# Cross-Polarization Modulation in DWDM Systems

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**Abstract**

*A statistical assessment of the nonlinear cross-polarization modulation is presented. The resulting distribution of the states of polarization is derived while accounting for various properties of the link.*

## Introduction

Cross-polarization modulation (XPolM) is a nonlinear process in which the state of polarization (SOP) of an optical signal in a (D)WDM system and the PMD vector of the fiber are randomly modulated on a timescale of the symbol duration. This generates crosstalk in polarization-multiplexed systems and degrades PMD compensation. Based on earlier work [1], we present a statistical model for XPolM which is valid for any modulation formats, channel load, dispersion maps and fiber PMD, and provide the necessary knowledge to assess the XPolM impact in arbitrary systems.

## Statistical Model of XPolM

To examine the statistical properties of XPolM in a wavelength channel of a WDM system, which we will denote the probe channel, it is beneficial to create a reference frame in which the effects of group velocity and birefringence at the carrier frequency of the probe are mathematically eliminated, as in [2]. We are then left with GVD, PMD and the (Kerr) nonlinearities. In the case of randomly birefringent fiber, the signal propagation can be calculated with the Manakov (PMD) equation. In this frame, the effect of XPolM is described as a rotation of the signal state of polarization around the sum of Stokes vectors of all wavelength channels in the WDM system [2,3].

This sum, $\mathbf{S}_\Sigma$, comprises many vectors which are random in length and orientation and varying at different rates, depending on the spectral distance from the probe channel. It may have a mean value, resulting in an irrelevant mean rotation, around which it fluctuates. These fluctuations are too fast to be compensable either optically or electrically, varying generally from bit to bit. These fluctuations can be described by the PDE

$$\frac{\partial \widehat{\mathbf{S}}}{\partial z} = \widetilde{\gamma}\, \mathbf{S}_\Sigma \times \widehat{\mathbf{S}}, \qquad (1)$$

where $\mathbf{S}_\Sigma$ is, in a simple but often valid approximation, isotropic.

Eq. (1) describes a series of rotations of $\widehat{\mathbf{S}}$ with random axis and angular velocity. Such a process has been examined by Roberts and Ursell [4] who derived the resulting Brownian distribution of SOPs, so called in acknowledgment of the resemblance to a Brownian motion on the surface of the unit (Poincaré) sphere. The symmetric Brownian distribution requires only a single parameter, $V$, which can be shown to be related to the DOP $\mathcal{D}$ as

$$\mathcal{D} = \exp\big(V/2\big). \qquad (2)$$

To quantify the expected XPolM magnitude, it suffices to determine the $V$ parameter for a fiber system.

## Estimation of XPolM Magnitude

The $V$ parameter is described in [4] as the variance of an equivalent plane motion. Its statistics are closely connected to the statistics of the Stokes sum $\mathbf{S}_\Sigma$. Under the assumption of an isotropic $\mathbf{S}_\Sigma$, it is possible, though somewhat laborious, to derive the following relation:

$$V = \frac{2}{3}\widetilde{\gamma}^2 \iint_0^L \mathcal{C}(z_1, z_2)\, dz_1\, dz_2, \qquad (3)$$

in which $\mathcal{C}(z_1, z_2)$ is the autocovariance function (ACF) of the stochastic process $\mathbf{S}_\Sigma$:

$$\mathcal{C}(z_1, z_2) = \Big\langle \mathbf{S}_\Sigma(z_1) \cdot \mathbf{S}_\Sigma(z_2) \Big\rangle - \Big\langle \mathbf{S}_\Sigma(z_1) \Big\rangle \cdot \Big\langle \mathbf{S}_\Sigma(z_2) \Big\rangle \qquad (4)$$

and $L$ is the length of the link. It follows that we can determine the XPolM magnitude from the statistical properties of the Stokes vectors of the various channels in the WDM system alone. The ACF of the sum is then just the sum of the ACFs of the individual channels (because they are uncorrelated). In a first-order approximation, we consider only the dominating effects of walk-off between channels due to GVD on the magnitude $|\mathbf{S}_n| = P_n$ of the Stokes vectors, SOP $\widehat{\mathbf{S}}_n$ decorrelation due to PMD, and attenuation –all of which are statistically independent– and thus write the ACF as the product

$$\mathcal{C}(z_1, z_2) = \sum_n \exp\Big[\mathcal{A}(z_1) + \mathcal{A}(z_2)\Big] \times \mathcal{C}_n^{\mathrm{WO}}(z_1, z_2)\, \mathcal{C}_n^{\mathrm{PMD}}(z_1, z_2) \qquad (5)$$

Herein, $\mathcal{A}(z)$ is a power normalization function accounting for attenuation $\alpha$ and gain $g$:

$$\mathcal{A}(z) \equiv \int_0^z \Big[g(\zeta) - \alpha(\zeta)\Big]\, d\zeta. \qquad (6)$$

The function $\mathcal{C}_n^{\mathrm{WO}}$ takes into account the walk-off between channel $n$ and the probe which occurs due to the different group velocities of both channels in the presence of GVD:

$$\mathcal{C}_n^{\mathrm{WO}}(z_1, z_2) = \frac{1}{T}\int_0^T P_n(z_1, t)\, P_n(z_2, t)\, dt - \Big\langle P_n \Big\rangle^2 \qquad (7)$$

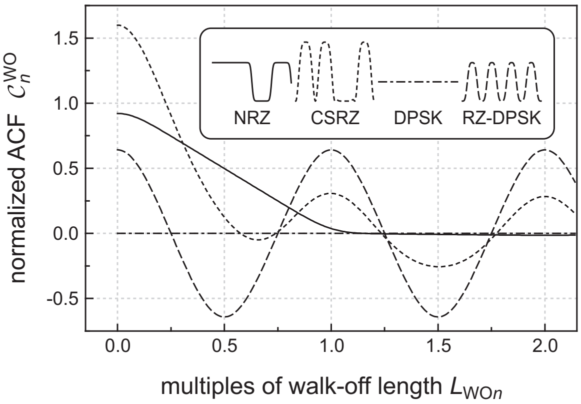


*Figure 1: Comparison of the walk-off ACF $\mathcal{C}_n^{WO}$ for several popular modulation formats.*

The walk-off rate depends largely on fiber dispersion $\beta_2$ and frequency separation $\Delta\omega_n$ between channel $n$ and the probe, and it is characterized by the walk-off length

$$L_{WOn} = T_S \Big/ \beta_2 \, \Delta\omega_n \qquad (8)$$

with symbol duration $T_S$. The walk-off-related ACFs for several popular modulation formats are shown in Fig. 1. It is obvious that different modulation formats will behave quite differently, e.g. (purely phase-modulated) DPSK channels ideally cause no XPolM at all.

The PMD-related ACF of the SOPs can be obtained from an expression found by Karlsson [5]:

$$\mathcal{C}_n^{PMD}(z_1, z_2) = \exp\left(-\frac{\pi \, \Delta\omega_n^2 \, D_{PMD}^2 \, |z_1 - z_2|}{8}\right), \qquad (9)$$

in which $D_{PMD}$ is the fiber PMD coefficient. The origin of this decorrelation lies in the frequency dependence of fiber birefringence. As every channel experiences a different magnitude of fiber birefringence, its SOP evolves differently. It is a random process in which the SOP eventually loses all memory of its initial orientation at $z_1$ and becomes equally probable to have any orientation on the Poincaré sphere.

**Results and Discussion**

Fig. 2 compares the DOP considering only walk-off ($D_{PMD} = 0$), additionally accounting for the SOP decorrelation due to PMD, and numerical simulations for a typical lightwave system. While the simple approximation with walk-off only overestimates the DOP for multi-span systems, the more sophisticated estimation according to (5) matches the numerical results well. Fig. 3 shows the combined dependence of the DOP on GVD and PMD for the NRZ DWDM system of Fig. 2 after propagation of 10 spans. While the approximations made here are less accurate for very small values of either GVD or PMD, it is clear that such systems in which GVD and/or PMD are very small will be most affected by XPolM.

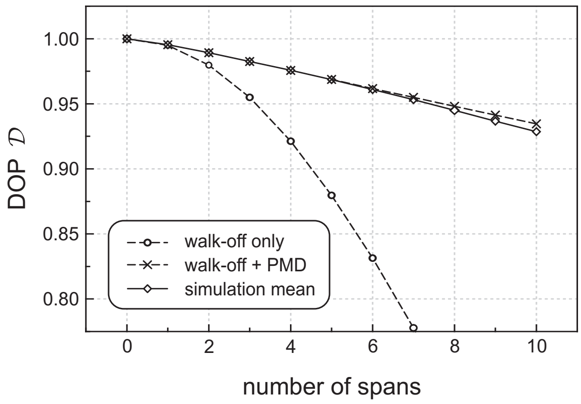


*Figure 2: Comparison of DOP estimates accounting for walk-off, additionally for SOP decorrelation, and simulation results ($N_{ch} = 11$, $D_{PMD} = 0.5\,ps/\sqrt{km}$ (all-order compensated), $D = 16\,ps/nm/km$ (fully in-line compensated), $P_{ch,avg} = 4\,mW$, $\widetilde{\gamma} = \frac{8}{9} \cdot 1.31 \cdot 10^{-3}\,(Wm)^{-1}$, $B = 10\,Gbps$ (NRZ), $\Delta f = 50\,GHz$).*

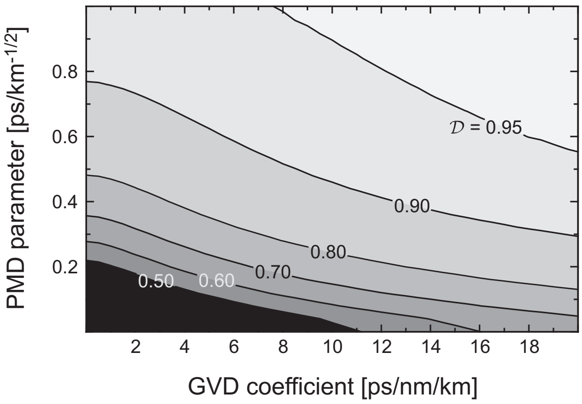


*Figure 3: XPolM-related reduction of the residual DOP (after all-order PMD compensation) as a function of fiber GVD coefficient $D$ and PMD parameter $D_{PMD}$.*

**Summary**

We have presented a statistical model of the XPolM-induced reduction of the DOP, which occurs in all kinds of DWDM systems. The so obtained DOP can be used to quantify the effects of XPolM and compare its magnitude in arbitrary optical links. Together with the resulting SOP distribution, which has also been presented herein, it becomes possible to determine the penalties due to XPolM in e.g. polarization-division multiplex systems.

**References**

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