} = \int_t^{t'} \phi_s ds$ and let r be the fraction of these observations that are either $\{s = A, z = H\}$ or $\{s = B, z = L\}$. Then by (13):

$$g_{t'} = \frac{(\bar{\omega}^r (1 - \bar{\omega})^{1-r})^n g_t}{(\bar{\omega}^r (1 - \bar{\omega})^{1-r})^n g_t + ((1 - \bar{\omega})^r \bar{\omega}^{1-r})^n (1 - g_t)} \quad (18)$$

Now compute $\mathbb{E}(\tau_{t'})$

$$\begin{aligned} \mathbb{E}(\tau_{t'}) &= 4(\bar{\mu} - 0.5) \mathbb{E}(|g_{t'} - 0.5|) \\ &> 4(\bar{\mu} - 0.5) \mathbb{E}(g_{t'} - 0.5) \\ &= 4(\bar{\mu} - 0.5) (g_t - 0.5) \\ &= \tau_t \end{aligned}$$

This first line is true by definition. The second line follows from the fact that if there is no illiquidity trap then $\bar{\phi} > 0$. Since the Poisson distribution is unbounded, there is a strictly

positive probability of n large enough and r low enough that, using (18), $g_{t'} < 0.5$. The third line follows from the law of iterated expectations and the last is again by definition.

Proof of Proposition 3

Let $M = \{m_j\}_{j=1}^{\infty}$ be a sequence where $m_j = 1$ if the j th observation $\{s, z\} \in \{\{A, H\} \cup (BL)\}$ and $m_j = 0$ otherwise. Given that μ is the same in both economies, the probabilities over M are the same in both economies and therefore so are probabilities over the paths of beliefs. Furthermore, let $F = \{f_j\}_{j=1}^{\infty}$ where $f_j \sim_{iid} U[0, 1]$ and for each M construct the arrival time t_j of the j th observation as $t_j = t_{j-1} - \frac{1}{\phi_{j-1}} \log(1 - f_j)$ where $\phi_{j-1} = \phi(K(\tau_{j-1}), x(\tau_{j-1}))$ and τ_{j-1} is the informativeness that results from the $j - 1$ th observation. Since ϕ is increasing in K and for given beliefs K will be higher in the economy with lower $c'(\cdot)$, t_j will be lower in the economy with lower $c'(\cdot)$. Therefore the result follows because by Proposition 2 informativeness increases over time.

Proof of Proposition 4

1. Assume w.l.o.g. $g_t > 0.5$. Equation (13) implies that after an unlikely observation, beliefs are:

$$g_{t'} = \frac{(1 - \bar{\omega}) g_t}{(1 - \bar{\omega}) g_t + \bar{\omega} (1 - g_t)} < g_t \quad (19)$$

If $g_{t'} \geq 0.5$, this immediately implies $\tau_{t'} < \tau_t$. If $g_{t'} < 0.5$, $\tau_{t'} < \tau_t$ holds unless

$$0.5 - g_{t'} \geq g_t - 0.5 \quad (20)$$

Assume (20) holds. Then:

$$\begin{aligned} 1 &\geq g_t + \frac{g_t}{g_t + \frac{\bar{\omega}}{1-\bar{\omega}}(1-g_t)} \\ &\geq \frac{\tau^*}{4(\bar{\mu} - 0.5)} + 0.5 + \frac{0.5 + \frac{\tau^*}{4(\bar{\mu} - 0.5)}}{0.5 + \frac{\tau^*}{4(\bar{\mu} - 0.5)} + \frac{\bar{\omega}}{1-\bar{\omega}} \left(0.5 - \frac{\tau^*}{4(\bar{\mu} - 0.5)}\right)} \\ \Rightarrow \tau^* &\leq \bar{\tau} \frac{\left(\frac{\bar{\omega}}{1-\bar{\omega}}\right)^{0.5} - 1}{\left(\frac{\bar{\omega}}{1-\bar{\omega}}\right)^{0.5} + 1} \end{aligned}$$

which contradicts (14). The second inequality follows from the assumption that $\tau_t \geq \tau^*$ and the last is just rearranging and using $\bar{\tau} \equiv 2(\bar{\mu} - 0.5)$. Hence (20) cannot hold and $\tau'_t < \tau_t$ after an unlikely observation. Since by Lemma 2 liquidity is increasing in τ , the result follows.

2. Assume w.l.o.g that $g_t > 0.5$. After n unlikely observations,

$$g'_t = \frac{g_t}{g_t + \left(\frac{\bar{\omega}}{1-\bar{\omega}}\right)^n (1 - g_t)}$$

so for

$$n > \frac{\log\left(\frac{2(\bar{\mu}-0.5)-\tau^*}{2(\bar{\mu}-0.5)+\tau^*}\right) + \log\left(\frac{2(\bar{\mu}-0.5)+\tau_t}{2(\bar{\mu}-0.5)-\tau_t}\right)}{\log\left[\frac{\bar{\omega}}{1-\bar{\omega}}\right]} \quad (21)$$

then $g'_t < \frac{\tau^*}{4(\bar{\mu}-0.5)} + 0.5$. If $g'_t > 0.5$, this implies $\tau'_t < \tau^*$. Furthermore, the argument from part 1 shows that if $g_t > 0.5$ and after the next observation $g'_t < 0.5$ then $\tau'_t < \tau^*$, so the inequality holds in this case as well.

3. This follows directly from the fact that the right hand side of (21) is increasing in τ_t .

Proof of Proposition 5

Using, (19), informativness after an unlikely observation is:

$$\begin{aligned} \tau'_t &= 4 \left(\frac{\frac{\tau_t}{4(\bar{\mu}-0.5)} + 0.5}{\frac{\tau_t}{4(\bar{\mu}-0.5)} + 0.5 + \frac{\bar{\omega}}{1-\bar{\omega}} \left(0.5 - \frac{\tau_t}{4(\bar{\mu}-0.5)}\right)} - 0.5 \right) (\bar{\mu} - 0.5) \\ &= \left(\frac{\tau_t - (2\bar{\omega} - 1) \bar{\tau}}{1 - (2\bar{\omega} - 1) \frac{\tau_t}{\bar{\tau}}} \right) \end{aligned}$$

This expression is decreasing in $\bar{\tau} = 2(\bar{\mu} - 0.5)$ and in $\bar{\omega} = \bar{\mu}\gamma + (1 - \bar{\mu})(1 - \gamma)$ and therefore is decreasing in $\bar{\mu}$.

Proof of Proposition 6

1. Equation (13) implies that, letting N_t be the cumulative number of observations up to and including time t and M_t be the number of those that are $\{s, z\} \in \{\{A, H\} \cup \{B, L\}\}$,

beliefs at t are

$$g_t = \frac{\bar{\omega}^{M_t} (1 - \bar{\omega})^{N_t - M_t} g_0}{\bar{\omega}^{M_t} (1 - \bar{\omega})^{N_t - M_t} g_0 + (1 - \bar{\omega})^{M_t} \bar{\omega}^{N_t - M_t} (1 - g_0)} \quad (22)$$

If there is no illiquidity trap then $\lim_{t \rightarrow \infty} N_t = \infty$ and the law of large numbers implies that $\frac{M_t}{N_t}$ converges almost surely to either $\bar{\omega}$ or $1 - \bar{\omega}$. Equation (22) implies that g_{t+1} converges almost surely to either 1 or 0, which implies $\tau_t \rightarrow \bar{\tau}$. The first order condition (10) implies $K_t \rightarrow \bar{K}$.

2. Immediate from the definition of an illiquidity trap.
3. Suppose $\tau > \tau^{IT}$ and assume that w.l.o.g $g > 0.5$. Suppose it were the case that an observation of $\{s, z\} \in \{\{A, L\} \cup \{B, H\}\}$ led to $g' < 0.5$. Using (13) we have

$$g' = \frac{(1 - \bar{\omega})g}{(1 - \bar{\omega})g + \bar{\omega}(1 - g)}$$

and replacing $\tau = 4(g - 0.5)(\bar{\mu} - 0.5)$ and $\tau' = 4(0.5 - g')(\bar{\mu} - 0.5)$, this implies

$$\tau' = \frac{2\bar{\tau}\bar{\omega} - (\tau + \bar{\tau})}{2\bar{\omega} - (2\bar{\omega} - 1)\left(\frac{\tau}{\bar{\tau}} + 1\right)} \quad (23)$$

If $\tau \geq \tau^{IT}$ and $\tau' \geq \tau^{IT}$, this implies

$$\tau^{IT} \leq \bar{\tau} \frac{\left(\frac{\bar{\omega}}{1 - \bar{\omega}}\right)^{0.5} - 1}{\left(\frac{\bar{\omega}}{1 - \bar{\omega}}\right)^{0.5} + 1}$$

which contradicts (14). Therefore if the illiquidity trap is large it cannot be the case that beliefs g cross 0.5 while remaining outside the illiquidity trap. Assume w.l.o.g. that $g_0 > 0.5$, and let $g^{IT} = \frac{\tau^{IT}}{4(\bar{\mu} - 0.5)} + 0.5$ be the beliefs at the edge of the illiquidity trap. Since g_t cannot cross the illiquidity trap, then

$$\Pr(g_t < 1 - g^{IT}) = 0 \quad \forall t \quad (24)$$

Furthermore, as long as g_t remains outside the illiquidity trap, the number of observations converges to infinity, so by the law of large numbers g_t must converge to either 0 or 1. Since it cannot converge to 0 because that would involve crossing the trap, then

for any ϵ :

$$\lim_{t \rightarrow \infty} \Pr(g_t > 1 - \epsilon | g_t > g^{IT}) = 1 \quad (25)$$

By the law of iterated expectations, $\mathbb{E}(g_t) = g_0$, which implies that

$$g_0 = \mathbb{E}(g_t | g_t \geq 1 - \epsilon) \Pr(g_t \geq 1 - \epsilon) + \mathbb{E}(g_t | g_t \in (g^{IT}, 1 - \epsilon)) \Pr(g_t \in (g^{IT}, 1 - \epsilon)) + \\ + \mathbb{E}(g_t | g_t \in [1 - g^{IT}, g^{IT}]) \Pr(g_t \in [1 - g^{IT}, g^{IT}]) + \mathbb{E}(g_t | g_t < 1 - g^{IT}) \Pr(g_t < 1 - g^{IT})$$

Taking limits and using (24) and (25):

$$g_0 = \lim_{t \rightarrow \infty} [\mathbb{E}(g_t | g_t \geq 1 - \epsilon) \Pr(g_t \geq 1 - \epsilon) + \mathbb{E}(g_t | g_t \in [1 - g^{IT}, g^{IT}]) \Pr(g_t \in [1 - g^{IT}, g^{IT}])]$$

and therefore

$$g_0 \leq \lim_{t \rightarrow \infty} \Pr(g_t \geq 1 - \epsilon) + g^{IT} \left(1 - \lim_{t \rightarrow \infty} \Pr(g_t \geq 1 - \epsilon)\right) \\ g_0 \geq (1 - \epsilon) \lim_{t \rightarrow \infty} \Pr(g_t \geq 1 - \epsilon) + (1 - g^{IT}) \left(1 - \lim_{t \rightarrow \infty} \Pr(g_t \geq 1 - \epsilon)\right)$$

Rearranging:

$$\frac{\tau_0 - \tau^*}{\bar{\tau} - \tau^*} \leq \lim_{t \rightarrow \infty} \Pr[\tau_t \geq (1 - 2\epsilon) \bar{\tau}] \leq \frac{\tau_0 + \tau^*}{(1 - 2\epsilon) \bar{\tau} + \tau^*}$$

4. Formula (22) can be rewritten as

$$g_t = \frac{\bar{\omega}^{D_t} g_0}{\bar{\omega}^{D_t} g_0 + (1 - \bar{\omega})^{D_t} (1 - g_0)}$$

where $D_t = 2M_t - N_t$ takes integer values. Suppose $g_0 = g^{IT} + \epsilon$ for $\epsilon > 0$. Then for any positive D_t , $g_t > g^{IT}$ so beliefs are outside the illiquidity trap. For $D_t = -1$, then equation (23) applies. If the illiquidity trap is not large and ϵ is small enough, this implies that $g_t < 0.5$ and beliefs are also outside the illiquidity trap. For $D_t < -1$, then g_t is even lower so it's also outside the illiquidity trap. Therefore no sequence of observations can lead the economy into the trap. Conversely, if $g_0 = \bar{\omega}$, then for $D_t = 1$ $g_t = 0.5$, which must be inside the trap and hence the trap is reached with positive probability.

Proof of Lemma 4

By (16) after an interval of length T with no observations starting at time t ,

$$g_{t+T} = 0.5 + \frac{g_t - 0.5}{e^{2\delta T}}$$

This implies that for any g_t if $T > \frac{1}{2\delta} \log\left(\frac{\bar{\tau}}{\tau^*}\right)$, then $\tau_{t+T} < \tau^*$. Since in a Poisson process an interval of length T with no observations happens with positive probability, then by the law of large numbers it happens almost surely. After it does, since there is an illiquidity trap no more ex-post signal are observed and $\tau_t \rightarrow 0$.

Proof of Proposition 7

Assume w.l.o.g. $g_t > 0.5$. By (16)

$$g_{t+T} = 0.5 + \frac{g_t - 0.5}{e^{2\delta T}} > 0.5$$

Equation (13) implies that beliefs are

$$g'_{t+T}(m=1) = \frac{\bar{\omega} g_{t+T}}{\bar{\omega}^m g_{t+T} + (1-\bar{\omega})^m (1-g_{t+T})}$$

if $\{s = A, z = L \text{ or } s = B, z = H\}$ is observed and

$$g'_{t+T}(m=0) = \frac{(1-\bar{\omega}) g_{t+T}}{(1-\bar{\omega}) g_{t+T} + \bar{\omega}^{1-m} (1-g_{t+T})}$$

otherwise. The fact that $g_{t+T} > 0.5$ therefore implies that

$$\tau'_{t+T}(m=1) > \tau'_{t+T}(m=0)$$

Taking expectations:

$$\begin{aligned}
\mathbb{E}(\tau'_{t+T} | \mu_{t+T} = \mu_t) &= \tau'_{t+T}(m=1) \Pr(m=1 | \mu_{t+T} = \bar{\mu}) \Pr(\mu_t = \bar{\mu}) \\
&\quad + \tau'_{t+T}(m=1) \Pr(m=1 | \mu_{t+T} = 1 - \bar{\mu}) \Pr(\mu_t = 1 - \bar{\mu}) \\
&= \tau'_{t+T}(m=0) \Pr(m=0 | \mu_{t+T} = \bar{\mu}) \Pr(\mu_t = \bar{\mu}) \\
&\quad + \tau'_{t+T}(m=0) \Pr(m=0 | \mu_{t+T} = 1 - \bar{\mu}) \Pr(\mu_t = 1 - \bar{\mu}) \\
&= \tau'_{t+T}(m=1) [\bar{\omega}g_t + (1 - \bar{\omega})(1 - g_t)] + \tau'_{t+T}(m=0) [(1 - \bar{\omega})g_t + \bar{\omega}(1 - g_t)]
\end{aligned} \tag{26}$$

Similarly

$$\mathbb{E}(\tau'_{t+T} | \mu_{t+T} \neq \mu_t) = \tau'_{t+T}(m=1) [\bar{\omega}(1 - g_t) + (1 - \bar{\omega})g_t] + \tau'_{t+T}(m=0) [(1 - \bar{\omega})(1 - g_t) + \bar{\omega}g_t] \tag{27}$$

Subtracting (27) from (26):

$$\mathbb{E}(\tau'_{t+T} | \mu_{t+T} = \mu_t) - \mathbb{E}(\tau'_{t+T} | \mu_{t+T} \neq \mu_t) = (\tau'_{t+T}(m=1) - \tau'_{t+T}(m=0)) (2\bar{\omega} - 1) (2g_t - 1) > 0$$

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