Anomalous modal structure in a waveguide with a photonic crystal core

Xiaofang Yu
Department of Applied Physics, Stanford University, Stanford, California 94305

Wah Tung Lau and Shanhui Fan
Department of Electrical Engineering, Stanford University, Stanford, California 94305

Received August 31, 2005; revised November 11, 2005; accepted November 17, 2005; posted December 1, 2005 (Doc. ID 64548)

We analyze a dielectric waveguide with a photonic crystal core. Using constant frequency contour analysis, we show that the modal behavior of this structure is drastically different from that of a conventional slab waveguide. In particular, at a given frequency the lowest-order guided mode can have an odd symmetry or can have more than one nodal plane in its field distribution. Also, there exist several single-mode regions with a different modal profile in each region. Finally, a single-mode waveguide for the fundamental mode with a large core and strong confinement can be realized. All these behaviors are confirmed by our three-dimensional finite-difference time-domain simulations. © 2006 Optical Society of America

OCIS codes: 130.2790, 160.3130.

For applications of dielectric waveguides, understanding and controlling their modal properties is of fundamental importance. Recent developments in photonic crystals have led to the invention of waveguide structures with unusual characteristics. Notable examples include air-core waveguides in which light is confined in low-index material[1,2] and endlessly single-mode dielectric-core photonic crystal fibers.[3] Also, photonic heterostructure waveguides with photonic crystal cores and claddings have been analyzed.[4] Recently, anomalous properties of the fundamental modes in coupled line-defect photonic crystal waveguides have been studied.[5] In this Letter we analyze a class of waveguide structures with a high-index photonic crystal core surrounded by a uniform low-index material [Fig. 1(a)]. In these structures, similar to conventional dielectric-slab waveguides, the guiding mechanism is provided by total internal reflections at the photonic crystal surfaces.[6] However, due to the complex spatial dispersion properties of photonic crystals, the modal structures of such waveguides are unusual. In particular, at certain frequency ranges the lowest-order guided mode might have an odd symmetry or might have more than one nodal plane in its field distribution. Also, there can exist several single-mode regions, each with a different modal profile. Finally, one can actually design a single-mode waveguide with a core dimension much larger than the operating wavelength, and yet the modes are still strongly confined in the core region. These properties could be important, for example, in suppressing modal competition in semiconductor lasers.

For concreteness, consider the waveguide shown in Fig. 1(a). The waveguide has a finite width d in the y direction. We are interested in the modes propagating along the x direction. The core consists of a square lattice of air holes (with lattice constant a, radius $0.42a$) introduced into a high-index dielectric slab with a dielectric constant $\epsilon=12$. The thickness of the slab is fixed to $a/2$ such that the guided modes remain nodeless in the z direction in the frequency region that we consider. The crystal is truncated along the (10) plane, which guarantees total internal reflection on the crystal–air interface.[6] To avoid surface modes, the crystal is truncated through the dielectric.[7]

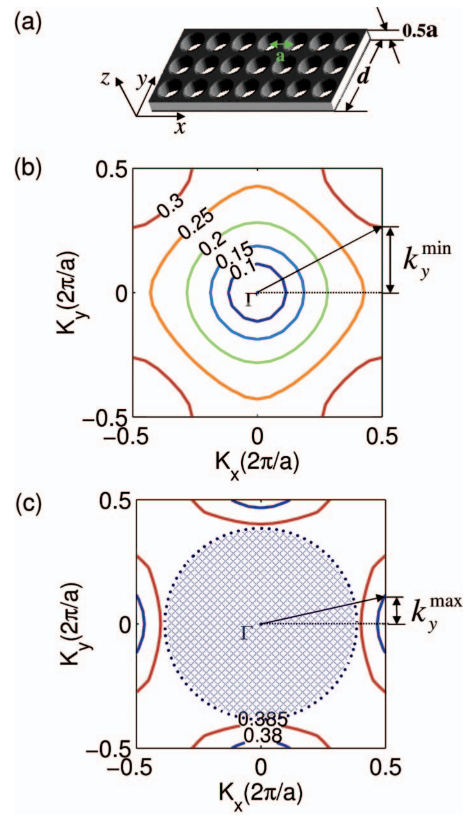

Fig. 1. (a) Waveguide with a photonic crystal core. The crystal consists of a square lattice of air holes introduced into a dielectric ($\epsilon=12$) slab waveguide suspended in air. The hole radius is $0.42a$, and the thickness is $0.5a$. (b) and (c) Constant frequency contours for the first and second TE-like bands of the corresponding infinite photonic crystal slab, respectively. In (c), regions that lie above the light cone are shaded.

0146-9592/06/060742-3/$15.00

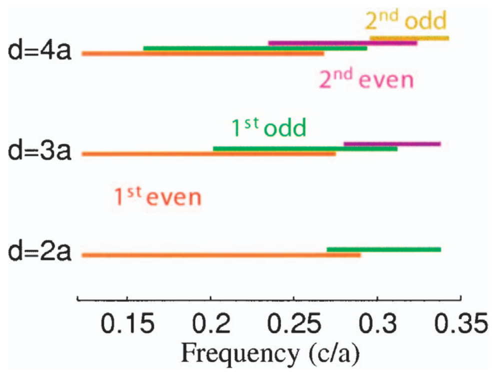

Fig. 2. Modal properties of the photonic crystal core waveguide shown in Fig. 1(a), with widths d of $2a$, $3a$, and $4a$. For each width, lines with different colors indicate the frequency ranges in which modes with different orders exist.

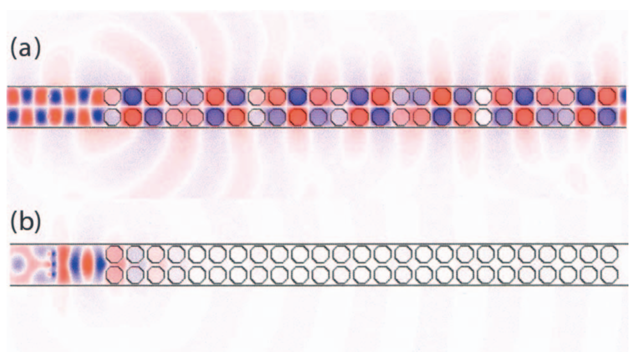

Fig. 3. Steady-state magnetic field (H_z-component) distributions at frequency $f=0.30(c/a)$ for a photonic crystal core waveguide excited by sources with (a) odd or (b) even symmetry. The width of the waveguide is $2a$. Red and blue represent large positive and negative amplitudes, respectively. The small radiation leakage in (a) is due to the modal mismatch between the PhC and the dielectric slab regions.

We seek to infer the behavior of such a waveguide, starting from the band structure of the infinite crystal. At a given frequency ω, to create a guided mode, the total internal reflection condition requires that the wave-vector component parallel to the interface satisfy[8]

$$k_{\parallel}(\omega) > n_{\text{air}}\frac{\omega}{c}. \tag{1}$$

Here $k_{\parallel}$ is restricted to the first Brillouin zone.[6] Also, for each guided mode, the phase accumulated as the wave travels a round trip between the two interfaces needs to satisfy the transverse resonance condition[8]:

$$2k_{\perp}d - \phi_c - \phi_s = 2m\pi, \tag{2}$$

where $k_{\perp}$ is the wave-vector component perpendicular to the interface and ϕ_c, ϕ_s are the phase shifts due to reflection at the interface. The number of nodes in the field pattern can be estimated as

$$m \approx \lfloor k_{\perp}d/\pi \rfloor. \tag{3}$$

Following the convention of the dielectric slab waveguides, a mode with m nodes in its field pattern is the mth-order mode.

When applying Eqs. (1)–(3), one should use the wave-vector values within the first Brillouin zone. This can be seen as follows: for an infinite photonic crystal, the propagating waves are in the Bloch form[9]

$$\vec{H}_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{\vec{k}}(\vec{r}), \tag{4}$$

where $\vec{H}_{\vec{k}}$ is the magnetic field of waves with wave vector $\vec{k}$, $u_{\vec{k}}(\vec{r})$ is a periodic function, i.e., $u_{\vec{k}}(\vec{r}+\vec{R}) = u_{\vec{r}}(\vec{r})$ ($\vec{R}$ is a crystal translation vector). For simplicity, we consider waveguides with widths that are integer multiples of the lattice constant ($d=ma$) [Eq. (2) in fact should apply for any choice of d]. The phase change accumulated by $u_{\vec{k}}(\vec{r})$ as waves travel a round trip in the transverse direction is therefore always a multiple of 2π. The condition for creating a waveguide mode therefore involves only the wave vector $\vec{k}$ that lies in the first Brillouin zone. In this case, the order of a mode refers to the number of nodes in its field envelope.

We compute the band structure of the bulk crystal [shown in Fig. 1(a)] by use of the MIT Photonic Bands package.[10] The constant frequency contours for the first two bands for the infinite crystal are shown in Figs. 1(b) and 1(c). We restrict our consideration to the TE-like modes[11] and the parts of the bands that lie below the light cone.

In the first band [Fig. 1(b)], there is a nonzero $k_y^{\min}$ for a given frequency f in the frequency range 0.25–$0.34(c/a)$. Hence, with an appropriate choice of

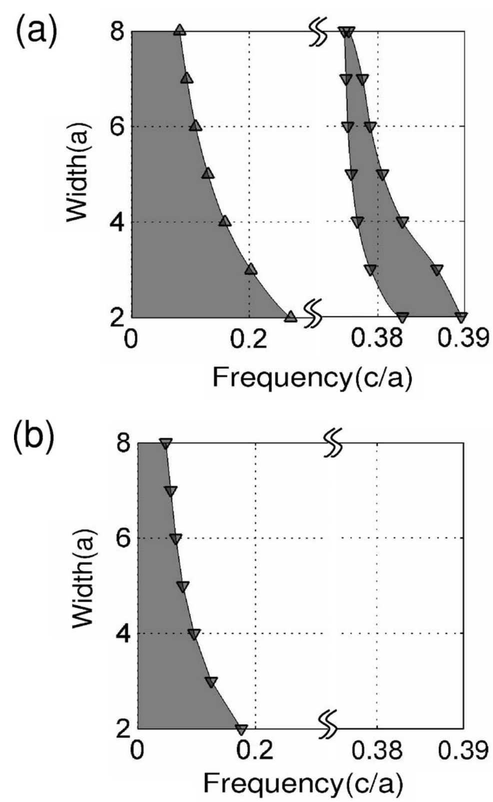

Fig. 4. Regions of frequency and width in which only a 0th-order mode is supported for (a) a photonic crystal core waveguide and (b) a conventional waveguide.

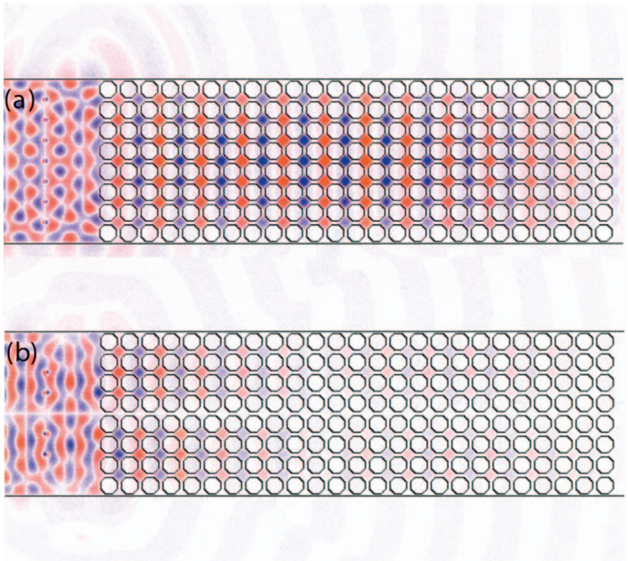

Fig. 5. Steady-state magnetic field distributions at frequency $f=0.37(c/a)$ for a photonic crystal core waveguide excited by sources with (a) even or (b) odd symmetry. The width of the waveguide is $8a$. The point dipoles are placed in positions that optimize the modal matching.

width d, $k_y d > \pi$ and the waveguide no longer supports the 0th-order mode (sometimes referred to as the fundamental mode). Instead, waveguides with different widths or at different frequencies can have very different lowest-order modes, as seen from the results of direct simulations of the dispersion relations of the waveguides (Fig. 2). In contrast, in a conventional symmetric dielectric slab waveguide, the lowest-order mode is always 0th-order mode at any frequency and width.

As an example, for a waveguide with a width of $2a$, the 0th-order mode does not exist in the frequency range 0.29–0.34(c/a). This can be directly visualized in three-dimensional finite-difference time-domain (FDTD) simulations, as shown in Fig. 3. At the steady-state frequency $f=0.3(c/a)$, a beam with an even modal profile cannot propagate through the waveguide, while a beam with a node in the horizontal plane is able to.

Figure 2 also reveals that with photonic crystals one can construct waveguides that are single mode, but the guided mode is not the 0th-order mode. Again, this is something that would be impossible to do with a conventional dielectric slab waveguide. As an example, the waveguide with $d=2a$ has two single-mode frequency regions: the frequency range 0.29–0.34(c/a), where it supports a single 1st-order mode, and the frequency range 0–0.27(c/a), where it supports a single 0th-order mode. In general, at a given frequency, by choosing the width such that wave vector $\vec{k}$ satisfies $(m-1)\pi < k_y d < (m+1)\pi$, we can design waveguides in which the mth-order mode will be the only guided mode.

We will now attempt to design a single-mode waveguide with a large cross section. This could be important for constructing large-core lasing structures in which modal competition is inherently suppressed.[12] To do so, we consider the second band [Fig. 1(c)]. When the frequency is close to the lower band edge, $k_y^{\max}$ could be infinitesimally small. Thus, the single-mode condition $k_y^{\max} d < \pi$ can still be satisfied even for a wide waveguide. Figure 4 compares regions of frequencies and width in which only the 0th-order mode is supported, for the photonic crystal core waveguides and the corresponding conventional dielectric waveguides. For the photonic crystal core waveguide, there is an additional single-mode frequency region in the frequency range of the second band.

To visualize the propagation characteristics of a waveguide in this frequency region, we choose $d=8a$ and perform three-dimensional FDTD simulations using sources with a frequency of 0.37(c/a) arranged with different symmetries. Only the fundamental mode can propagate in the photonic crystal core waveguide, and all higher-order modes are suppressed (Fig. 5). Also, since k_x is large, the fundamental mode is strongly confined in the waveguide region and decays rapidly in the air. In contrast, the corresponding conventional waveguide allows 18 guided modes at the same frequency.

This work was funded in part by Army Research Office–Multi-University Research Initiative grant DAAD19-03-1-0227 and by a David and Lucile Packard Fellowship. The simulations were performed through the support of a National Science Foundation–National Research Allocation Committee program. X. Yu (xfyu@stanford.edu) gratefully acknowledges the support of Herb and Jane Dwight Fellowships.

References

1. A. Mekis, J. C. Chen, I. Kurland, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **77**, 3787 (1996).
2. R. F. Cregan, B. J. Mangan, J. C. Knight, T. A. Birks, P. St. J. Russell, P. J. Roberts, and D. C. Allan, Science **285**, 1537 (1999).
3. J. C. Knight, T. A. Birks, P. St. J. Russell, D. M. Atkin, Opt. Lett. **21**, 1547 (1996).
4. M. Charbonneau-Lefort, E. Istrate, M. Allard, J. Poon, and E. H. Sargent, Phys. Rev. B **65**, 125318 (2002).
5. C. Martijn de Sterke, L. C. Botten, A. A. Asatryan, T. P. White, and R. C. McPhedran, Opt. Lett. **29**, 1384 (2004).
6. X. Yu and S. Fan, Phys. Rev. E **70**, 036612 (2004).
7. W. T. Lau and S. Fan, Appl. Phys. Lett. **81**, 3915 (2002).
8. C. R. Pollock, *Fundamentals of Optoelectronics* (Irwin, 1995).
9. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton, 1995).
10. S. G. Johnson and J. D. Joannopoulos, Opt. Express **8**, 273 (2001).
11. S. G. Johnson, S. Fan, P. R. Villeneuve, L. A. Kolodziejski, and J. D. Joannopoulos, Phys. Rev. B **60**, 5751 (1999).
12. A. E. Siegman, *Lasers* (University Science, 1986).