Complex Dynamics of Autonomous Communication Networks and the Intelligent Communication Paradigm

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Abstract. Dynamics of arbitrary communication system is analysed as unreduced interaction process. The applied generalised, universally nonperturbative method of effective potential reveals the phenomenon of dynamic multivaluedness of competing system configurations forced to permanently replace each other in a causally random order, which leads to universally defined dynamical chaos, complexity, fractality, self-organisation, and adaptability. We demonstrate the origin of huge, exponentially high efficiency of the unreduced, complex network dynamics and specify the universal symmetry of complexity as the fundamental guiding principle for creation and control of such qualitatively new kind of networks and devices.

1 Introduction

Any communication system can be considered as a particular case of general dynamical system formed by many interacting units. If the system components are permitted to freely interact without *strict* external control, then such unreduced interaction process leads inevitably to complex-dynamical, essentially nonlinear and chaotic structure emergence, or generalised (dynamically multivalued) self-organisation [1–3], extending the conventional, basically regular self-organisation concept. The usual technology and communication practice and paradigm rely, however, on very strong human control and totally regular, predictable dynamics of controlled systems and environment, where unpredictable events can only take the form of undesirable failures or noise.

Growing volumes and complication of communication system links and functions lead inevitably to increasing probability of undesirable deviations from the pre-programmed regular behaviour, largely compromising its supposed advantages. On the other hand, such increasingly useful properties as intrinsic system creativity and autonomous adaptability to changing environment and individual user demands should certainly involve another, much less regular and more diverse kind of behaviour. In this paper we analyse these issues in a rigorous way by presenting the unreduced, nonperturbative analysis of an arbitrary system of interacting entities and show that such *unreduced interaction process* possesses the natural, dynamically derived properties of chaoticity, creativity (autonomous structure formation ability), adaptability, and exponentially high efficiency, which can be consistently unified into the totally universal concept of *dynamic complexity* [1]. This concept and particular notions it unifies represent essential extension with respect to respective results of the usual theory always using one or another version of perturbation theory that strongly reduces real interaction processes and leads inevitably to regular kind of dynamics (even in its versions of chaoticity). We shall specify these differences in our analysis and demonstrate the key role of unreduced, interaction-driven complexity, chaoticity and self-organisation in the superior operation properties, as it has already been demonstrated for a large scope of applications [1–8].

We start, in Sect. 2, with a mathematical demonstration of the fact that the unreduced interaction process within *any* real system leads to intrinsic, genuine, and omnipresent *randomness* in the system behaviour, which can be realised in a few characteristic regimes and leads to the *universally* defined *dynamic complexity.* We outline the change in strategy and practice of communication system construction and use, which follows from such unreduced analysis of system interactions. The *universality* of our analysis is of special importance here, since the results can be applied at various *naturally entangled* levels of communication system operation. In particular, we demonstrate the complex-dynamic origin of the huge, *exponentially high efficiency growth* of the unreduced, causally random system dynamics, with respect to the standard, basically regular system operation (Sect. 3). Finally, the dynamically derived, *universal symmetry, or conservation, of complexity* is introduced as the new guiding principle and tool of complex system dynamics that should replace usual, regular programming. The *paradigm of intelligent communication systems* is thus specified, since we show also [1, 5] that the property of *intelligence* can be consistently described as high enough levels of the unreduced dynamic complexity. This "intelligent communication" is the most complete, inevitable realisation, and in fact a synonym, of the truly *autonomous* communication dynamics and its expected properties.

2 Complex Dynamics of Unreduced Interaction Process

We begin with a general expression of multi-component system dynamics (or many-body problem), called here *existence equation*, fixing the fact of interaction between the system components, and generalising various model equations:

$$\left\{ \sum_{k=0}^{N} \left[h_k \left(q_k \right) + \sum_{l>k}^{N} V_{kl} \left(q_k, q_l \right) \right] \right\} \Psi \left(Q \right) = E \Psi \left(Q \right) \ , \tag{1}$$

where $h_k \left(q_k \right)$ is the "generalised Hamiltonian" of the k-th system component in the absence of interaction, q_k is the degree(s) of freedom of the k-th component (expressing its "physical nature"), $V_{kl} \left(q_k, q_l \right)$ is the (generally arbitrary) interaction potential between the k-th and l-th components, $\Psi \left(Q \right)$ is the system state-function, $Q \equiv \{q_0, q_1, ..., q_N\}$, E is the eigenvalue of the generalised Hamiltonian, and summations are performed over all (N) system components. The generalised Hamiltonian, eigenvalues, and interaction potential represent a suitable measure of dynamic complexity defined below and encompassing practically all “observable” quantities (action, energy, momentum, current, etc.) at any level of dynamics. Therefore (1) can express the unreduced interaction configuration at any level of communication network of arbitrary initial structure. It can also be presented in a particular form of time-dependent equation by replacing the generalised Hamiltonian eigenvalue E with the partial time derivative operator (for the case of explicit interaction potential dependence on time).

One can separate one of the degrees of freedom, e.g. $q_0 \equiv \xi$, corresponding to a naturally selected, usually “system-wide” entity, such as “embedding” configuration (system of coordinates) or common “transmitting agent”:

$$\left\{ h_0\left(\xi\right) + \sum_{k=1}^{N} \left[h_k\left(q_k\right) + V_{0k}\left(\xi, q_k\right) \right] + \sum_{l>k}^{N} V_{kl}\left(q_k, q_l\right) \right\} \Psi\left(\xi, Q\right) = E \Psi\left(\xi, Q\right), \tag{2}$$

where now $Q \equiv \{q_1, ..., q_N\}$ and $k, l \geq 1$.

We then express the problem in terms of known free-component solutions for the “functional”, internal degrees of freedom of system elements ($k \geq 1$):

$$h_k\left(q_k\right) \varphi_{kn_k}\left(q_k\right) = \varepsilon_{n_k} \varphi_{kn_k}\left(q_k\right) \ , \tag{3}$$

$$\Psi\left(\xi, Q\right) = \sum_n \psi_n\left(\xi\right) \varphi_{1n_1}\left(q_1\right) \varphi_{2n_2}\left(q_2\right) ... \varphi_{Nn_N}\left(q_N\right) \equiv \sum_n \psi_n\left(\xi\right) \Phi_n\left(Q\right), \tag{4}$$

where $\{\varepsilon_{n_k}\}$ are the eigenvalues and $\{\varphi_{kn_k}\left(q_k\right)\}$ eigenfunctions of the k-th component Hamiltonian $h_k\left(q_k\right)$, forming the complete set of orthonormal functions, $n \equiv \{n_1, ..., n_N\}$ runs through all possible eigenstate combinations, and $\Phi_n\left(Q\right) \equiv \varphi_{1n_1}\left(q_1\right) \varphi_{2n_2}\left(q_2\right) ... \varphi_{Nn_N}\left(q_N\right)$ by definition. The system of equations for $\{\psi_n\left(\xi\right)\}$ is obtained then in a standard way, using the eigen-solution orthonormality (e.g. by multiplication by $\Phi_n^*\left(Q\right)$ and integration over Q):

$$\begin{aligned} \left[h_0\left(\xi\right) + V_{00}\left(\xi\right)\right] \psi_0\left(\xi\right) + \sum_n V_{0n}\left(\xi\right) \psi_n\left(\xi\right) &= \eta \psi_0\left(\xi\right) \\ \left[h_0\left(\xi\right) + V_{nn}\left(\xi\right)\right] \psi_n\left(\xi\right) + \sum_{n' \neq n} V_{nn'}\left(\xi\right) \psi_{n'}\left(\xi\right) &= \eta_n \psi_n\left(\xi\right) - V_{n0}\left(\xi\right) \psi_0\left(\xi\right), \end{aligned} \tag{5}$$

where $n, n' \neq 0$ (also below), $\eta \equiv \eta_0 = E - \varepsilon_0$, $\eta_n = E - \varepsilon_n$, $\varepsilon_n = \sum_k \varepsilon_{n_k}$,

$$V_{nn'}\left(\xi\right) = \sum_k \left[V_{k0}^{nn'}\left(\xi\right) + \sum_{l>k} V_{kl}^{nn'} \right] , \tag{6}$$

$$V_{k0}^{nn'}\left(\xi\right) = \int_{\Omega_Q} dQ \Phi_n^*\left(Q\right) V_{k0}\left(q_k, \xi\right) \Phi_{n'}\left(Q\right) \ , \tag{7}$$

$$V_{kl}^{nn'}\left(\xi\right) = \int_{\Omega_Q} dQ \Phi_n^*\left(Q\right) V_{kl}\left(q_k, q_l\right) \Phi_{n'}\left(Q\right) \ , \tag{8}$$

and we have separated the equation for $\psi_0(\xi)$ describing the generalised "ground state" of the system elements, i. e. the state with minimum complexity. The obtained system of equations expresses the same problem as the starting equation (2), but now in terms of "natural", dynamic variables, and therefore it can be obtained for various starting models, including time-dependent and formally "nonlinear" ones (see below for a rigorous definition of *essential* nonlinearity).

We try now to approach the solution of the "nonintegrable" system of equations (5) with the help of the generalised effective, or optical, potential method [9], where one expresses $\psi_n(\xi)$ through $\psi_0(\xi)$ from the equations for $\psi_n(\xi)$ using the standard Green function technique and then inserts the result into the equation for $\psi_0(\xi)$, obtaining thus the *effective existence equation* that contains *explicitly* only "integrable" degrees of freedom (ξ) [1–8]:

$$h_0(\xi)\,\psi_0(\xi) + V_{\text{eff}}(\xi;\eta)\,\psi_0(\xi) = \eta\psi_0(\xi) \ , \tag{9}$$

where the operator of *effective potential (EP)*, $V_{\text{eff}}(\xi;\eta)$, is given by

$$V_{\text{eff}}(\xi;\eta) = V_{00}(\xi) + \hat{V}(\xi;\eta)\,, \quad \hat{V}(\xi;\eta)\,\psi_0(\xi) = \int\limits_{\Omega_\xi} d\xi' V(\xi,\xi';\eta)\psi_0(\xi')\,, \tag{10}$$

$$V(\xi,\xi';\eta) = \sum_{n,i} \frac{V_{0n}(\xi)\,\psi^0_{ni}(\xi)\,V_{n0}(\xi')\,\psi^{0*}_{ni}(\xi')}{\eta - \eta^0_{ni} - \varepsilon_{n0}} \ , \quad \varepsilon_{n0} \equiv \varepsilon_n - \varepsilon_0 \ , \tag{11}$$

and $\left\{\psi^0_{ni}(\xi)\right\}$, $\left\{\eta^0_{ni}\right\}$ are complete sets of eigenfunctions and eigenvalues of a *truncated* system of equations:

$$\left[h_0(\xi) + V_{nn}(\xi)\right]\psi_n(\xi) + \sum_{n'\neq n} V_{nn'}(\xi)\psi_{n'}(\xi) = \eta_n\psi_n(\xi) \ . \tag{12}$$

One should use now the eigenfunctions, $\{\psi_{0i}(\xi)\}$, and eigenvalues, $\{\eta_i\}$, of the formally "integrable" equation (9) to obtain other state-function components:

$$\psi_{ni}(\xi) = \hat{g}_{ni}(\xi)\,\psi_{0i}(\xi) \equiv \int\limits_{\Omega_\xi} d\xi' g_{ni}(\xi,\xi')\,\psi_{0i}(\xi') \ , \tag{13}$$

$$g_{ni}(\xi,\xi') = V_{n0}(\xi') \sum_{i'} \frac{\psi^0_{ni'}(\xi)\,\psi^{0*}_{ni'}(\xi')}{\eta_i - \eta^0_{ni'} - \varepsilon_{n0}} \ , \tag{14}$$

and the total system state-function, $\Psi(q_0, q_1, ..., q_N) = \Psi(\xi, Q)$ (see (4)):

$$\Psi(\xi,Q) = \sum_i c_i \left[\Phi_0(Q) + \sum_n \Phi_n(Q)\,\hat{g}_{ni}(\xi)\right]\psi_{0i}(\xi) \ , \tag{15}$$

where the coefficients c_i should be found from the state-function matching conditions at the boundary where interaction effectively vanishes. The measured quantity, generalised as structure density $\rho(\xi, Q)$, is obtained as the state-function squared modulus, $\rho(\xi, Q) = |\Psi(\xi, Q)|^2$ (for "wave-like" complexity levels), or as the state-function itself, $\rho(\xi, Q) = \Psi(\xi, Q)$ (for "particle-like" structures) [1].

Since the EP expression in the effective problem formulation (9)-(11) depends essentially on the eigen-solutions to be found, the problem remains "nonintegrable" and formally equivalent to the initial formulation (1), (2), (5). However, it is the effective version of a problem that leads to its unreduced solution and reveals the nontrivial properties of the latter [1–8]. The most important property of the unreduced interaction result (9)-(15) is its *dynamic multivaluedness* meaning that one has a *redundant* number of different but individually complete, and therefore *mutually incompatible*, problem solutions, each of them describing an *equally real* system configuration. We call each such locally complete solution (and real system configuration) *realisation* of the system and problem. Plurality of system realisations follows from the unreduced EP expressions due to the nonlinear and self-consistent dependence on the solutions to be found, reflecting the physically real and evident plurality of possible combinations of interacting eigen-modes [1–8]. It is important that dynamic multivaluedness emerges only in the unreduced problem formulation, whereas the standard theory, including EP method applications (see e.g. [9]) and the scholar "science of complexity" (theory of chaos, self-organisation, etc.), resorts invariably to one or another version of perturbation theory, whose approximation, used to obtain an "exact", closed-form solution, totally "kills" redundant solutions by eliminating just those nonlinear dynamical links and retains *only one*, "averaged" solution, usually expressing only *small* deviations from initial, pre-interaction configuration. This *dynamically single-valued*, or *unitary*, problem reduction forms the basis of the whole canonical science paradigm.

Since we have many *incompatible* system realisations that tend to appear from the same, driving interaction, we obtain the key property of *causal, or dynamic, randomness* in the form of permanently *changing* realisations that replace each other in the *truly random* order. Therefore dynamic multivaluedness, rigorously derived simply by unreduced, correct solution of a real many-body (interaction) problem, provides the *universal dynamic origin* and *meaning* of the *omnipresent, unceasing* randomness in the system behaviour, also called *(dynamical) chaos* (it is essentially different from any its unitary version, reduced to an "involved regularity" or *postulated* external "noise"). This means that the genuine, truly complete *general solution* of an arbitrary problem (describing a *real* system behaviour) has the form of *dynamically probabilistic* sum of measured quantities for particular system realisations:

$$\rho(\xi, Q) = \sum_{r=1}^{N_{\Re}}{}^{\oplus} \rho_r(\xi, Q) \ , \tag{16}$$

where summation is performed over all system realisations, $N_{\Re}$ is their number (its maximum value is equal to the number of system components, $N_{\Re} = N$), and the sign $\oplus$ designates the special, dynamically probabilistic meaning of the sum described above. It implies that any measured quantity (16) is *intrinsically unstable* and its current value *will* unpredictably change to another one, corresponding to another, *randomly* chosen realisation. Such kind of behaviour is readily observed in nature and actually explains the living organism behaviour [1, 4, 5], but is thoroughly avoided in the unitary theory and technological systems (including communication networks), where it is correctly associated with linear "noncomputability" and technical failure (we shall consider below this *limiting* regime of real system dynamics). Therefore the universal dynamic multivaluedness thus revealed by the rigorous problem solution forms the fundamental basis for the transition to "bio-inspired" and "intelligent" kind of operation in artificial, technological and communication systems, where causal randomness can be transformed from an obstacle to a qualitative advantage (Sect. 3).

The rigorously derived randomness of the generalised EP formalism (9)-(16) is accompanied by the *dynamic definition of probability*. Because the elementary realisations are equivalent in their "right to appear", the dynamically obtained, *a priori probability*, α_r, of an elementary realisation emergence is given by

$$\alpha_r = \frac{1}{N_{\Re}} \ , \quad \sum_r \alpha_r = 1 \ . \tag{17}$$

However, a real observation may fix uneven groups of elementary realisations because of their multivalued self-organisation (see below). Therefore the dynamic probability of observation of such general, compound realisation is determined by the number, N_r, of elementary realisations it contains:

$$\alpha_r\left(N_r\right) = \frac{N_r}{N_{\Re}} \quad \left(N_r = 1, ..., N_{\Re}; \ \sum_r N_r = N_{\Re}\right), \quad \sum_r \alpha_r = 1 \ . \tag{18}$$

An expression for *expectation value*, $\rho_{\exp}\left(\xi, Q\right)$, can easily be constructed from (16)-(18) for statistically long observation periods:

$$\rho_{\exp}\left(\xi, Q\right) = \sum_r \alpha_r \rho_r\left(\xi, Q\right) \ . \tag{19}$$

It is important, however, that our dynamically derived randomness and probability need not rely on such "statistical", empirically based result, so that the basic expressions (16)-(18) remain valid even for a *single* event of realisation emergence and *before* any event happens at all.

The realisation probability distribution can be obtained in another way, involving *generalised wavefunction* and *Born's probability rule* [1, 3, 5, 8, 10]. The wavefunction describes the system state during its transition between "regular", "concentrated" realisations and constitutes a particular, "intermediate" realisation with spatially extended and "loose" (chaotically changing) structure, where the system components transiently disentangle before forming the next "regular" realisation. The intermediate, or "main", realisation is explicitly obtained in the unreduced EP formalism [1, 3, 5, 8, 10] and provides, in particular, the causal, totally realistic version of the quantum-mechanical wavefunction at the lowest, "quantum" levels of complexity. The "Born probability rule", now also causally derived and extended to any level of world dynamics, states that the realisation probability distribution is determined by the wavefunction values (their squared modulus for the "wave-like" complexity levels) for the respective system configurations. The generalised wavefunction (or distribution function) satisfies the universal Schrödinger equation (Sect. 3), rigorously derived from the dynamic quantization of complex dynamics [1, 3, 5, 8, 10], while Born's probability rule follows from the *dynamic* "boundary conditions" mentioned in connection to the state-function expression (15) and actually satisfied just during each system transition between a "regular" realisation and the extended wavefunction state. Note also that it is this "averaged", weak-interaction state of the wavefunction, or "main" realisation, that actually remains in the dynamically single-valued, one-realisation "model" and "exact-solution" paradigm of the unitary theory, which explains both its partial success and fundamental limitations.

Closely related to the dynamic multivaluedness is the property of *dynamic entanglement* between the interacting components, described in (15) by the dynamically weighted products of state-function components depending on various degrees of freedom (ξ, Q). It provides a rigorous expression of the tangible *quality* of the emerging system structure and is absent in unitary models. The obtained *dynamically multivalued entanglement* describes a "living" structure, permanently changing and probabilistically *adapting* its configuration, which provides a well-specified basis for "bio-inspired" technological solutions. The properties of dynamically multivalued entanglement and adaptability are further amplified due to the extended *probabilistic fractality* of the unreduced general solution [1, 4, 5], obtained by application of the same EP method to solution of the truncated system of equations (12) used in the first-level EP expression (11).

We can now consistently and universally define the unreduced *dynamic complexity*, C, of any real system (or interaction process) as arbitrary growing function of the total number of *explicitly obtained* system realisations, $C = C(N_{\Re})$, $dC/dN_{\Re} > 0$, or the rate of their change, equal to zero for the unrealistic case of only one system realisation, $C(1) = 0$. Suitable examples are provided by $C(N_{\Re}) = C_0 \ln N_{\Re}$, generalised energy/mass (proportional to the temporal rate of realisation change), and momentum (proportional to the spatial rate of realisation emergence) [1, 5, 8, 10]. It becomes clear now that the whole *dynamically single-valued* paradigm and results of the canonical theory (including its versions of "complexity" and *imitations* of "multi-stability" in *abstract*, mathematical "spaces") correspond to exactly *zero* value of the unreduced dynamic complexity, which is equivalent to the effectively zero-dimensional, point-like projection of reality in the "exact-solution" perspective.

Correspondingly, *any* dynamically single-valued "model" is strictly regular and *cannot* possess any true, intrinsic randomness (chaoticity), which should instead be introduced artificially (and inconsistently), e.g. as a *regular* "amplification" of a "random" (by convention) *external* "noise" or "measurement error". By contrast, our unreduced dynamic complexity is practically synonymous to the equally universally defined and genuine *chaoticity* (see above), since multiple system realisations, appearing and disappearing only in the *real* space (and *forming* thus its tangible, changing structure [1, 3, 5, 8]), are redundant (mutually incompatible), which is the origin of *both* complexity and chaoticity. The genuine dynamical chaos thus obtained has its complicated internal structure (contrary to the ill-defined unitary "stochasticity") and always contains *partial regularity*, which is dynamically, inseparably entangled with truly random elements.

The universal dynamic complexity, chaoticity, and related properties involve the *essential, or dynamic, nonlinearity* of the unreduced problem solution and corresponding system behaviour. It is provided by the naturally formed dynamical links of the developing interaction process, as they are expressed in the (eventually fractal) EP dependence on the problem solutions to be found (see (9)-(11)). It is the *dynamically emerging* nonlinearity, since it appears even for a formally "linear" initial problem expression (1)-(2), (5), whereas the usual, mechanistic "nonlinearity" is but a perturbative approximation to the essential nonlinearity of the unreduced EP expressions. The essential nonlinearity leads to the irreducible *dynamic instability* of any system state (realisation), since both are determined by the same dynamic feedback mechanism.

Universality of our description leads, in particular, to the unified understanding of the whole diversity of existing dynamical regimes and types of system behaviour [1, 2, 5]. One standard, limiting case of complex (multivalued) dynamics, called *uniform, or global, chaos*, is characterised by sufficiently different realisations with a homogeneous distribution of probabilities (i.e. $N_r \approx 1$) and $\alpha_r \approx 1/N_{\Re}$ for all r in (18)) and is obtained when the major parameters of interacting entities (suitably represented by frequencies) are similar to each other (which leads to a "strong conflict of interests" and resulting "deep disorder"). The complementary limiting regime of *multivalued self-organisation, or self-organised criticality (SOC)* emerges for sufficiently different parameters of interacting components, so that a small number of relatively rigid, low-frequency components "enslave" a hierarchy of high-frequency and rapidly changing, but configurationally similar, realisations (i.e. $N_r \sim N_{\Re}$ and realisation probability distribution is highly inhomogeneous). The difference of this extended, multivalued self-organisation (and SOC) from the usual, unitary version is essential: despite the rigid *external* shape of the system configuration in this regime, it contains the intense "internal life" and *chaos* of permanently changing "enslaved" realisations (which are *not* superposable unitary "modes"). Another important advance with respect to the unitary "science of complexity" is that the unreduced, multivalued self-organisation unifies the extended versions of a whole series of separated unitary "models", including SOC, various versions of "synchronisation", "control of chaos", "attractors", and "mode locking". All the intermediate dynamic regimes between those two limiting cases of uniform chaos and multivalued SOC (as well as their multi-level, fractal combinations) are obtained for intermediate values of interaction parameters. The point of transition to the strong chaos is expressed by the *universal criterion of global chaos onset*:

$$\kappa \equiv \frac{\Delta\eta_i}{\Delta\eta_n} = \frac{\omega_\xi}{\omega_q} \cong 1 \ , \tag{20}$$

where κ is the introduced *chaoticity* parameter, $\Delta\eta_i$, ω_ξ and $\Delta\eta_n \sim \Delta\varepsilon$, ω_q are energy-level separations and frequencies for the inter-component and intracomponent motions, respectively. At $\kappa \ll 1$ one has the externally regular multivalued SOC regime, which degenerates into global chaos as κ grows from 0 to 1, and the maximum irregularity at $\kappa \approx 1$ is again transformed into a multivalued SOC kind of structure at $\kappa \gg 1$ (but with a "reversed" system configuration).

One can compare this transparent and universal picture with the existing diversity of separated and incomplete unitary criteria of chaos and regularity. Only the former provide a real possibility of understanding and control of communication tools of arbitrary complexity, where more regular regimes can serve for desirable direction of communication dynamics, while less regular ones will play the role of efficient search and adaptation means. This combination forms the basis of any "biological" and "intelligent" kind of behaviour [1, 4, 5] and therefore can constitute the essence of the *intelligent communication paradigm* supposed to extend the now realised (quasi-) regular kind of communication, which corresponds to the uttermost limit of SOC ($\kappa \to 0$). While the latter *inevitably* becomes inefficient with growing network sophistication (where the chaos-bringing resonances of (20) *cannot* be avoided any more), it definitely lacks the "intelligent power" of unreduced complex dynamics to generate meaning and adaptable structure development.

3 Huge efficiency of complex communication dynamics and the guiding role of the symmetry of complexity

The *dynamically probabilistic fractality* of the system structure emerges naturally by the unreduced interaction development itself [1, 4, 5]. It is obtained mathematically by application of the same EP method (9)-(14) to solution of the truncated system of equations (12), then to solution of the next truncated system, etc., which gives the irregular and *probabilistically moving* hierarchy of realisations, containing the intermittent mixture of global chaos and multivalued SOC, which constitute together a sort of *confined chaos*. The total realisation number $N_{\Re}$, and thus the power, of this autonomously branching interaction process with a *dynamically parallel* structure grows *exponentially* within any time period. It can be estimated in the following way [5].

If our system of inter-connected elements contains N_{unit} "processing units", or "junctions", and if each of them has n_{conn} real or "virtual" (possible) links, then the total number of interaction links is $N = n_{\mathrm{conn}} N_{\mathrm{unit}}$. In most important cases N is a huge number: for both human brain and genome interactions N is greater than 10^{12}, and being much more variable for communication systems, it will typically scale in similar "astronomical" ranges. The key property of *unreduced, complex* interaction dynamics, distinguishing it from any unitary version, is that the maximum number $N_{\Re}$ of realisations actually taken by the system (also per time unit) and determining its real "power" P_{real} (of search, memory, cognition, etc.) is given by the number of *all possible combinations of links*, i.e.

$$P_{\mathrm{real}} \propto N_{\Re} = N! \to \sqrt{2\pi N}\left(\frac{N}{e}\right)^{N} \sim N^{N} \ggg N \ . \tag{21}$$

Any unitary, sequential model of the same system (including its *mechanistically* "parallel" and "complex" modes) would give $P_{\mathrm{reg}} \sim N^{\beta}$, with $\beta \sim 1$, so that

$$P_{\mathrm{real}} \sim (P_{\mathrm{reg}})^{N} \ggg P_{\mathrm{reg}} \sim N^{\beta} \ . \tag{22}$$

Thus, for $N \sim 10^{12}$ we have $P_{\mathrm{real}} \gg 10^{10^{13}} \gg 10^{10^{12}} \sim 10^{N} \to \infty$, which is indeed a "practical infinity", also with respect to the unitary power of $N^{\beta} \sim 10^{12}$.

These estimates demonstrate the true power of complex (multivalued) communication dynamics that remains suppressed within the unitary, quasi-regular operation mode dominating now in man-made technologies. The huge power values for complex-dynamical interaction correlate with the new *quality* emergence, such as *intelligence* and *consciousness* (at higher levels of complexity) [5], which has a direct relation to our *intelligent* communication paradigm, meaning that such properties as *sensible*, context-related information processing, personalised *understanding* and autonomous *creativity* (useful self-development), desired for the new generation networks, are inevitable *qualitative* manifestations of the above "infinite" power.

Everything comes at a price, however, and a price to pay for the above qualitative advantages is rigorously specified now as irreducible *dynamic randomness*, and thus unpredictability of operation details in complex information-processing systems. We only rigorously confirm here an evident conclusion that *autonomous* adaptability and genuine *creativity* exclude any detailed, regular, predictable pre-programming in principle. But what then can serve as a guiding principle and practical strategy of construction of those qualitatively new types of communications networks and their "intelligent" elements? We show in our further analysis of complex-dynamic interaction process that those guiding rules and strategy are determined by a general law of complex (multivalued) dynamics, in the form of *universal symmetry, or conservation, of complexity* [1, 3, 5]. This universal "order of nature" and evolution law unifies the extended versions of all (correct) conservation laws, symmetries, and postulated principles (which are causally derived and realistically interpreted now). Contrary to any unitary symmetry, the universal symmetry of complexity is *irregular* in its structure, but always *exact* (never "broken"). Its "horizontal" manifestation (at a given level of complexity) implies the actual, dynamic symmetry between realisations, which are really taken by the system, constituting the system dynamics (and evolution) and replacing the abstract "symmetry operators". Therefore the conservation, or symmetry, of system complexity totally determines its dynamics and explains the deep "equivalence" between the emerging, often quite dissimilar and chaotically changing system configurations [3].

Another, "vertical" manifestation of the universal symmetry of complexity is somewhat more involved and determines emergence and development of different levels of complexity within a real interaction process. System "potentialities", or (real) power to create new structure at the very beginning of interaction process (before any actual structure emergence) can be universally characterised by a form of complexity called *dynamic information* and generalising the usual "potential energy" [1, 3, 5]. During the interaction process development, or structure creation, this potential, latent form of complexity is progressively transformed into its explicit, "unfolded" form called *dynamic entropy* (it generalises kinetic, or heat, energy). The universal *conservation of complexity* means that this important transformation, determining every system dynamics and evolution, happens so that the sum of dynamic information and dynamic entropy, or *total complexity*, remains unchanged (for a given system or process). This is the absolutely universal formulation of the symmetry of complexity, that includes the above "horizontal" manifestation and, for example, extended and unified versions of the first and second laws of thermodynamics (i.e. conservation of energy and its permanent degradation). It also helps to eliminate the persisting (and inevitable) series of confusions around the notions of information, entropy, complexity, and their relation to real system dynamics in the unitary theory (thus, really expressed and processed "information" corresponds rather to a particular case of our generalised dynamic entropy, see [1, 5] for further details).

It is not difficult to show [1, 3, 5, 8] that the natural, universal measure of dynamic information is provided by the (generalised) action $\mathcal{A}$ known from classical mechanics, but now acquiring a much wider, essentially nonlinear and causally complete meaning applicable at any level of complexity. One obtains then the universal differential expression of complexity conservation law in the form of generalised Hamilton-Jacobi equation for action $\mathcal{A} = \mathcal{A}(x, t)$:

$$\frac{\Delta \mathcal{A}}{\Delta t}\Big|_{x=\text{const}} + H\left(x, \frac{\Delta \mathcal{A}}{\Delta x}\Big|_{t=\text{const}}, t\right) = 0 \ , \tag{23}$$

where the *Hamiltonian*, $H = H(x, p, t)$, considered as a function of emerging space coordinate x, momentum $p = (\Delta \mathcal{A}/\Delta x)|_{t=\text{const}}$, and time t, expresses the unfolded, entropy-like form of differential complexity, $H = (\Delta S/\Delta t)|_{x=\text{const}}$ (note that the discrete, rather than usual continuous, versions of derivatives and variable increments here reflect the naturally quantized character of unreduced complex dynamics [1, 3, 5, 8]). Taking into account the dual character of multivalued dynamics, where every structural element contains permanent transformation from the localised, "regular" realisation to the extended configuration of the intermediate realisation of generalised wavefunction and back (Sect. 2), we obtain the universal Schrödinger equation for the wavefunction (or distribution function) $\Psi(x, t)$ by applying the causal, dynamically derived quantization procedure [1, 3, 5, 8, 10] to the generalised Hamilton-Jacobi equation (23):

$$\frac{\partial \Psi}{\partial t} = \hat{H}\left(x, \frac{\partial}{\partial x}, t\right)\Psi \ , \tag{24}$$

where $\mathcal{A}_0$ is a characteristic action value (equal to Planck's constant at quantum levels of complexity) and the Hamiltonian operator, $\hat{H}$, is obtained from the Hamiltonian function $H = H(x, p, t)$ of equation (23) with the help of causal quantization (we also put here continuous derivatives for simplicity).

Equations (23)-(24) represent the universal differential expression of the symmetry of complexity showing how it directly determines dynamics and evolution of any system or interaction process (they justify also our use of the Hamiltonian form for the starting existence equation, Sect. 2). This universally applicable Hamilton-Schrödinger formalism can be useful for rigorous description of any complex network and its separate devices, provided we find the *truly complete* (dynamically multivalued) general solution to particular versions of equations (23)-(24) with the help of unreduced EP method (Sect. 2).

We have demonstrated in that way the fundamental, analytical basis of description and understanding of complex (multivalued) dynamics of real communication networks and related systems, which can be further developed in particular applications in combination with other approaches. The main *practical proposition* of the emerging intelligent communication paradigm is to open the way for the *free, self-developing structure creation* in communication networks and tools with strong interaction (including self-developing internet structure, intelligent search engines, and distributed knowledge bases). The liberated, autonomous system dynamics and structure creation, "loosely" governed by the hierarchy of system interactions as described in this report, should essentially exceed the possibilities of usual, deterministic programming and control.

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Annex I:

Brief (poster) presentation of the report at WAC 2004 (Berlin, 18-19 October 2004)

Report presented at WAC 2004, http://www.autonomic-communication.org/wac/

Complex Dynamics of Autonomous Communication Networks and the Intelligent Communication Paradigm

THE CONSTRUCTIVE POWER OF INTERACTION-DRIVEN CHAOS IN AUTONOMOUS NETWORK DYNAMICS

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The dynamics of arbitrary communication system is analysed as unreduced interaction process. The applied generalised, universally nonperturbative method of effective potential reveals the phenomenon of dynamic multivaluedness of competing system configurations forced to permanently replace each other in a dynamically random order, which leads to universally defined dynamical chaos, complexity, fractality, self-organisation, and adaptability.
We demonstrate, within our ***mathematically rigorous analysis***, the origin of the huge, exponentially high efficiency of the unreduced, complex network dynamics and specify the universal symmetry of complexity as the fundamental guiding principle for creation and control of such qualitatively new kind of networks and devices.

[1] A.P. Kirilyuk, “Dynamically Multivalued, Not Unitary or Stochastic, Operation of Real Quantum, Classical and Hybrid Micro-Machines”, E-print physics/0211071 at http://arXiv.org.

[2] A.P. Kirilyuk, *Universal Concept of Complexity by the Dynamic Redundance Paradigm: Causal Randomness, Complete Wave Mechanics, and the Ultimate Unification of Knowledge* (Kyiv: Naukova Dumka: 1997). For a non-technical review see also: E-print physics/9806002 at http://arXiv.org.

[3] A.P. Kirilyuk, “Dynamically Multivalued Self-Organisation and Probabilistic Structure Formation Processes”, *Solid State Phenomena*, **97-98** (2004) 21-26. E-print physics/0405063 at http://arXiv.org.

Generalised dynamic equation for many-body interaction problem:

$$\left\{\sum_{k=0}^{N}\left[h_k\left(q_k\right)+\sum_{l>k}^{N}V_{kl}\left(q_k,q_l\right)\right]\right\}\Psi\left(Q\right)=E\Psi\left(Q\right)$$

or

$$\left\{h_0\left(\xi\right)+\sum_{k=1}^{N}\left[h_k\left(q_k\right)+V_{0k}\left(\xi,q_k\right)\right]+\sum_{l>k}^{N}V_{kl}\left(q_k,q_l\right)\right\}\Psi\left(\xi,Q\right)=E\Psi\left(\xi,Q\right)$$

The unreduced (nonperturbative) general solution is ***always probabilistic*** (phenomenon of ***dynamic multivaluedness = intrinsic chaoticity***):

$$\rho\left(\xi,Q\right)=\sum_{r=1}^{N_{\Re}}{}^{\oplus}\rho_r\left(\xi,Q\right)$$

$\oplus$ designates ***dynamically probabilistic*** sum over ***incompatible*** realisations

Two limiting regimes of complex dynamics:
multivalued self-organisation (or SOC) and ***uniform (global) chaos***

Universal criterion of global (strong) chaos:

$$\kappa\equiv\frac{\Delta\eta_i}{\Delta\eta_n}=\frac{\omega_\xi}{\omega_Q}\simeq 1$$

or ***resonance*** between the main system motions

Criterion of quasi-regularity (self-organisation): $\kappa\ll 1$ or $\kappa\gg 1$.

As network complexity grows one ***cannot avoid resonance (“clash”)***, $\kappa\sim 1$ and therefore essential ***dynamic randomness becomes inevitable***:

Highly complicated interaction networks cannot be close to regularity

Ordinary, unitary dynamic “models” and approaches are inapplicable

Let’s transform the unitary model *defect* (system failure)
into the complex-dynamic system *advantage* (superior power and qualities)

Essentially chaotic dynamics gives ***huge advantage in efficiency***

Chaotic network efficiency is determined
by the number of ***all combinations of links***, or

$$N! \simeq \sqrt{2\pi N}(N/e)^N \sim N^N$$

for N links, where N is usually large itself in a full-scale system

One obtains exponentially big advantage with respect to unitary efficiency
growing only as $N^{\beta}\,(\beta \sim 1)$

This huge advantage is the expression of ***autonomous creativity***
of unreduced (multivalued) interaction dynamics,
with the inevitable "payment" for it by (controlled) chaoticity of results

Intrinsically chaotic operation
with huge efficiency and autonomous creativity/adaptability
is the new, genuine, rigorously substantiated
paradigm of intelligent, truly autonomous communication

Particular aspects and applications:

(1) Knowledge-based structure of intelligent communication networks
FEASIBLE TODAY, AS SELF-DEVELOPING KNOWLEDGE STRUCTURES (SOFTWARE)

(2) Intrinsically adapted to complex-dynamic problem solution
NATURAL BIO AND ARTIFICIAL BIO-INSPIRED SYSTEMS

(3) Intelligent network (and its users!) becomes more intelligent
KNOWLEDGE-BASED NETWORK SUPPORTS ITS OWN DEVELOPMENT

(4) understanding and development of natural and *true* artificial
intelligence and *consciousness*
(GENUINE) MACHINE CONSCIOUSNESS PARADIGM

(5) Universal symmetry of complexity as *unified guiding principle*
IRREGULAR, BUT NEVER "BROKEN" TRANSFORMATION OF POTENTIALITY INTO
STRUCTURE, OR *DYNAMIC INFORMATION* INTO *DYNAMIC ENTROPY*
→ COMPLEXITY CORRESPONDENCE PRINCIPLE

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APPENDIX: Mathematical Details

From Perturbative Model Reduction to Unreduced Problem Solution

Arbitrary interaction process (system) dynamics in terms of the (free) component eigen-modes:

$$h_0(\xi)\psi_n(\xi)+\sum_{n'}V_{nn'}(\xi)\psi_{n'}(\xi)=\eta_n\psi_n(\xi)\ , \tag{a}$$

where the total system state-function is

$$\Psi(q_0,q_1,...,q_N)\equiv\Psi(\xi,Q)=\sum_{n\equiv(n_1,n_2,...,n_N)}\psi_n(q_0)\varphi_{1n_1}(q_1)\varphi_{2n_2}(q_2)...\varphi_{Nn_N}(q_N)\equiv\sum_n\psi_n(\xi)\Phi_n(Q)\ . \tag{b}$$

After finding eigen-solutions of the system of equations (A), $\{\psi_{ni}(\xi),\eta_{ni}\}$, the *general solution* of a problem is their combination, such as

$$\Psi(\xi,Q)=\sum_n c_i\psi_{ni}(\xi)\Phi_n(Q)\ . \tag{c}$$

In reality $\{\psi_{ni}(\xi),\eta_{ni}\}$ are always found from a perturbative approximation:

$$\left[h_0(\xi)+V_{nn}(\xi)+\tilde{V}_n(\xi)\right]\psi_n(\xi)=\eta_n\psi_n(\xi)\ ,\quad V_0(\xi)<\tilde{V}_n(\xi)<\sum_{n'}V_{nn'}(\xi)\ . \tag{d}$$

<u>The unreduced general solution of the same problem</u>
is the *dynamically probabilistic* sum over *redundant* system realisations:

$$\rho(\xi,Q)\equiv|\Psi(\xi,Q)|^2=\sum_{r=1}^{N_{\Re}}{}^{\oplus}\rho_r(\xi,Q)\ ,\quad \rho_r(\xi,Q)=|\Psi_r(\xi,Q)|^2\ , \tag{A.1}$$

$$\Psi_r(\xi,Q)=\sum_i c_i^r\left[\Phi_0(Q)\psi_{0i}^r(\xi)+\sum_{n,i'}\frac{\Phi_n(Q)\psi_{ni'}^0(\xi)\int_{\Omega_\xi}d\xi'\psi_{ni'}^{0*}(\xi')V_{n0}(\xi')\psi_{0i}^r(\xi')}{\eta_i^r-\eta_{ni'}^0-\varepsilon_{n0}}\right], \tag{A.2}$$

where $\left\{\psi_{0i}^r(\xi),\,\eta_i^r\right\}$ are eigen-solutions of the *effective* equation:

$$h_0(\xi)\psi_0(\xi)+V_{\text{eff}}(\xi;\eta)\psi_0(\xi)=\eta\psi_0(\xi)\ , \tag{B.1}$$

$$V_{\text{eff}}\left(\xi;\eta_i^r\right)\psi_{0i}^r(\xi)=V_{00}(\xi)\psi_{0i}^r(\xi)+\sum_{n,i'}\frac{V_{0n}(\xi)\psi_{ni'}^0(\xi)\int_{\Omega_\xi}d\xi'\psi_{ni'}^{0*}(\xi')V_{n0}(\xi')\psi_{0i}^r(\xi')}{\eta_i^r-\eta_{ni'}^0-\varepsilon_{n0}}\ , \tag{B.2}$$

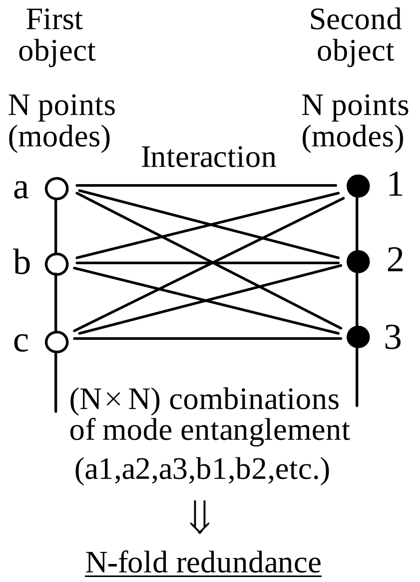
and $\left\{\psi_{ni}^0(\xi),\,\eta_{ni}^0\right\}$ are eigen-solutions of the truncated system of equations:

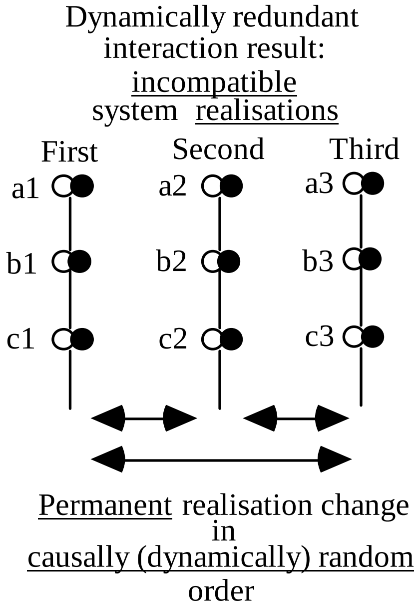
$$h_0(\xi)\psi_n(\xi)+\sum_{n'}V_{nn'}(\xi)\psi_{n'}(\xi)=\eta_n\psi_n(\xi)\ ,\ \underline{n,n'\neq0}\ . \tag{C}$$

Elementary length $\Delta x=\lambda=\Delta\eta_i^r$, time $\Delta t=\Delta x/v_0$, action $A_0=V_{\text{eff}}\Delta t$ (D)

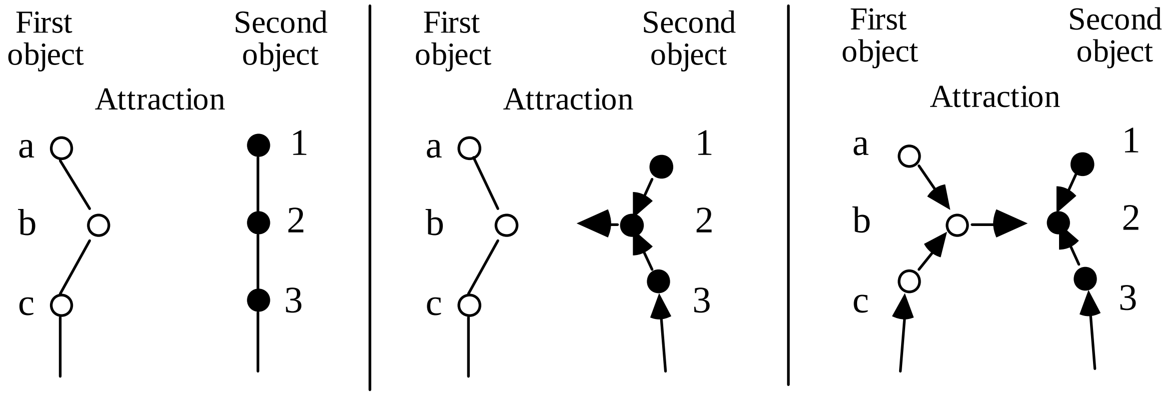
Dynamic Redundance (Multivaluedness) as the Origin of Causal Randomness in Arbitrary System with Interaction. Dynamic Entanglement of Interacting Entities & Fractality

Interaction between two many-body (‘many-point’) entities (objects):





Permanent dynamic instability in any system with interaction by dynamic feedback loops in the unreduced interaction development:

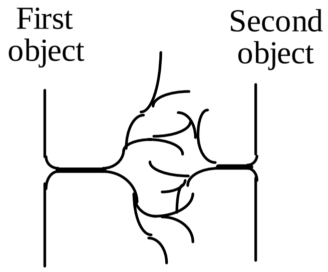


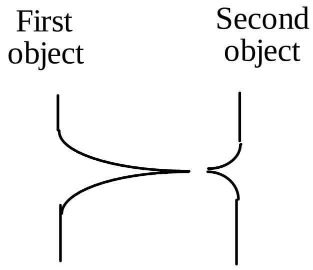
Universal fractality:

Dynamic (autonomous) formation

Causal (purely dynamic) randomness (irregularity)

Intrinsic adaptability





Annex II:

Knowledge-Based Structure of Intelligent Communication Networks

(panel extension of the report at WAC 2004)

Knowledge-Based Structure

of Intelligent Communication Networks

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- **Existing networks are based on hardware tools**
  regular, rigid structure, nonintelligent behaviour
  inevitable first step of development (now accomplished)
- **Knowledge, not tools, is the network purpose**
  very limited solution by “advanced” search tools (Google)
- **Knowledge-based network is the next step**
  structure guided by (developing) knowledge
  permanently changeable, irregular, autonomously adaptable
  complex-dynamic interaction, intelligent network
  mathematical structure of probabilistic dynamic fractal

Knowledge-Based Structure
of Intelligent Communication Networks

- **Full solution needs qualitatively new tools**

  dynamically adaptable, interactive software
  "Intelligent Communication Paradigm" (presented here)
  genuine intelligence as a high level of dynamic complexity

- **A reduced solution is feasible today and useful**

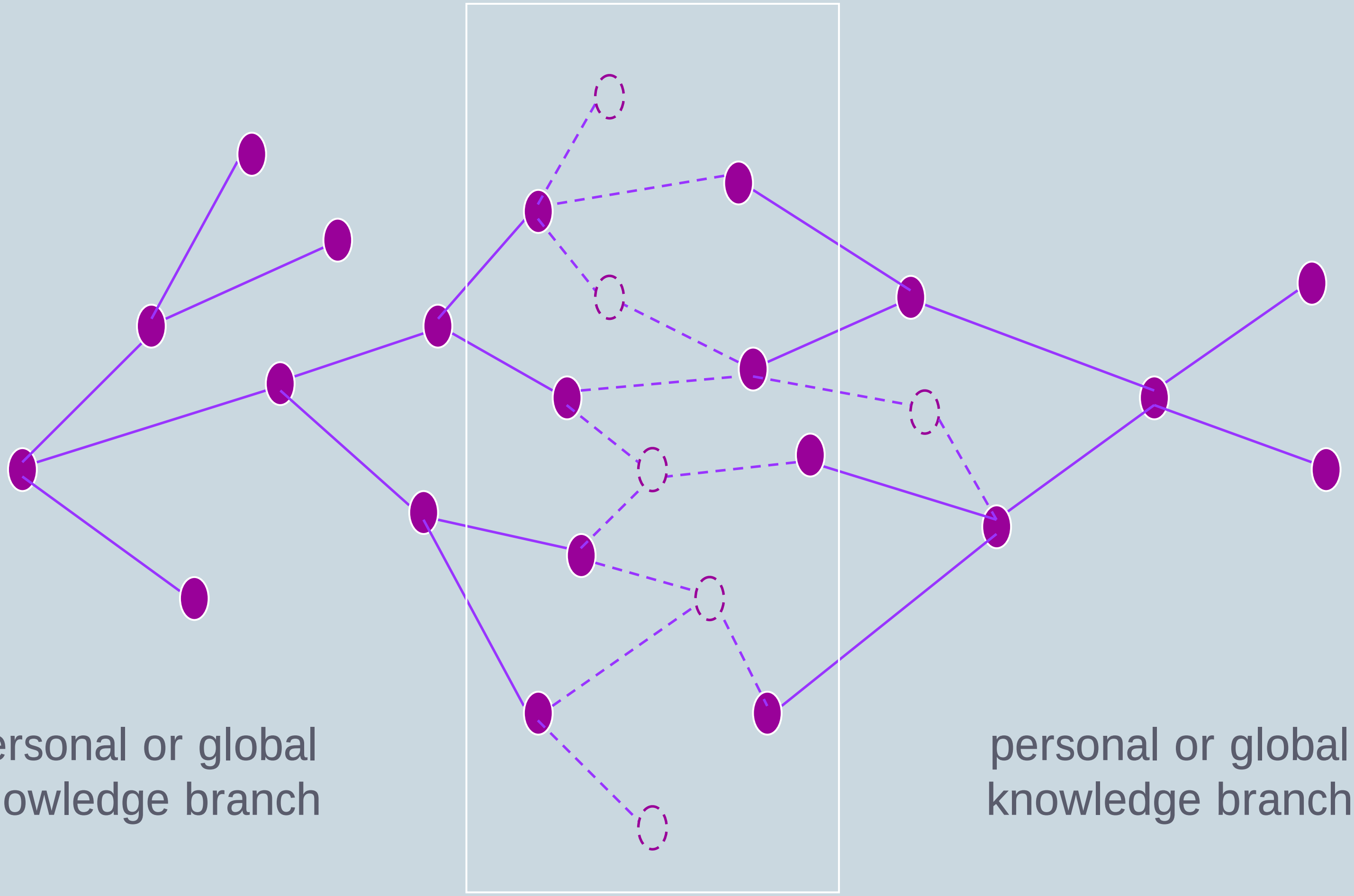
  hierarchic, knowledge-guided structure of software
  "tree of knowledge" in the web and personal tools
  permanent interaction and autonomous change

- **The reduced solution prepares the full solution**

  mechanical "intelligence" as a way to genuine intelligence
  users of intelligent network become more intelligent
  knowledge-based network supports its own development

Knowledge-Based Structure

of Intelligent Communication Networks



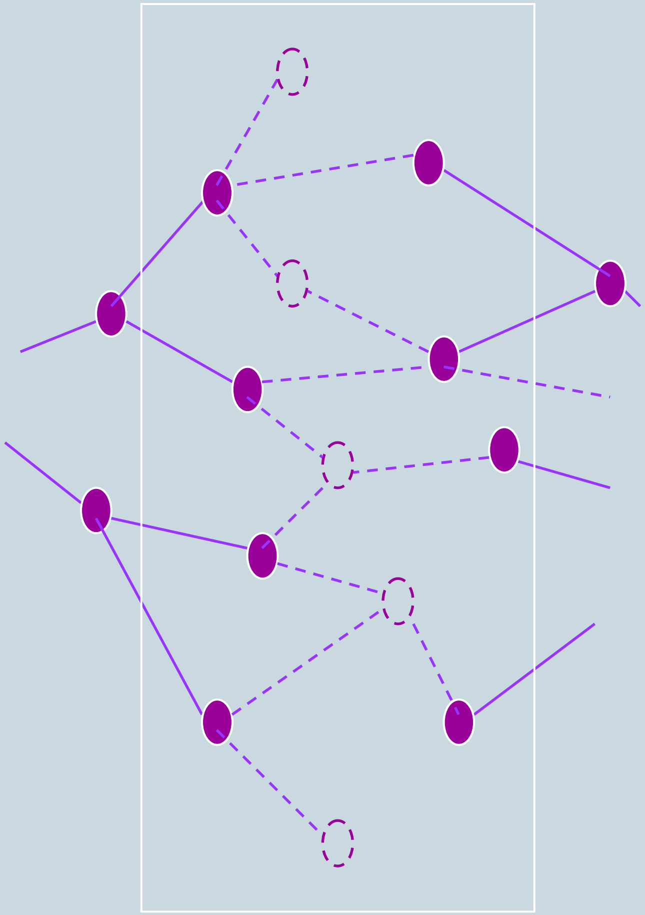
personal or global
knowledge branch

personal or global
knowledge branch

autonomous development
due to branch interaction

Knowledge-Based Structure

of Intelligent Communication Networks



branch interaction process

(1) Available “interaction space”

(2) Self-amplifying (chaotic) interaction rule

(3) Based on the unreduced complexity and
“complexity correspondence principle”
↓
Universal criterion of intelligent operation