

# Phase-Noise Generation in an Amplitude Limiter Using Saturation of a Fiber-Optic Parametric Amplifier

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## Abstract

Generation of phase noise in output signals of an amplitude limiter using saturation of FOPA is analyzed. It is shown that optimum pump power exists for minimum output phase noise.

## Introduction

Phase-preserving amplitude regeneration improves performance of long-distance transmission of phase-shift keying optical signals because it can reduce nonlinear phase noise that is caused by the translation of amplitude to phase noise via the nonlinearity of transmission fibers. One promising candidate for such a phase-preserving amplitude regenerator or limiter is a fiber-optic parametric amplifier (FOPA) operated in saturation [1]. Because the FOPA relies on ultrafast nonlinearity in fibers, it suppresses pulse-to-pulse amplitude fluctuation in high-speed signals [2]. Although signal-phase variation in the course of amplitude stabilization by the FOPA is smaller than other types of amplitude regenerators based on self-phase modulation (SPM), some residual phase noise is inevitably induced by the nonlinear process in the FOPA [3]. This should be avoided as far as possible especially when the signal is repeatedly regenerated in the system. In this paper, the phase noise generation in the optical limiter based on saturation of FOPA is semi-analytically analyzed. Contributions from pump amplitude noise to the output signal-phase noise via the cross-phase modulation (XPM) recently discussed in [4] are quantified and it is shown that optimum pump power exists for minimum output signal-phase noise.

## Analysis of Phase Variation

The analysis of the saturated single-pump FOPA in this paper is based on the following set of coupled differential equations [5]:

$$dA_p/dz = -(\alpha/2)A_p - 2\gamma A_p A_s A_i \sin \theta \quad (1a),$$

$$dA_s/dz = -(\alpha/2)A_s + \gamma A_p^2 A_i \sin \theta \quad (1b),$$

$$dA_i/dz = -(\alpha/2)A_i + \gamma A_p^2 A_s \sin \theta \quad (1c),$$

$$d\theta/dz = \Delta\beta + \gamma[2A_p^2 - A_s^2 - A_i^2 + (A_p^2 A_i / A_s + A_p^2 A_s / A_i - 4A_s A_i) \cos \theta] \quad (1d),$$

where  $A_p \exp(i\theta_p)$ ,  $A_s \exp(i\theta_s)$ , and  $A_i \exp(i\theta_i)$  are the amplitudes of pump, signal, and idler, respectively,  $\theta = \Delta\beta z - 2\theta_p + \theta_s + \theta_i$  with  $\Delta\beta = \beta_2(2\pi\Delta\nu)^2$ , where  $\beta_2$  and  $\Delta\nu$  are the GVD coefficient at the pump wavelength and the frequency separation between the signal and the pump, respectively.  $\alpha$  and  $\gamma$  are the loss and nonlinearity coefficients of the fiber. Solving (1) with

appropriate initial conditions  $A_i(0)=0$  and  $\theta(0)=\pi/2$ , we have solutions of the amplified signal  $A_s(L)\exp[i\theta_s(L)]$ , the generated idler  $A_i(L)\exp[i\theta_i(L)]$ , and the depleted pump  $A_p(L)\exp[i\theta_p(L)]$  at the fiber output  $z=L$ .

For the analysis of generation of phase noise, the expressions  $A_m(z)=A_{m0}(z)+\Delta A_m(z)$  and  $\theta_m(z)=\theta_{m0}(z)+\Delta\theta_m(z)$  ( $m= p, s, \text{ or } i$ ) are substituted into (1) with  $A_{m0}$  and  $\theta_{m0}$  are the solutions of (1). By solving the linearized coupled differential equations for  $\Delta A_m$  and  $\Delta\theta_m$  ( $m= p, s, \text{ or } i$ ), we obtain relations between the deviations of amplitudes and phases at the input and output ends of the nonlinear fiber such as

$$\Delta A_s(L) = a_{ss}\Delta A_s(0) + a_{sp}\Delta A_p(0),$$

$$\Delta\theta_s(L) = b_{ss}\Delta A_s(0) + b_{sp}\Delta A_p(0).$$

Coefficients  $a_{ss}$  and  $b_{ss}$  represent amplitude-noise reduction and SPM-induced phase noise by the amplitude limiter.  $a_{sp}$  and  $b_{sp}$ , on the other hand, represent induced amplitude and phase noise caused by pump noise via gain modulation and XPM, respectively. By using these coefficients we can calculate signal-to-noise ratio (SNR) of the output signal amplitude  $\rho_{\text{sig,out}} = A_{s0}^2(L) / \langle \Delta A_s^2(L) \rangle$  and variance of the signal phase noise  $\sigma_{\text{phase}}^2 = \langle \Delta\theta_s^2(L) \rangle$  in terms of amplitude SNRs of the input signal and pump,  $\rho_{\text{sig,in}} = A_{s0}^2(0) / \langle \Delta A_s^2(0) \rangle$  and  $\rho_{\text{pump,in}} = A_{p0}^2(0) / \langle \Delta A_p^2(0) \rangle$

It is noted that (1a)-(1d) becomes inaccurate when the generation of frequency components other than signal, pump, and idler is significant. In the following numerical results an additional frequency component at  $2\omega_s - \omega_p$  is included in the formulation, which is needed especially for the analysis of low pump-power operation.

## Numerical Results

Parameters of the fiber used in the numerical analysis are  $L=1\text{km}$ ,  $\gamma=12\text{W/km}$ , and the dispersion slope  $dD/d\lambda = 0.03 \text{ ps/nm}^2/\text{km}$ . Difference between the pump and the zero-dispersion wavelengths is  $\lambda_p - \lambda_0=3\text{nm}$ . Effect of fiber loss is neglected.

Fig.1 shows saturation behavior of the FOPA, where output signal power  $A_s^2(L)$  is plotted versus input signal power  $A_s^2(0)$  for three different pump powers  $P_p = A_p^2(0)$ . Signal frequency is set at the peak position in the unsaturated gain spectra for each

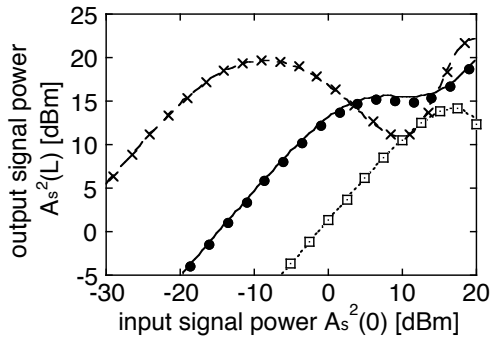


Figure 1: Saturation behavior of a FOPA. Curves with crosses, dots, and squares are for  $P_p=400$ ,  $200$ , and  $50$ mW, respectively. Continuous curves are the results based on the coupled differential equations including four frequency components while symbols are exact numerical results.

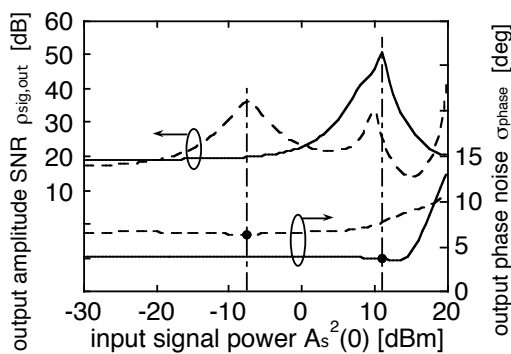


Figure 2: Amplitude SNR and standard deviation of phase noise of the output signal from the FOPA with  $P_p=200$ mW (solid curves) and  $400$ mW (dashed curves). Vertical lines indicate optimum operation points of the limiter for each pump power.

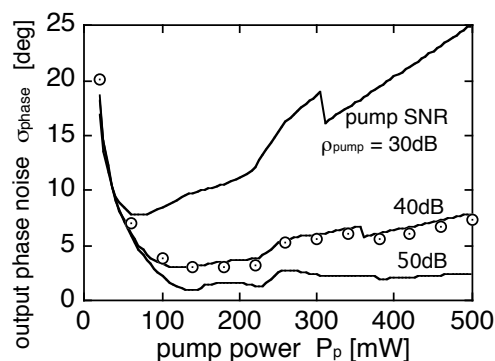


Figure 3: Output phase noise induced by the input amplitude noise versus pump power. Contributions from initial amplitude noise both of signal and of pump are included.

pump power,  $\Delta v = (-2\gamma P_p / \beta_2)^{1/2} / (2\pi)$ . It is shown that the input signal power at which the output signal power

starts to saturate is increased as the pump power and, therefore, the unsaturated gain are decreased.

Fig.2 shows the output signal SNR  $\rho_{sig,out}$  and phase noise  $\sigma_{phase}$  versus input signal power for two different pump powers  $P_p=200$  and  $400$ mW. SNRs of input signal and pump amplitudes are fixed at  $20$  and  $40$ dB, respectively. Fig.2 shows that output SNR takes maxima at input powers at which the output signal power levels off as shown in Fig.1. It is also shown that the output phase noise is significantly larger for larger pump power, which is due to the XPM-mediated phase noise [4]. At the optimum operation point of the limiter, at which the output SNR becomes maximum, the output phase noise is smaller for the lower pump power. At further lowered pump power, however, saturation of output signal power takes place at yet higher input signal power. Then the contribution of SPM-induced phase noise becomes dominant, which is shown by a growth in output phase noise as the input signal power becomes larger than  $\sim 10$ dBm in Fig.2.

Fig.3 shows the output phase noise at optimum input signal power versus pump power. Input signal SNR is fixed at  $20$ dB. It is shown that optimum pump power exists at which the XPM and SPM-induced phase noises compromise. Kinks in the curves are due to discontinuous change in optimum input signal power that happens when multiple peaks exist in the curve of the output signal SNR versus input power such as those shown in Fig.2. In Fig.3 results of totally numerical calculation using the split-step Fourier method are also plotted as circles for the pump SNR of  $40$ dB. The calculation assumes a NRZ binary PSK format for the input signal and the circular Gaussian noise is added to the complex amplitude. At the output of the limiter, phase noise induced by the initial amplitude noise is evaluated at bit centers. The result agrees well with that obtained by the above-discussed semi-analytical approach.

**Conclusion**

Phase noise generation by an all-optical amplitude limiter using saturation of FOPA was analyzed. It was shown that the amplitude noise on high-power pump induces large phase noise to the amplitude-limited output signal. There exists optimum pump power for minimum output phase noise.

**References**

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