What is the Relatedness of Mathematics and Art and why we should care?

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There have been a wide range of any human activities concerning the term of “Art and Mathematics”. Regarding directly to the historical root, there are a great deal of discussions on art and mathematics and their connections. As we can see from the artworks in the Renaissance Age, let’s say Leonardo da Vinci (1452-1519) did a lot of art pieces in the mathematical precisions. Even more, the works of Luca Pacioli (1145-1514), Albrecht Dürer (1471-1528), and M.C. Escher (1898-1972) have shown us that there are geometrical patterns laid upon artistic artifacts. It is clear that at some time, some mathematicians have become artists, often while pursuing their mathematics (Malkevitch, 2003).

There should be three options arise when we talk about the relatedness between mathematics and art, namely:

1. The art of mathematical concept

As stated by Aristotle in his Metaphysica,

> the mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.

there are obviously certain beautiful parts in mathematics. This is a way of seeing mathematical works in the perspective of beauty, and any non-mathematical expressions may come up when we understand a mathematical concept. Great mathematical works are often to be celebrated as a state of the art, and as written by mathematician Godfrey H. Hardy (1941),

> The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

There are certain things in the matter of beauty when we talk about a mathematical concept. For a short example, we can see a very nice and beautiful mathematical model of neural network, when we see how it tries to approach a biological system and in the other hand is used to stunningly predict a financial time-series as a statistical tool (Situngkir & Surya, 2004). Yes, there is beauty in mathematical concepts and no doubt of it among them understand, notwithstanding, the beauty is absent in the eyes of laypersons.

2. The mathematical view of artworks

This perspective comes from what we can see from an artwork in the perspective of structural pattern. An artwork may come from anything, ideas, traditions, etc. and unintendedly can be seen with inclusion of geometrical pattern which is mathematical. The concept of symmetry is very famous when we talk about "tribal" art. For example, when we see Indonesian traditional works, batik, for example, it is obvious that we meet a lot of symmetrical pattern in it.

Figure 1
Self-similarity geometry in Batik: Javanese Batik (*top*) and Papuan Asmat batik (*bottom*)

As figured in figure 1, there are pattern of the so called mathematical term "self similarity" in them. A more advance of this concept can also be seen in architectural works, e.g. ancient religious temples. If we see the Hindi Prambanan Temple, it is obvious that the concept of the self-similarity exists. There are patterns of small parts of the giant temple depicting the whole big part of it (see Situngkir & Hariadi, 2005 for more visualization). We can see this in figure 2. Surely, in this term we are not talking about an aesthetic of an object at all but the mathematical beauty and vibe visualized in it.

Figure 2
Pattern of self-similarity in Prambanan Temple

The concept of the self-similarity is well-known in a mathematical domain, fractal. However, in its original form, there are many fascinating and beautiful visualization as we gaze a figure of fractal. Interestingly, this concept is not only seen in visual art, but also in music. A part of Johann Sebastian Bach (1685-1750), the Trias Harmonica for 8 canon instruments, shows how the composer wrote a very short musical note to be played creatively by musicians. The notation gives all the necessary pitch and rhythmic information, but the operations of transposition, time and pitch intervals, and symmetry operations must be determined by the performers. Somehow, this short note depicted all the playable melodies, pitch, and whatnot fascinatingly (Solomon, 2002).

Of course, we cannot be sure that the pattern is made up by the awareness of such concept, but to day we can see the artworks in such mathematical order. However, as noted by Malkevitch (2003),

> whereas an artist may choose to create a pattern with absolute and strict adherence in all details to have symmetry in the pattern, this is not all that common for "tribal" artists or artisans. Thus, if one looks carefully at a rug which at first view looks very symmetrical, it is common to see that at a more detailed level it is not quite totally symmetric either in the use of the design or of the colors used in different parts of the design. One can see the small liberties that are taken either because of the difficulty of making patterns exact by hand or because the

> artist wants consciously to make such small variations. In analyzing the symmetry of such a pattern it probably makes sense to idealize what the artist has done before applying some mathematical classification of the symmetry involved.

However, this kind of art analysis is somehow become interesting to extract a lot of information hidden in many tradition artifacts and artworks. It is common that to day we do not know a lot about how some ancient things or buildings were made, but we are aware that some lost civilizations were having some interesting techniques that can inspire us to day in modern time. Here, mathematical perspective on artworks is a challenging method to do that.

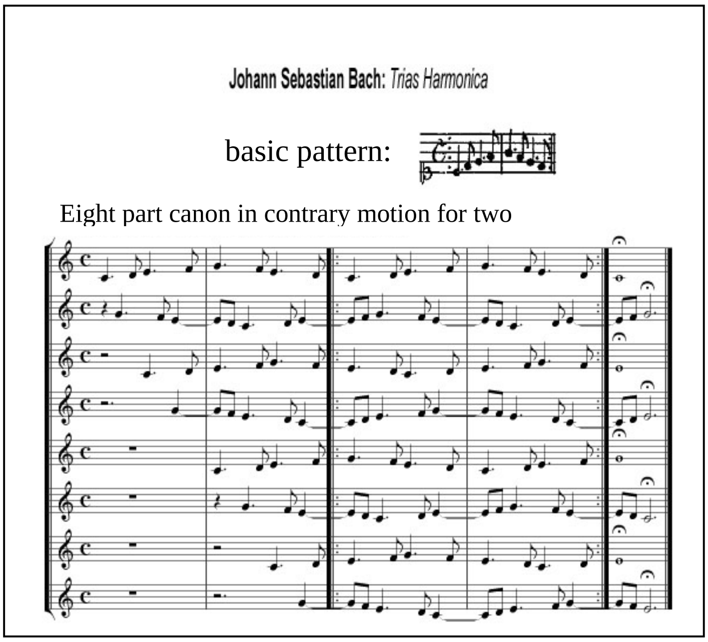

Figure 3
A solution of Bach's music, the Trias Harmonica (courtesy by Solomon, 2002).

3. Artworks from Mathematical Concepts

It is common that some mathematical and scientific concepts have given the inspirational contributions to a lot of artists around the world in all times. Probably, it started by Leonardo da Vinci on showing the geometrical aspects of human body in his famous Vitruvian Man. It shows a man within a circle and a square - an illustration of the proportional canon of ancient Roman architect, Vitruvius.

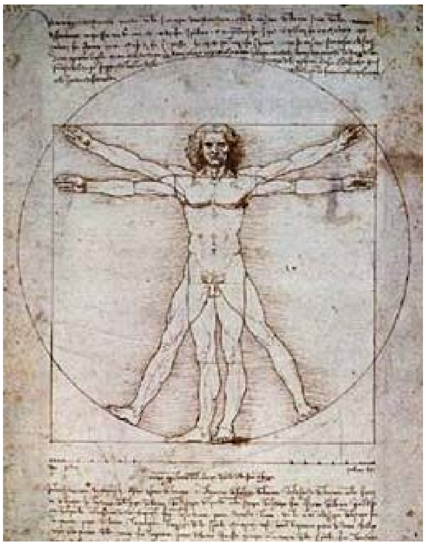

Figure 4
The famous Vitruvian Man by Leonardo da Vinci

Another artists inspired by geometrical concepts are M. C. Escher. The concept of Escher can also be attributed as generative art, making artworks that can be generated from such artistic rules of drawing. One of his famous works and a technique of generating such art by Escher are shown in figure 5. The work of Escher can be understood as a work of tessellation, a pattern made up of one or more shapes, completely covering a surface without any gaps or overlaps. From the figure we can see that the pattern of reptile can be "generated" by using the hexagonal tessellation.

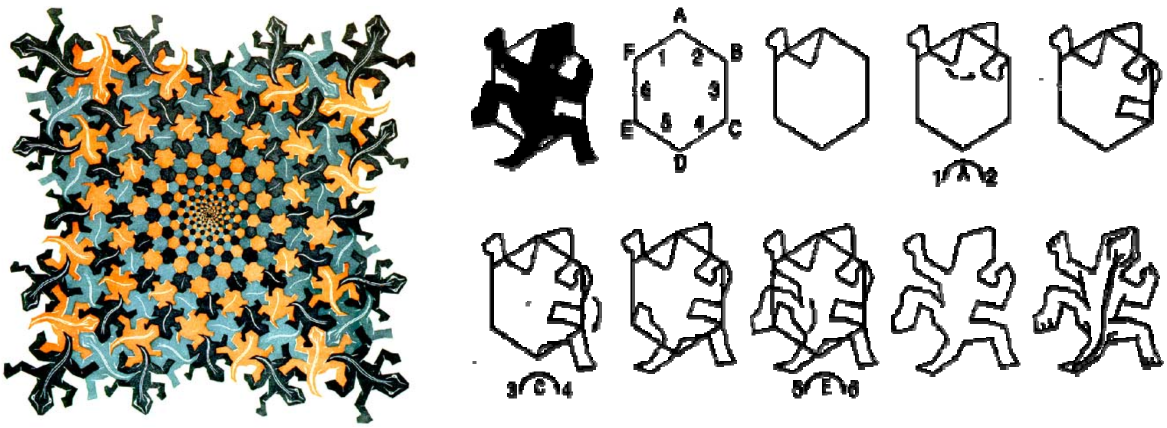

Figure 5
Escher's Development II (*left*) and the drawing of reptile tessellation (*right*)

However, the recent development of mathematics and computations has brought us to a standard visual art development, the work of generative art. Simply this is a stream of art invigorated by the geometry of self-similarity, fractals, and cellular automata. The artists find the basic pattern or rule and let the computation do the rest of it. This is a very exciting field of art to be the place of unlimited creative imagination suit with.

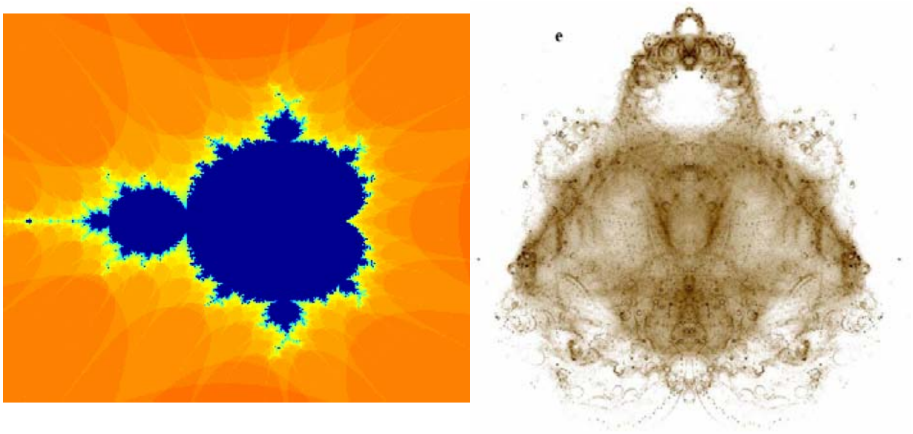

Figure 6
Mandelbrot fractal (*left*) based on simple equation of complex number and its artistic modification known as Buddhabrot (*right*) as created by Melinda Green (1993).

One nice example is created by Melinda Green (1993) showing how the Mandelbrot set can be graphically turned into the image of Buddha-like (named Buddhabrot). The Mandelbrot set is made of the set of points *c* in complex plane for which iterated in the simple equation

$$z_{n+1} = z_n^2 + c$$

where $z_0 = 0$.

The resulting Buddhabrot is rendered by making 2-dimensional array of counters, one for each pixel. Then, a random sampling of points *c* is iterated through the Mandelbrot function, and, for points which do escape within a chosen number of iterations, and are thus not in the Mandelbrot set, the counters for each pixel that the z value landed on are incremented (once per hit). After a large number of values c have been iterated, image colors (or color saturation/brightness) are then chosen based on the values recorded in the array (Wikipedia, 2005).

Furthermore, musical art is also become an interesting field. A well-known way to create music from mathematical and computational concept is cellular automata. This way, artist may use the computer MIDI-mapper in order to create music or at minimum inspire the musical creation. One interesting work of art is made by Eduardo Rick Miranda (2002) made a musical creation by using CAMUS (cellular automata music creator). One way to create music in his term is the Demon Cyclic Cycle automata model. It is made by 2 dimensional cellular automata with random initial condition in several states of automata and iteratively computed resulting stable pattern. Interestingly, the resulting music is in the color of jazz and classical musical orchestration. The musical creation in this term is still in development and somehow giving a lot of possibilities in musical creation.

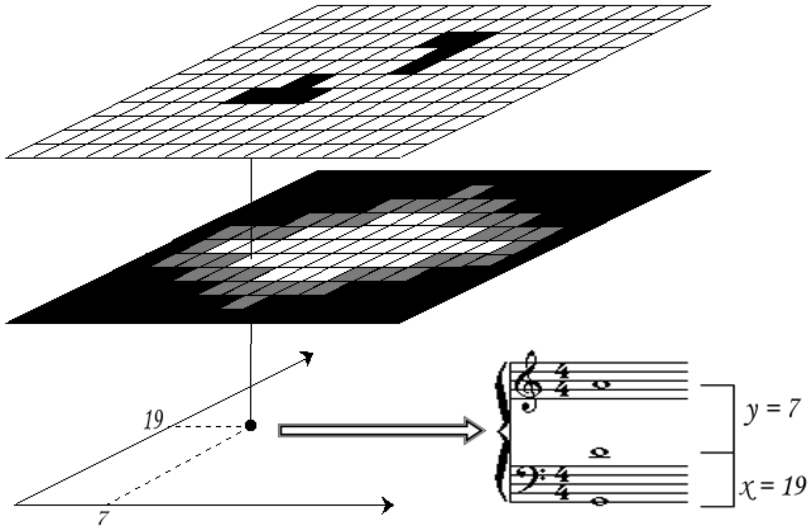

Figure 7
The illustration of music creation by using CAMUS. The upper layer is the Game of Life and the lower is Demon Cyclic cellular automata. Both states of the automata then mapped into Cartesian plane to be transformed into the proper musical notes (adapted from Miranda, 2002).

From the short discussion above, we can see the three points of relatedness between mathematical concepts and the realm of artworks. There are more examples we can find in the future as the development is still going on. However, a lot of developments in this inter-field fall into one of these three options. It is obvious that those kinds of works and projects will always show the very primary fascinating relation generally between science and art.

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