BOOLEAN OPERATIONS OF TRIANGULATED SOLIDS AND THEIR APPLICATIONS IN THE 3D GEOLOGICAL MODELLING

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ABSTRACT:

In this paper we present a new method for Boolean operations (intersection, union and difference) of triangulated solids, and its applications in three-dimensional (3D) computerized geological modelling (3DGM). This method is adapted to arbitrary (convex or concave), close geological solids. Three major steps are included in this method: (1) Calculate intersection points between triangles. Our work is mainly concerned with coplanar triangles, while non-planar cases are dealt with previous algorithm (Möller, 1997). Bounding box technique is employed to speed up this process. (2) Retriangulate the intersected triangles with the point-by-point insertion triangulation method, in order to keep consistent between pairs of solids. (3) Inclusion test between triangles and solids. This work can be reduced to testing whether a point is inside a solid or not. We extend the "Crossing Number Method" from 2D to 3D, and deal with some particular cases when the ray passes vertexes or edges of triangles. Boolean operations of triangulated solids have broad applications in engineering, geology, biological research, medical imaging, etc. We are concerned with the applications of this method and geological knowledge in 3DGM. Several examples, such as intrusions, ore bodies and tunnels, bifurcated geological bodies, are presented to validate the method.

1. INTRODUCTION

Solid Modelling has broad applications in industrial design (Braid, 1975; Krouse, 1985), geological modelling (Bak and Mill, 1989; Lemon and Jones, 2003; Wu, 2005) and other fields. Boundary Representation (B-Rep), such as TIN (Triangulated Irregular Network), is one of common representation methods of solid models. In this paper we present a new method for Boolean operations of B-Rep solids (TIN). The solid can be arbitrary (convex or concave), but not self-intersected and cannot have holes. The types of Boolean operations include Union, Intersection and Difference (Figure 1).

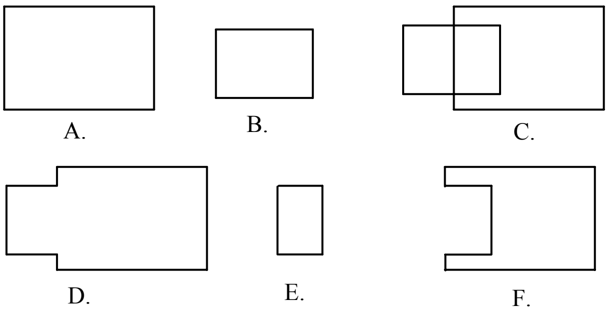


Figure 1. Boolean operations: (A) Body A; (B) Body B; (C) Spatial relationship of A and B; (D) Union ($A \cup B$); (E) Intersection ($A \cap B$); (F) Difference ($A - B$).

The broad applications of computers in geological fields such as oil, mine make the urgent need of 3DGM. Because of geological complexity, uncertainty, and limitations of available useful data (Turner, 1989; Kelk, 1992; Simon, 1994; Mallet, 1997), 3DGM is a complicated process. Especially in multi-body modelling, the complex spatial topological relationships of geological bodies have to be maintained. So the use of geological knowledge in 3DGM is very important, even pre-requisite in many cases.

From the perspective of geology, geological modelling is a process of analysing and deducing the diverse geological history events and their characteristics from all kinds of "information" reserved in strata and the current geological phenomena, and generally it follows the rule of "uniformitarianism" and employs the method of "strata history comparison". This method in the research of geology can also be applied in 3DGM. For example, we can first build the basic geological models ignoring the complex process in the geological history, and then build the complex body respectively, at last "insert" them into the basic geological bodies by Boolean operations of body objects. This is like the recurrence of the geological evolvement process (Wang, 2003). This paper presents the applications of this method in the modelling of intrusions, ore bodies and tunnels, bifurcated geological bodies.

2. ALGORITHMS OF SOLID BOOLEAN OPERATIONS

As introduced in above sections, Boolean operations of solids include union, intersection and difference. The key algorithms of these three operations are similar, so we just take the difference operation for an example. Suppose that there are solids, A and B, we will compute the Difference operation between A and B, namely A-B.

2.1 Major steps

This algorithm includes three major steps: (1) Calculate intersection points between triangles. To every triangle of one solid, we should compute its intersection points with all of the triangles of other solids. If intersection points are more than one, we should take the link lines of the points as constrained edges for triangulation in the next step. The whole process can be speeded up by the filtration with bounding box technique (Aftosmis et al., 1998; Zhou, 1999). Here we don't use the method of neighbour triangle searching (Lo and Wang, 2004), because it cannot improve the efficiency when we deal with arbitrary solids which may have several intersected parts. (2)

Retriangulate the intersected triangles for keeping consistent between pairs of intersected solids and making sure that there are no triangles crossing the intersection lines. With the constraint of new points and constrained edges, intersected triangles are retriangulated by the point-by-point insertion triangulation method (Lawson, 1978; Tsai, 1993). (3) Inclusion test between triangles and solids. Delete the solid A's triangles which are inside B, and add the B's triangles which are inside A. This can be reduced to the test between point and solid.

2.2 Calculate intersection points between triangles

There are two cases when calculating intersection points between triangles: the two triangles are coplanar or non-planar. The latter case is dealt with the algorithm proposed by Möller (1997), and our work is concerned with the coplanar cases.

2.2.1 Non-planar triangles intersection (Möller, 1997)

Suppose that triangle T_1 and T_2 lie in planar π_1 and π_2 respectively, and π_1 intersects with π_2 at line L. The process of calculation of intersection points is showed as follows (Figure 2):

(1) Filtering: compute plane equation of π_2, take every vertex coordinates of T_1 into the equation, if the results have same sign, positive or negative, the vertexes of T_1 must be on the same side of π_2 and it certainly does not intersect with π_2; if the results are equal to zero, T_1 and π_2 are coplanar. The relation between T_2 and π_1 can be computed in the same way.

(2) If the triangles are not be filtered, they may intersect, then compute the intersection line L of π_1 and π_2.

(3) By projecting vertexes of triangles to L, we can calculate out that T_1 and π_2 intersect at $I_1^1I_1^2$, T_2 and π_1 intersect at $I_2^1I_2^2$.

(4) Computing to judge whether $I_1^1I_1^2$ and $I_2^1I_2^2$ are overlay. If they are intersected, calculate the intersection points further.

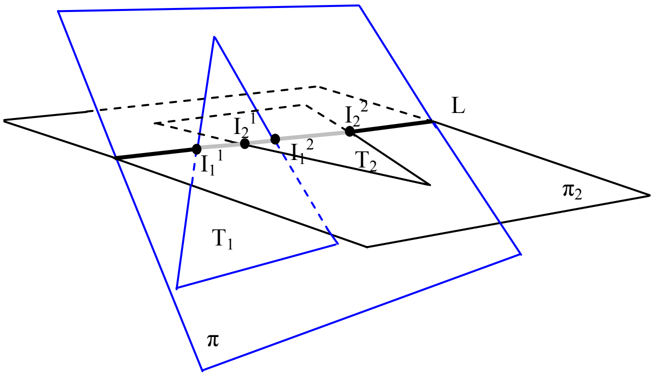


Figure 2. Calculate intersection points between non-planar triangles

2.2.2 Co-planar triangles intersection

The coplanar case is complicated and it can be classified by the number of intersection points, there are ten classes (Figure 3)

- 0 intersection points：(1), (2)
- 1 intersection points：(3), (4)
- 2 intersection points：(5), (6), (7), (8)
- 3 intersection points：(9), (10), (11), (12)
- 4 intersection points：(13), (14), (15)
- 5 intersection points：(16)，(17)
- 6 intersection points：(18)
- share one edge：(19), (20), (21), (22), (23), (24)
- share two edges：(25), (26)
- share three edges：(27)

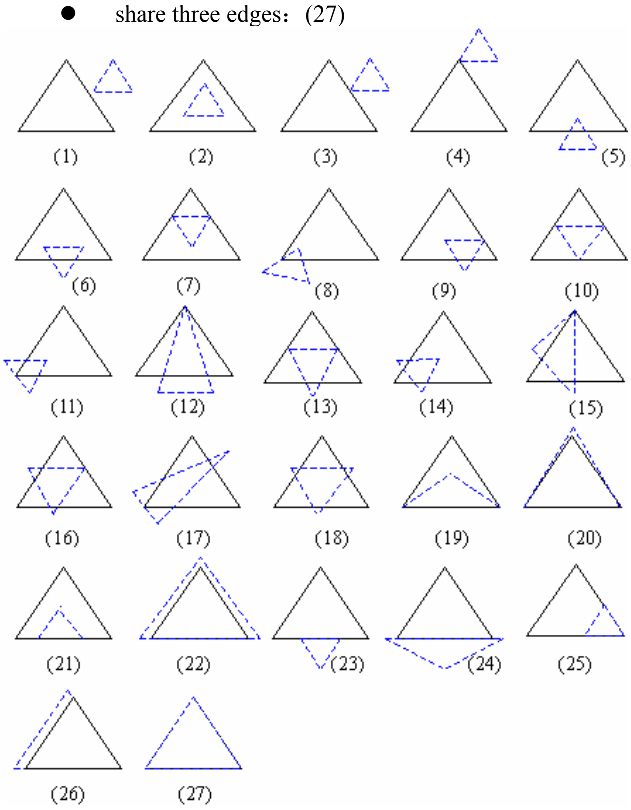


Figure 3. Classification of co-planar triangles intersection

Since it is trouble to deal with these cases respectively, we propose a method to deal with them uniformly. We find out that the shape of the overlay region of intersection triangles is a convex region which degenerates to a point or a line in few cases, so we can calculate the convex region (Graham, 1972) from the intersection points and other triangles' vertexes which lie in the current operating triangle, and then every edge of the convex region will be a constrained edge for triangulation. Take triangle 1($A_1B_1C_1$) and triangle 2($A_2B_2C_2$) in Figure 4 as an example, they are co-planar intersection and the former is the current operating triangle. We first compute their intersection points P_1, P_2, P_3 and P_4， and then find B_2 which is one vertex of triangle 2 and lies in the triangle 1, thirdly compute the convex region by these points, at last we get convex region of P_1-P_2-P_3-P_4-B_2, add points P_1, P_2, P_3, P_4, B_2 and constrained edges P_1P_2, P_2P_3, P_3P_4, P_4B_2, B_2P_1 to triangle 1, the new added data will be constraint for the next step of triangulation.

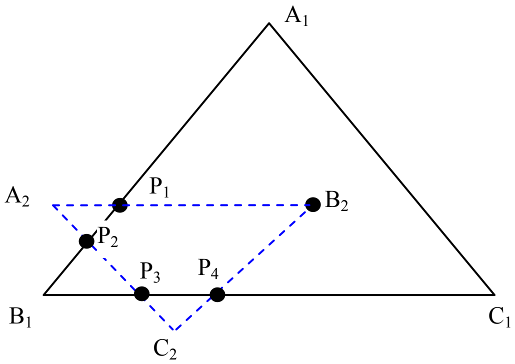


Figure 4. Intersection of co-planar triangles

2.3 Retriangulation of intersected triangles

Insert the constrained points and edges added in the former step and retriangulate them by using point-by-point insertion triangulation method (Lawson, 1978; Tsai, 1993), and make sure that the new generated triangle edges will not cross the new added constrained edge (Vigo and Pla, 1995). Here we deal with all of the triangles not using edges swap method (Lo and Wang, 2005) at the intersection lines, because of the considering of multi-value projection problem.

2.4 Contain testing between triangles and solids

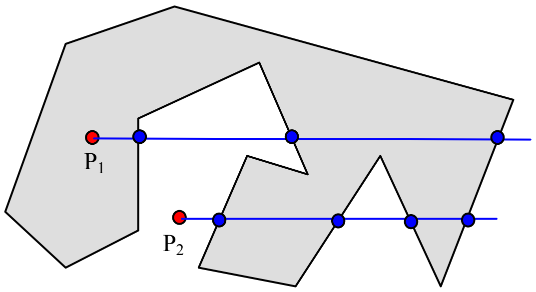


Figure 5. Point inclusion in polygon test by "crossing number method" In 2D. The ray from Point P1 has 3 intersection points with polygon and 3 is odd, so it is in polygon; P2 has 4 points and is outside the polygon.

Because intersected triangles have been dealt with, we can guarantee that the relationship between a triangle and a solid must be one of the follows: inside the solid, on the solid or outside the solid. This work can be reduced to testing whether one point is inside a solid or not. If three vertexes of a triangle are inside the solid, it must be in the solid; if they all are outside the solid, it must be out; we cannot make sure weather a triangle is on surface or not if its 3 vertexes are on the solid's surface for the existence of concave solids. In this case, we have to compute the relationship between inner point of triangle and the solid, and then the triangle is on surface if the inner point does.

In 2D, one method of "Jordan Curve theorem" is used to define whether a point is in a polygon: lead a ray from the test point, and the number of the intersection points between the ray and the polygon's boundary edges must be odd. As showed in Figure 5, p_1 has 3 intersection points, so it is in the polygon; p_2 has 4 intersection points, so it is outside. This method of point inclusion in polygon test is commonly called "crossing number method" (Shimrat, 1962; Haines, 1994).

We extend this method from 2D to 3D to test whether a point is in a solid or not. Lead an arbitrary ray from this point, such as a line along positive Z axis and has a length of two times of the height of the bounding box, and then count the intersection points number between the ray and the solid boundary. The test point would be in the solid if the number is odd and out if even. Special approaches are proposed when the ray intersects with triangle at vertexes or on edges, because there would be several same intersected points in this case. To solve this problem, we project the intersection point and all the triangles intersected at this point to XOY plane and compute the convex region, if the point is in the region, the number of intersected points at this point is regarded as 1, or 0 if is out. If one triangle is parallel with Z axis, ignore it and continue computing.

After the inclusion test, we do the next operations:
(1) Delete A's triangles which lie in B, including the ones lie on B's boundary.

(2) Add the B's triangles which lie in A to solid A, including the ones lie on the A's boundary, which are marked for the next step to decide if add it in final.

(3) Delete the added triangles which should not be added to A. It has two steps: (a) Build the topological relationships of triangles of A (Hrádek et al., 2003), delete the triangles which have wrong topological relationship (3 or more triangles share an edge) and have been marked in step2 (the B's triangles but lie on A's boundary and have been added to A). (b) Because it just deletes the triangles directly linked with original solid A in the former step, there would be some separated triangles which also should be deleted. For this, build topological relationships first, and find the triangles which have at least one edge without linked triangles, push them into a stack and mark a deleting sign, then pop a triangle from the stack and find its neighbour triangles, mark them and push them into the stack, recursively dealing with the triangles in the stack until it becomes empty, at last delete all the triangles which have the deleting sign.

3. APPLICATIONS IN 3D GEOLOGICAL MODELING

The Boolean operations of solids, combined with geological knowledge, have broad applications in geological modelling. Generally, the process of the application has 3 major steps:

(1) Reconstructing the basic geological bodies, such as the basic strata.

(2) Reconstructing the complex geological bodies, such as intrusions.

(3) Taking Boolean operations between complex and basic geological bodies and getting the final solids.

Three examples, such as intrusions, ore bodies and tunnels, bifurcated geological bodies, are presented as follows.

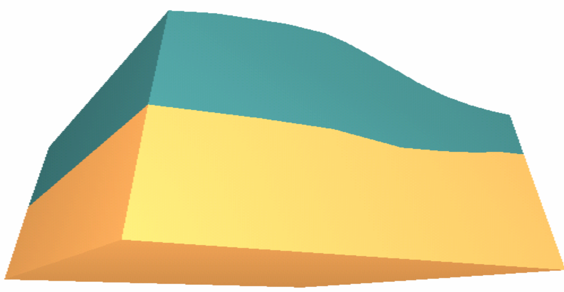
3.1 Intrusions

The magma intrudes the wall rock, alternates the rock and forms the intrusion finally.

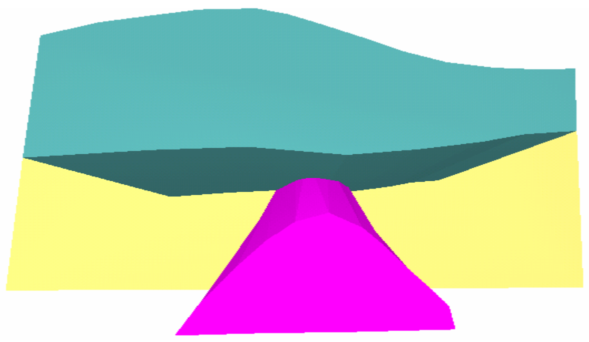
(1) As showed in Figure 6A, geological bodies A and B, wall rock, are built first.

(2) Then reconstruct the intrusion model C, whose relationships with A and B are showed in Figure 6B. It divides B into 2 parts and intrude to A. B is showed in transparency to show them clearly.

(3) Take the Difference operation between solids, A-C (delete A's triangles which lie in C and add C's triangles which lie in A to solid A) and B-C (delete B's triangles which lie in C and add C's triangles which lie in B to solid B). Figure 6C is the rendering result figure and the Figure 6D is wire frame figure.



A.



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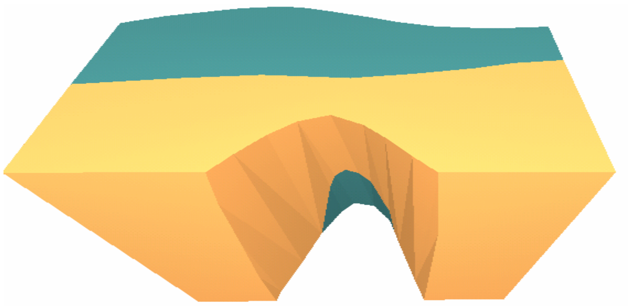
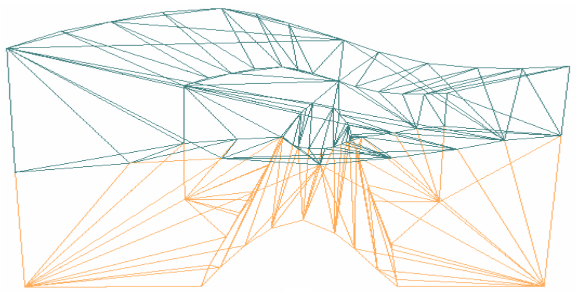


C.



D.

Figure 6. Reconstruction of intrusion models based on solid Difference operations: (A) Basic geological bodies A and B; (B) Intrusion C, and the relationships with A and B; (C) Results of A-C, B-C (rendering figure); (D) Wire frame result

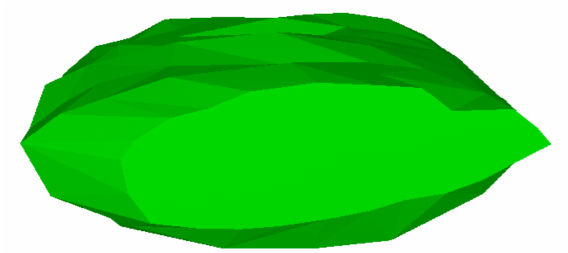
3.2 Ore bodies and tunnels

This method can also be applied in geological engineering project. For example, we can build ore bodies and tunnels independently, then apply the difference Boolean operation to them and get the practical representation of ore bodies with tunnels. Based on this, we can calculate the volume, reserves and other parameters of the ore body based this model.

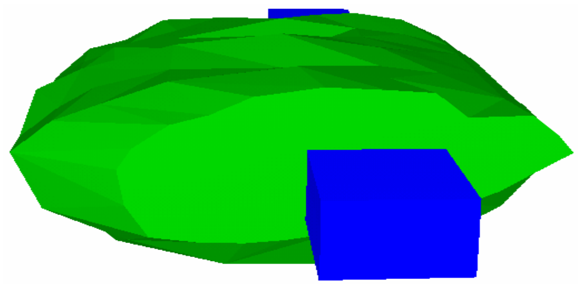
(1) Showed in Figure 7A, reconstruct ore body model A.

(2) Build the tunnel Model B (Figure 7B).

(3) Figure 7C is the rendering figure after taking A-B operation; Fig.7d is the wire frame figure.



A.



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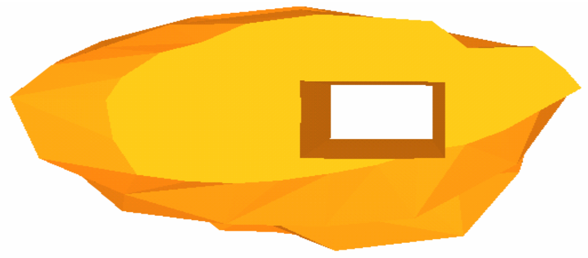

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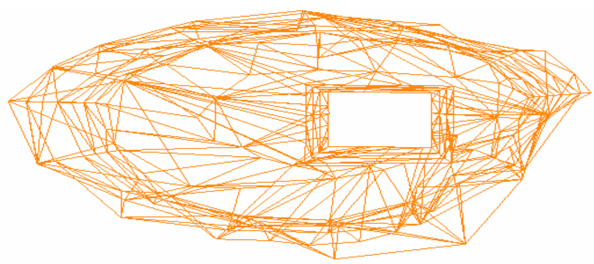


D.

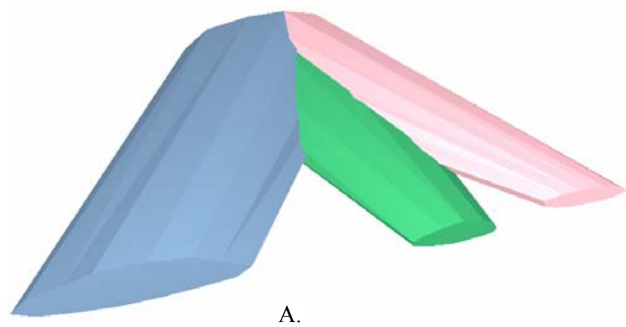
Figure 7. Build ore and tunnel models based solid Difference operations: (A) Ore body A; (B) Tunnel B and the relationship with A; (C) Result of A-B (rendering figure); (D) Wire frame result figure.

3.3 Bifurcated geological bodies

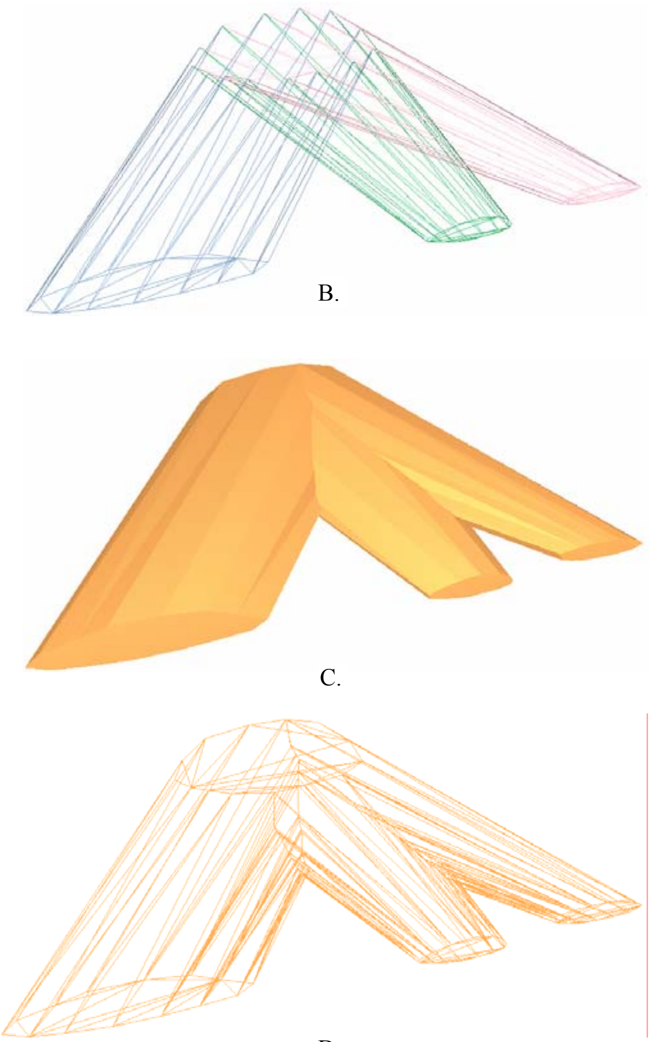
Bifurcated geological bodies are a kind of common geological phenomena, such as furcation of river. Bifurcations lead to that there are different parts of a geological body in different cross-sections. As showed in Figure 8, the geological body is one part in a section but 3 parts in the other.

(1) As showed in Figure 8A, first build 3 individual bodies A, B and C respectively.

(2) Figure 8B is the rendering figure after taking $A \cup B \cup C$ operation (Unite A's triangles which lie out the B and C, B's triangles lie out A and C, and C's triangles lie out the A and B); Figure 8C is the wire frame figure.



A.



B.

C.

D.

Figure 8. Build bifurcations models based on Union operations: (A) Basic geological bodies A, B, C and their spatial relationships; (B) Basic geological bodies showed in line frame; (C) Result of $A \cup B \cup C$ (rendering figure); (D) Wire frame result figure.

4. CONCLUSION

In this paper a new method for Boolean operations (Union, Intersection and Difference) of solids is proposed. And the main steps and algorithms of the method are discussed in detail.

We also present the applications of this method in the 3D geological modelling. This method combined with geology knowledge can reconstruct complex geological bodies and human's engineering activities. Three examples, intrusions, ore bodies and tunnels, bifurcated geological bodies, show that the method has the ability to build complex geological models.

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REFERENCES

Aftosmis, M.J., Berger, M.J., Melton, J.E., 1998. Robust and efficient Cartesian mesh generation for component-based geometry, AIAA J. 36 (6), pp.952–960.

Bak, P.R.G., Mill, A.J.B., 1989. Three dimensional representation in a Geoscientific Resource Management System for the minerals industry. In: Raper, J. (Ed.), Three Dimensional Applications in Geographic Information Systems. Taylor & Francis, New York, pp. 155–182.

Braid, I.C., 1975. The synthesis of solids bounded by many faces. Communications of the ACM 18 (4), 209–216.

Graham, R.L., 1972. An Efficient Algorithm for Determining the Convex Hull of a Finite Point Set, Info. Proc. Lett. 1 (4), pp.132-133.

Haines, Eric.,1994. Point in Polygon Strategies. Graphics Gems IV, ed. Paul Heckbert, Academic Press, pp. 24-46.

Hrádek, J., Kuchař, M., Skala, V., 2003. Hash functions and triangular mesh reconstruction. Computers & Geosciences 29 (6), 741-751.

Kelk, B., 1992. 3-D Modelling with Geoscientific Information Systems: the problem. In: Turner A. K. (Ed), Three-dimensional Modeling with Geoscientific Information System. Kluwer Academic Publishers, Dordrecht, The Netherlands, pp.29-37.

Krouse, J.K., 1985. Solid modeling catches on. Machine Design 57 (3), 60–64.

Lawson, C.L., 1977. Software for C^1 Surface Interpolation. Mathematical Software III. J. Rice, Ed. New York: Academic Press, pp.161-194.

Lemon, A.M., Jones, N.L., 2003. Building solid models from boreholes and user-defined cross-sections. Computers & Geosciences 29 (5), 547-555.

Lo, S.H., Wang, W.X., 2004. A fast robust algorithm for the intersection of triangulated surfaces, Eng. Comput. 20 (2004) 11–21.

Lo, S.H., Wang, W.X., 2005. Finite element mesh generation over intersecting curved surfaces by tracing of neighbours. Finite Elements in Analysis and Design 41 (4), pp.351-370.

Mallet, J.L., 1997. Discrete modeling for natural objects. Mathematical Geology 29 (2), 199–219.

Möller, T., 1997. A fast triangle-triangle intersection test. Journal of Graphics Tools, 2(2), pp.25-30.

Shimrat, M., 1962. Algorithm 112, Position of Point Relative to Polygon, Comm. ACM 5(8), pp.434.

Simon, W.H, 1994. 3D Geoscience Modeling: Computer Techniques for Geological Characterization. Springer-Verlag, pp.27-36.

Tsai, V.J.D., 1993. Delaunay Triangulations in TIN Creation: an overview and a Linear-time Algorithm. Int. J. of GIS, 7 (6): pp.501-524.

Turner, A.K., (Ed.), 1992. Three-Dimensional Modeling with Geoscientific Information Systems. Kluwer Academic Publishers, Dordrecht, The Netherlands, 443pp.

Vigo, M., Pla, N., 2000. Computing Directional Constrained Delaunay Triangulations, Computers & Graphics 24, pp. 181-190.

Wang, Y., 2003. Algorithm Research on Three Dimensional Vector Data Creation and Prototype Implementation. Dissertation, Peking University, pp.75-76 (in Chinese).

Wu, Q., Xu, H., Zou, X.K., 2005. An effective method for 3D geological modeling with multi-source data integration. Computers & Geosciences 31 (1), 35–43.

Zhou, Y, Suri, S., 1999. Analysis of a bounding box heuristic for object intersection. Journal of the ACM (JACM), 46 (6), pp. 833-857.