Principal Mode Coefficients for Multimode Fibers

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Abstract

For the first time to our knowledge the computation of the expansion coefficients of the principal modes of multimode fibers is reported.

Introduction

Multimode fibers (MMFs) traditionally considered for many years as a transmission medium with limited bandwidth performance as compared to singlemode fibers have recently demonstrated their feasibility to support broadband transmission by means of several techniques, including subcarrier multiplexing (SCM) [1], mode group multiplexing (MGM) [2], optical frequency multiplication [3], and, orthogonal frequency division multiplexing (OFDM) [4].

To fully understand the performance of MMF links it is essential to analyze the effect of modal dispersion and its interplay with mode coupling. Although this is traditionally carried out using coupled mode theory the resulting equations do not lead to an easy interpretation of the results. Very recently however it has been shown by Fan and Kahn [5] that multimode fibers support principal modes (PMs) which play a similar role than that of the principal states of polarization (PSPs) in a singlemode fiber [6]. PMs do not suffer from modal dispersion to first order of frequency variation and form orthogonal bases both at the input and the output of the fiber. They constitute therefore an ideal tool for deriving a more amenable formalism for the analysis of propagation through MMFs.

There are however differences between PSPs and PMs. One of the most important is that, both PMs at the fiber input and output depend on the fiber link length z and this is also true as far as the expansion coefficients (those required to express the input electric field to the fiber in terms of the input PM base) are concerned. Without the knowledge of the later, the technique based on the expansion of the signal in terms of the input PM is of limited application. To the best of our knowledge this has not been reported so far.

In this paper we provide, for the first time to our knowledge, a very simple derivation of such coefficients which is based on the derivation of the MMF link transfer function using the PM model and then comparing the result with that previously obtained by us [7] using the coupled mode theory. The results confirm that the expansion coefficients depend on the MMF link length as expected. These results open new perspectives for the use of the PM model to the analysis of both digital and analog MMF systems limited by intermodal dispersion.

Principal Modes Coefficient derivation

To this purpose we express the input field to the multimode fiber in terms of its input principal modes $\{|a_i(z)\rangle\}$, i=1,2….N:

$$E_{in}(t) = u(t)\sum_{i=1}^{N} c_i(z)|a_i(z)\rangle \tag{1}$$

where $c_i(z)$ is the (z-dependent) coefficient corresponding to the principal mode $|a_i(z)\rangle$ and $u(t)$ is the information bearing time domain signal which is given by:

$$u(t) = \sqrt{S(t)}f(t) \tag{2}$$

with $S(t)$ representing the modulating signal and $f(t)$ the continuous-wave optical carrier. The output electrical field from the multimode fiber link will be given by:

$$E_{out}(t) = \sum_{i=1}^{N} c_i(z)\left[u(t) * h_i(t)\right]|b_i(z)\rangle \tag{3}$$

where $h_i(t)$ represents the MMF impulse response corresponding to the principal mode "i" and $\{|b_i(z)\rangle\}$ are the output principal modes of the multimode fiber. We consider the case where the impulse response for the principal mode "i" is given by:

$$h_i(t) = \frac{1}{\sqrt{2\pi j\beta_2 z}} e^{-\frac{(t-\tau_i)^2}{2j\beta_2 z}} \tag{4}$$

In other words, each principal mode has a different group delay τ_i and also experiences the effect of the fiber first order chromatic dispersion parameter β_2.

To compute the MMF transfer function we consider an analog single tone RF modulating signal:

$$\sqrt{S(t)} = \sqrt{P}\left[1 + \frac{m_o}{8}(1 + j\alpha_c)e^{j\Omega t} + \frac{m_o}{8}(1 + j\alpha_c)e^{-j\Omega t}\right] \tag{5}$$

where P is the RF power, $m_o << 1$ the modulation index, α_c the source chirp parameter and Ω the frequency of the modulating RF tone. If we assume as in [7] that the source autocorrelation function is given by a Gaussian function:

$$R(t'-t'') = e^{-\frac{(t'-t'')^2}{2\sigma_c^2}} \qquad (6)$$

where: $\sigma_c \approx 1/(\sqrt{2}W)$ is the source RMS coherence time and W is the source RMS linewidth. From (1)-(6) one gets the following transfer function for the MMF link:

$$H(\Omega) = \sqrt{1+\alpha_c^2}\, e^{-\frac{1}{2}\left(\frac{\Omega\beta_2 z}{\sigma_c}\right)^2} \cos\left(\frac{\Omega^2\beta_2 z}{2} - a\tan(\alpha_c)\right) \cdot \sum_{k=1}^{N} \left|c_k \varepsilon_k^b\right|^2 e^{j\Omega\tau_k} \qquad (7)$$

If we compare (7) with the transfer function obtained in [7] using the coupled mode theory for mode groups k=1,2,…N we finally deduce the expression for the expansion coefficients as:

$$\left|c_k \varepsilon_k^b\right|^2 = 2k(C_{kk} + G_{kk}) \qquad (8)$$

where C_{kk} is the light injection coefficient [see Eq. (5) of [7] for definition] and G_{kk} is the modal-coupling coefficient defined for power transitions only between adjacent mode groups. Both have been defined in terms of the relevant MM fiber parameters in [7].

Results

Equation (8) shows that the principal mode expansion coefficients depend on both the light injection condition at the input end of the MMF and the coupling between the mode groups propagated through the fiber. The evaluation of the intermodal coupling influence is shown in Fig.1, where the dependence of the coefficients $\left|c_k \varepsilon_k^b\right|^2$ (normalized to their maximum value) with the normalized distance *z*/*D* for different values of the variance of the core deformation function *f*(*z*) have been plotted for a silica 62.5-µm core-diameter multimode fiber with a parabolic core grading, i.e. $\alpha = 2$. *D* represents the mode coupling correlation length. It has been assumed a uniform distribution of the light injection coefficients C_{kk} and an optical source emitting at 1310 nm. As it was expected from (8) and the results of [7] the coefficients $\left|c_k \varepsilon_k^b\right|^2$ behave as an inverse function of the distance, converging more rapidly to a constant value for decreasing values of the coupling length variance σ^2. The influence of the offset launch technique is evaluated in Fig. 2 for a 500 m MMF link with the same characteristics as the previously simulated one. The variance of the core deformation function has been fixed to $\sigma^2 = 510^{-3}$ m^2 and the correlation length to D = 1e-3 m. Fig. 2 shows the normalized expansion coefficients versus the principal mode number "*k*" for different Gaussian distributions of the C_{kk} coefficients centered at *k* = 1, *N*/4, *N*/2, 3*N*/4 and *N*, (where in this case we obtain *N* = 12), compared with the uniform one. Firstly, we observe the quadratic dependence with the principal mode number power to its lower-order adjacent mode group. Secondly, it is shown that the principal mode number dependence follows the behavior of the mode injection distribution since the maximum expansion coefficients values (near 0 dB) are related to the applied offset values.

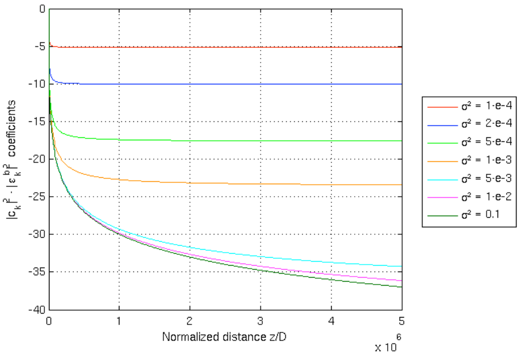

Figure 1: . Influence of the normalized distance z/D on the expansion coefficients for uniform launch distribution.

The influence of the graded-index exponent α and the optical emission wavelength λ on the expansion coefficients were also analyzed, concluding that their effect was almost negligible.

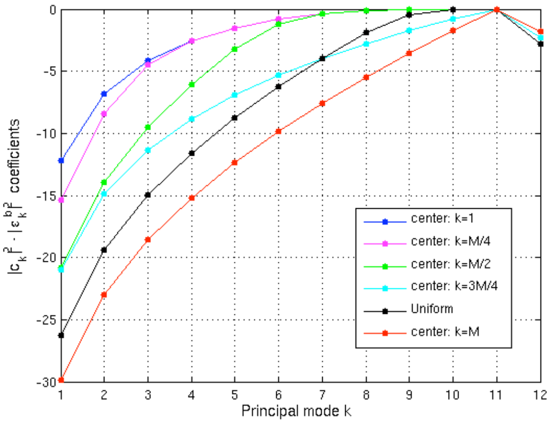

Figure 2: Influence of the offset launch technique on the expansion coefficients in a 500 m link.

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