Omnidirectional reflection from a one-dimensional photonic crystal

Joshua N. Winn, Yoel Fink, Shanhui Fan, and J. D. Joannopoulos

Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139

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We demonstrate that one-dimensional photonic crystal structures (such as multilayer films) can exhibit complete reflection of radiation in a given frequency range for all incident angles and polarizations. We derive a general criterion for this behavior that does not require materials with very large indices. We perform numerical studies that illustrate this effect.

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Low-loss periodic dielectrics, or photonic crystals, allow the propagation of light to be controlled in otherwise difficult or impossible ways.[1–4] In particular, a photonic crystal can be a perfect mirror for light from any direction, with any polarization, within a specified frequency range. It is natural to assume that a necessary condition for such omnidirectional reflection is that the crystal exhibit a complete three-dimensional photonic bandgap, that is, a frequency range within which there are no propagating solutions of Maxwell's equations. Here we report that this assumption is false—in fact a one-dimensional photonic crystal will suffice. We introduce a general criterion for omnidirectional reflection for all polarizations and apply it to the case of a dielectric multilayer film. Previous attempts to attain high reflectance for a wide range of incident angles involved dielectric films with high indices of refraction, high special dispersion properties, or multiple contiguous stacks of films.[5–9]

A one-dimensional photonic crystal has an index of refraction that is periodic in the y coordinate and consists of an endlessly repeating stack of dielectric slabs, which alternate in thickness from d_1 to d_2 and in index of refraction from n_1 to n_2. Incident light can be either s polarized ($\mathbf{E}$ is perpendicular to the plane of incidence) or p polarized (parallel). Because the medium is periodic in y and homogeneous in x and z, the electromagnetic modes can be characterized by a wave vector $\mathbf{k}$, with k_y restricted to $0 \le k_y \le \pi/a$. We may suppose that $k_z = 0$, $k_x \ge 0$, and $n_2 > n_1$ without loss of generality. The allowed mode frequencies ω_n for each choice of $\mathbf{k}$ constitute the band structure of the crystal. The continuous functions $\omega_n(\mathbf{k})$, for each n, are the photonic bands.

For an arbitrary direction of propagation, it is convenient to examine the projected band structure, which is shown in Fig. 1 for a quarter-wave stack with $n_1 = 1$ and $n_2 = 2$. To make this plot we first computed the bands $\omega_n(k_x, k_y)$ for the structure, using a numerical method to solve Maxwell's equations in a periodic medium.[10] (In fact, for the special case of a multilayer film, an analytic expression for the dispersion relation is available.[11]) Then, for each value of k_x, the mode frequencies ω_n for all possible values of k_y were plotted. Thus in the gray regions there are electromagnetic modes for some value of k_y, whereas in the white regions there are no electromagnetic modes, regardless of k_y.

One obvious feature of Fig. 1 is that there is no complete bandgap. For any frequency there exists some electromagnetic mode with that frequency—the normal-incidence bandgap is crossed by modes with $k_x > 0$. This is a general feature of one-dimensional photonic crystals.

However, the absence of a complete bandgap does not preclude omnidirectional reflection. The criterion is not that there be no propagating states within the crystal; rather, the criterion is that there be no propagating states that can couple to an incident propagating wave. As we argue below, the latter criterion is equivalent to the existence of a frequency range in which the projected band structures of the crystal and the ambient medium have no overlap.

The electromagnetic modes in the ambient medium obey $\omega = c(k_x{}^2 + k_y{}^2)^{1/2}$, where c is the speed of light in the ambient medium, so generally $\omega > ck_x$. The whole region above the solid diagonal light lines $\omega = ck_x$ is filled with the projected bands of the ambient medium.

If a semi-infinite crystal occupies $y < 0$ and the ambient medium occupies $y > 0$, the system is no longer periodic in the y direction and the electromagnetic modes of the system can no longer be classified by a single value of k_y. These modes must be written as

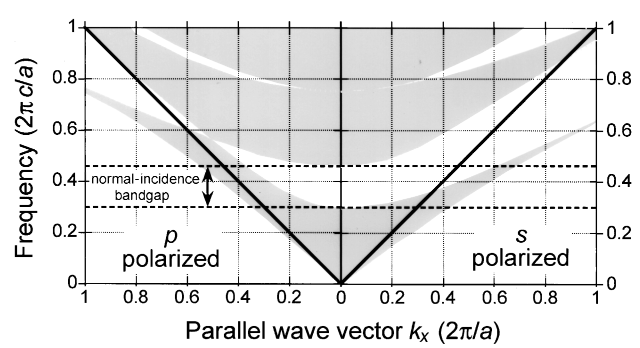

Fig. 1. Projected band structure for a quarter-wave stack with $n_1 = 1$ and $n_2 = 2$. Electromagnetic modes exist only in the shaded regions. The s-polarized modes are plotted to the right of the origin, and the p-polarized to the left. The dark lines are the light lines $\omega = ck_x$. Frequencies are reported in units of $2\pi c/a$.

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a weighted sum of plane waves with all possible k_y. However, k_x is still a valid symmetry label. The angle of incidence θ upon the interface at $y = 0$ is related to k_x by $\omega \sin\theta = ck_x$.

For there to be any transmission through the semi-infinite crystal at a particular frequency, there must be an electromagnetic mode available at that frequency that is extended for both $y > 0$ and $y < 0$. Such a mode must be present in the projected photonic band structures of both the crystal and the ambient medium. (The only states that could be present in the semi-infinite system that were not present in the bulk system are surface states, which decay exponentially in both directions away from the surface and are therefore irrelevant to the transmission of an external wave). Therefore, the criterion for omnidirectional reflection is that there exist a frequency zone in which the projected bands of the crystal have no states with $\omega > ck_x$.

In Fig. 1, the lowest two p bands cross at a point above the line $\omega = ck_x$, preventing the existence of such a frequency zone. This crossing occurs at the Brewster angle $\theta_B = \tan^{-1}(n_2/n_1)$, at which there is no reflection of p-polarized waves at any interface. At this angle there is no coupling between waves with k_y and $-k_y$, a fact that permits the band crossing to occur.

This difficulty vanishes when we lower the bands of the crystal relative to those of the ambient medium by raising the indices of refraction of the dielectric films. Figure 2 shows the projected band structure for the case $n_1 = 1.7$ and $n_2 = 3.4$. In this case there is a frequency zone in which the projected bands of the crystal and ambient medium do not overlap, namely, from the filled circle ($\omega a/2\pi c = 0.21$) to the open circle ($\omega a/2\pi c = 0.27$). This zone is bounded above by the normal-incidence bandgap and below by the intersection of the top of the first gray region for p-polarized waves with the light line.

Between the frequencies corresponding to the filled and open circles there will be total reflection from any incident angle for either polarization. For a finite number of films the transmitted light will diminish exponentially with the number of films. The calculated transmission spectra for a finite system of ten films (five periods) are plotted in Fig. 3 for various angles of incidence. The calculations were performed with transfer matrices.[12] The stop band shifts to higher frequencies with more-oblique angles, but there is a region of overlap that remains intact for all angles.

The graphic criterion for omnidirectional reflection is that the filled circle be lower than the open circle (the second band at $k_x = 0$, $k_y = \pi/a$). Symbolically,

$$\omega_{p1}\left(k_x = \frac{\omega_{p1}}{c}, k_y = \frac{\pi}{a}\right) < \omega_{p2}\left(k_x = 0, k_y = \frac{\pi}{a}\right), \tag{1}$$

where $\omega_{pn}(k_x, k_y)$ is the p-polarized band structure function for the multilayer film. Note that the left-hand side is a self-consistent solution for frequency ω_{p1}. The difference between these two frequencies is the range of omnidirectional reflection.

We calculated this range (when it exists) for a comprehensive set of film parameters. Since all the mode wavelengths scale linearly with $d_1 + d_2 = a$, we need consider only three parameters for a multilayer film: n_1, n_2, and d_1/a. To quantify the range of omnidirectional reflection (ω_1, ω_2) in a scale-independent manner, we report the range–midrange ratio, which is defined as $(\omega_2 - \omega_1)/\tfrac{1}{2}(\omega_2 + \omega_1)$.

For each choice of n_1 and n_2/n_1, there is a value of d_1/a that maximizes the range–midrange ratio. That choice can be computed numerically. Figure 4 is a contour plot of the ratio, as n_1 and n_2/n_1 are varied, for the maximizing value of d_1/a.

An approximate analytic expression for the optimal zone of omnidirectional reflection can be derived:

$$\frac{\Delta\omega}{2c} = \frac{a\cos\left(-\sqrt{\dfrac{A-2}{A+2}}\right)}{d_1 n_1 + d_2 n_2} - \frac{a\cos\left(-\sqrt{\dfrac{B-2}{B+2}}\right)}{d_1\sqrt{n_1{}^2 - 1} + d_2\sqrt{n_2{}^2 - 1}}, \tag{2}$$

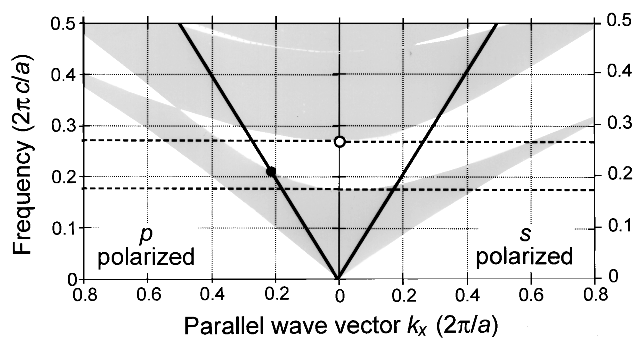

Fig. 2. Projected band structure for a quarter-wave stack with $n_1 = 1.7$ and $n_2 = 3.4$, with the same conventions as in Fig. 1.

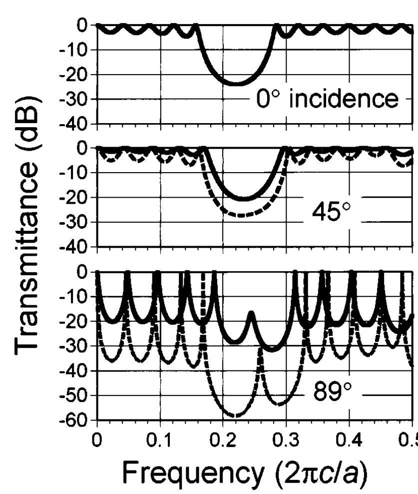

Fig. 3. Calculated transmission spectra for a quarter-wave stack of ten films ($n_1 = 1.7$, $n_2 = 3.4$) for three angles of incidence. Solid curves, p-polarized waves; dashed curves, s-polarized waves. The overlapping region of high reflectance (>20 dB) corresponds to the region between the open and filled circles of Fig. 2.

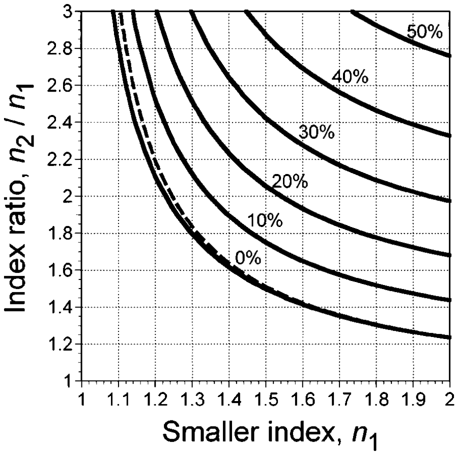

Fig. 4. Range–midrange ratio for omnidirectional reflection, plotted as contours. For the solid contours the optimal value of d_1/a was chosen. The dashed curve is the 0% contour for the case of a quarter-wave stack. For the general case of an ambient medium with index $n_0 \neq 1$, the abscissa becomes n_1/n_0.

where

$$A \equiv \frac{n_2}{n_1} + \frac{n_1}{n_2}\,, \qquad B \equiv \frac{n_2\sqrt{n_1{}^2 - 1}}{n_1\sqrt{n_2{}^2 - 1}} + \frac{n_1\sqrt{n_2{}^2 - 1}}{n_2\sqrt{n_1{}^2 - 1}}\,. \tag{3}$$

In deriving Eq. (2) we assumed that the optimal film is approximately a quarter-wave stack. Numerically we find this to be an excellent approximation for the entire range of parameters depicted in Fig. 4; the frequencies as predicted by this approximation are within 0.5% of the exact frequencies. As a result the optimization of d_1/a results in a range–midrange ratio very close to that which results from a quarter-wave stack with the same indices: $d_1/a = n_2/(n_2 + n_1)$. In Fig. 3, the 0% contour for quarter-wave stacks is plotted as a dashed curve, which is very close (always within 2% in the indices of refraction) to the numerically optimized contour.

It can be seen from Fig. 4 that, for omnidirectional reflection, the index ratio should be reasonably high (>1.5) and the indices themselves somewhat higher (by >1.5) than that of the ambient medium. The former condition increases the band splittings, and the latter depresses the frequency of the Brewster crossing. An increase in either factor can partially compensate for the other. The materials should also have a long absorption length for the frequency range of interest, especially at grazing angles, where the path length of the reflected light along the crystal surface is long.

Although we have illustrated our arguments by use of multilayer films, the notions in this Letter apply generally to any periodic dielectric function $n(y)$. What is required is the existence of a zone of frequencies in which the projected bands of the crystal and ambient medium have no overlap.

However, the absence of a complete bandgap does have physical consequences. In the frequency range of omnidirectional reflection there exist propagating solutions of Maxwell's equations, but they are states with $\omega < ck_x$ and decrease exponentially away from the crystal boundary. If such a state were launched from within the crystal, it would propagate to the boundary and reflect, just as in total internal reflection.

Likewise, although it might be arranged that the propagating states of the ambient medium do not couple to the propagating states of the crystal, any evanescent states in the ambient medium will couple to them. For this reason, a point source of waves placed very close ($d < \lambda$) to the crystal surface could indeed couple to the propagating state of the crystal. Such restrictions, however, apply only to a point source, and one can easily overcome them by simply adding a low-index cladding layer to separate the point source from the film surface.

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J. D. Joannopoulos' e-mail address is joannop@mit.edu.

References

1. E. Yablonovitch, Phys. Rev. Lett. **58,** 2059 (1987).
2. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (U. Princeton Press, Princeton, N.J., 1995).
3. J. Pendry, J. Mod. Opt. **41,** 209 (1994).
4. J. D. Joannopoulos, P. R. Villeneuve, and S. Fan, Nature (London) **386,** 143 (1997).
5. P. Baumeister, Opt. Acta **8,** 105 (1961).
6. J. D. Rancourt, *Optical Thin Films User Handbook* (SPIE Optical Engineering Press, Bellingham, Wash., 1996), pp. 68–71.
7. P. Yeh, *Optical Waves in Layered Media* (Wiley, New York, 1988), pp. 161–163.
8. K. V. Popov, J. A. Dobrowolski, A. V. Tikhonravov, and B. T. Sullivan, Appl. Opt. **36,** 2139 (1997).
9. M. H. MacDougal, H. Zhao, P. D. Dapkus, M. Ziari, and W. H. Steier, Electron. Lett. **30,** 1147 (1994).
10. R. D. Meade, A. M. Rappe, K. D. Brommer, and J. D. Joannopoulos, Phys. Rev. B **77,** 8434 (1993).
11. P. Yeh, A. Yariv, and C.-S. Hong, J. Opt. Soc. Am. **67,** 423 (1977).
12. E. Hecht, *Optics,* 2nd ed. (Addison-Wesley, Reading, Mass., 1987), pp. 373–378.