A Review of the 10-40 GHz Auction

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In February 2008, Ofcom's 10-40 GHz auction concluded. This was Ofcom's first combinatorial clock auction. The auction used an innovative format intended to encourage an efficient assignment of the 27 lots. Each of the ten bidders won one or more lots. All 27 lots were assigned. This note briefly reviews the auction.

After examining the auction data, I find no evidence that the auction was less than fully efficient. The bidders had an opportunity to express their true preferences, and there was little incentive for them to do otherwise as a result of the second price rule. Thus, I expect that the bids did reflect true preferences, in which case the outcome was fully efficient.

The bids of a few bidders were non-monotonic—a larger amount was bid for a package that was a strict subset of another package. In some cases this appeared to be the result of constraints from the activity rule, suggesting that some bidders failed to bid on the largest profitable package during the clock rounds. These binding activity rule constraints may have reduced auction revenues, but I doubt efficiency was compromised.

I begin with a description of the 10-40 GHz setting. Then I examine the principal stage of the auction, including both the clock rounds and the supplementary bids. Then I discuss the assignment stage.

1 Summary of environment and auction format

The 10-40 GHz auction included ten national lots at 10 GHz, two national and three sub-national lots at 28 GHz, and six national lots each at 32 GHz and 40 GHz. The spectrum is suitable for many possible uses, such as fixed wireless access or backhaul. Some applications require aggregating multiple contiguous lots. For this reason a package auction was desirable, especially given the desire for the auction to be technology neutral. Since national lots within a particular band were believed to be nearly perfect substitutes, it made sense to simplify the auction with the use of generic national lots in each of the four bands.

The 10-40 GHz auction was Ofcom's first use of the combinatorial clock auction format. The combinatorial clock auction is a simple yet powerful package auction. It enables the auction to determine the successful technology, rather than the regulator. Since at every point in the auction bidders are bidding on mutually exclusive packages of lots, there is no exposure problem. A bidder never runs the risk of winning just some of what it needs. Also, bids are binding commitments, and any of the bidder's bids throughout the entire auction may be part of the winning set. This provides a strong incentive for a bidder to bid in a way that is consistent with its preferences. The combinatorial clock auction allows package bids without introducing the complexity that is often associated with combinatorial auctions. Rather the auction begins with a simple and familiar price discovery process, followed by a final round of bidding. Only at

[1] This note was funded by Ofcom. The views expressed are my own.

the end of the supplementary round does the auctioneer need to solve an optimization problem to determine the quantity of spectrum won by each bidder. The optimization takes just seconds for a problem the size of the 10-40 GHz auction. Finally, to determine the specific assignments, winners express preferences over the various specific assignments consistent with the generic awards.

The combinatorial clock auction has an activity rule to encourage price discovery. Without an activity rule, a bidder may wait until the supplementary round before submitting serious bids, and thereby undermine price discovery. The auction used the eligibility point rule—as prices increase the package size can stay the same or decrease; it cannot increase.

Under the eligibility point rule, a bidder's best strategy in the clock stage is not the natural one of bidding on its most profitable package given the round prices. Rather, the best strategy is to bid on its largest package that is still profitable. Only with this strategy can the bidder be sure that it will be able to bid its full value for desirable packages in the supplementary round. If the bidder instead bid on its most profitable package in each round of the clock stage, the bidder likely would face severe constraints in its supplementary bids for packages larger than its final clock package. Sometimes these constraints can result in nonmonotonic bids in which the bidder submits a higher bid for a package that is a strict subset of a larger package.

2 Principal stage

The outcome of the principal stage is shown in Table 1. Remarkably, all ten bidders won at least one lot. This would suggest that the threshold problem—the difficultly that small bidders sometimes face in package auctions—was not significant. There is no evidence that small bidders failed to top some large package bids, because of a failure to bid full values.

Table 1. Principal stage outcome

Bidder	10GHz nat	28GHz nat	28GHz sub1	28GHz sub2	28GHz sub3	32GHz nat	40GHz nat	Bid	Opp. cost	Base price
Arqiva		2						1,599,000	260,500	260,500
BT						1		1,001,000	179,000	179,000
Digiweb	2							142,000	39,000	39,000
Faultbasic					1			750,000	30,000	30,000
MLL						1	1	250,000	179,000	179,000
Orange						2		2,999,999	261,000	261,000
RedM				1				34,000	10,000	10,000
TMobile	8					2	1	8,500,000	319,000	319,000
Transfinite			1					97,000	20,000	20,000
UKBB							4	420,000	120,000	120,000
Total	10	2	1	1	1	6	6	15,792,999	1,417,500	1,417,500

For each winning package, the bid amount, the opportunity cost, and the base price are shown. All prices throughout this note are in pounds. The opportunity cost is the Vickrey price. Notice that for each of the ten bidders the base price is equal to the opportunity cost. This means that none of the bidders could have gained by manipulating the bids. Bidding true value was the best strategy.

2.1 Clock rounds

The auction had 17 clock rounds. The rounds are shown in Table 2. For each of the seven categories, the price, demand, and excess demand is given. The auctioneer did a good job of adjusting prices in response to excess demand. This is seen by fairly smooth reduction of excess demand throughout the clock rounds. Remarkably, the clock stage concluded with six of the seven categories in supply and demand balance. Often in package auctions, the clock stage concludes with a larger number of categories with excess supply.

Table 2. Clock rounds

	10GHz nat			28GHz nat			28GHz sub1			28GHz sub2			28GHz sub3			32GHz nat			40GHz nat		
Round	Price	demand	excess	price	demand	excess	price	demand	excess	price	demand	excess	price	demand	excess	price	demand	excess	price	demand	excess
1	10,000	21	11	60,000	4	2	20,000	3	2	10,000	1	0	30,000	3	2	60,000	19	13	30,000	8	2
2	12,000	23	13	72,000	5	3	24,000	2	1	10,000	2	1	36,000	3	2	72,000	17	11	36,000	7	1
3	14,000	23	13	86,000	6	4	29,000	1	0	11,000	2	1	43,000	3	2	86,000	16	10	40,000	5	-1
4	17,000	18	8	103,000	6	4	29,000	1	0	13,000	2	1	52,000	2	1	103,000	14	8	40,000	8	2
5	20,000	18	8	124,000	5	3	29,000	2	1	16,000	2	1	62,000	2	1	124,000	14	8	44,000	7	1
6	24,000	20	10	149,000	6	4	32,000	2	1	19,000	2	1	74,000	2	1	149,000	12	6	48,000	6	0
7	29,000	18	8	179,000	4	2	38,000	2	1	23,000	2	1	89,000	1	0	179,000	12	6	48,000	9	3
8	35,000	18	8	215,000	4	2	46,000	1	0	28,000	2	1	89,000	1	0	215,000	10	4	55,000	8	2
9	42,000	18	8	258,000	3	1	46,000	1	0	34,000	1	0	89,000	1	0	258,000	8	2	66,000	11	5
10	50,000	18	8	310,000	3	1	46,000	1	0	34,000	1	0	89,000	1	0	310,000	7	1	79,000	11	5
11	60,000	11	1	372,000	3	1	46,000	2	1	34,000	1	0	89,000	1	0	341,000	9	3	95,000	6	0
12	63,000	11	1	446,000	3	1	51,000	2	1	34,000	1	0	89,000	1	0	409,000	8	2	95,000	8	2
13	66,000	11	1	535,000	2	0	61,000	2	1	34,000	1	0	89,000	1	0	491,000	7	1	105,000	9	3
14	69,000	10	0	535,000	1	-1	73,000	2	1	34,000	1	0	89,000	2	1	540,000	7	1	126,000	8	2
15	69,000	10	0	535,000	3	1	88,000	1	0	34,000	1	0	98,000	3	2	594,000	5	-1	151,000	5	-1
16	69,000	10	0	589,000	3	1	88,000	2	1	34,000	2	1	118,000	2	1	594,000	5	-1	151,000	4	-2
17	69,000	10	0	707,000	2	0	97,000	1	0	37,000	1	0	130,000	1	0	594,000	6	0	151,000	5	-1

Although the final clock prices are not a good indicator of base prices, the clock prices are a good indicator of the relative prices paid for packages. Table 3 compares the bid, the base price, and the final clock price for each bidder's winning package. Notice that the ratio of the base price and the winning bid varies widely across bidders—from 4% to 72%. In sharp contrast, the ratio of the base price and the final clock prices is fairly steady across bidders, ranging from 17% to 30%. This means that the final clock prices were a good indicator of relative prices.

A disadvantage of the eligibility point activity rule is that the final clock prices tend to be high relative to base prices. The reason is that during the clock stage bidders optimally bid on the largest profitable package given the round prices, rather than the most profitable package. Thus, the bidders continue to bid on larger packages when the smaller packages are more profitable. Excess demand is greater and clock prices are pushed higher than they would be if bidders bid on their most profitable package in each round. The 10-40 GHz auction clearly illustrates this tendency. Final clock prices were roughly five times higher than the base prices.

Table 3. Comparison of bid, base price, and final clock price

Bidder	Bid	Base price	Final clock	Base / bid	Base / clock
Arqiva	1,599,000	260,500	1,414,000	16%	18%
BT	1,001,000	179,000	594,000	18%	30%
Digiweb	142,000	39,000	138,000	27%	28%
Faultbasic	750,000	30,000	130,000	4%	23%
MLL	250,000	179,000	745,000	72%	24%
Orange	2,999,999	261,000	1,188,000	9%	22%
RedM	34,000	10,000	37,000	29%	27%
TMobile	8,500,000	319,000	1,891,000	4%	17%
Transfinite	97,000	20,000	97,000	21%	21%
UKBB	420,000	120,000	604,000	29%	20%
Total	15,792,999	1,417,500	6,838,000	9%	21%

The clock rounds appeared to be helpful for both price discovery and assignment discovery. Table 4 shows the final clock bid and the winning bid for each bidder. In many cases, the bidder's winning package is the same or nearly the same as the bidder's final package bid. In nine of the ten cases, the winning bid was a supplementary bid. Only two of the ten bidders exited the clock stage before round 17.

Table 4. Comparison of final clock bids and winning bids

			10GHz nat		28GHz nat		28GHz sub1		28GHz sub2		28GHz sub3		32GHz nat		40GHz nat	
Bidder	Round	Bid	price	q	price	q	price	q	price	q	price	q	price	q	price	q
Arqiva	17	1,451,000	69,000		707,000	2	97,000		37,000	1	130,000		594,000		151,000	
Arqiva	SB	1,599,000	-		-	2	-		-		-		-		-	
BT	17	896,000	69,000		707,000		97,000		37,000		130,000		594,000	1	151,000	2
BT	SB	1,001,000	-		-		-		-		-		-	1	-	
Digiweb	17	138,000	69,000	2	707,000		97,000		37,000		130,000		594,000		151,000	
Digiweb	SB	142,000	-	2	-		-		-		-		-		-	
Faultbasic	17	281,000	69,000		707,000		97,000		37,000		130,000	1	594,000		151,000	1
Faultbasic	SB	750,000	-		-		-		-		-	1	-		-	
MLL	8	215,000	35,000		215,000	1	46,000		28,000		89,000		215,000		55,000	
MLL	SB	250,000	-		-		-		-		-		-	1	-	1
Orange	17	1,782,000	69,000		707,000		97,000		37,000		130,000		594,000	3	151,000	
Orange	SB	2,999,999	-		-		-		-		-		-	2	-	
RedM	8	28,000	35,000		215,000		46,000		28,000	1	89,000		215,000		55,000	
RedM	SB	34,000	-		-		-		-	1	-		-		-	
TMobile	17	1,891,000	69,000	8	707,000		97,000		37,000		130,000		594,000	2	151,000	1
TMobile	SB	8,500,000	-	8	-		-		-		-		-	2	-	1
Transfinite	17	97,000	69,000		707,000		97,000	1	37,000		130,000		594,000		151,000	
Transfinite	SB	97,000	-		-		-	1	-		-		-		-	
UKBB	17	151,000	69,000		707,000		97,000		37,000		130,000		594,000		151,000	1
UKBB	13	420,000	66,000		535,000		61,000		34,000		89,000		491,000		105,000	4

Note: Final clock bids are in white; winning bids are in yellow.

2.2 Supplementary bids

Supplementary bids are intended for bidders to increase their clock bids to full value and add bids on any additional packages that are relevant but were not bid on during the clock rounds. Bidding behavior varied a great deal across bidders.

Table 5 shows the total number of bids and the number of supplementary bids submitted by each bidder. Only BT and TMobile submitted a large number of supplementary bids. All the bidders but BT and TMobile simply increased clock bids, and added a handful of supplementary bids on packages closely related to their bids in the latter part of the clock stage. This is what one would expect from bidders following the safe strategy of bidding on the largest profitable package in the clock stage and then bidding full values on relevant packages in the supplementary round.

Table 5. Number of package bids and supplementary bids

Bidder	Number of bids	Supplementary bids
Arqiva	28	22
BT	545	545
Digiweb	6	1
Faultbasic	12	12
MLL	15	15
Orange	4	4
RedM	2	2
TMobile	107	107
Transfinite	5	1
UKBB	14	3
Total	738	712

Faultbasic's supplementary bids are non-monotonic (Table 6). In particular, Faultbasic's winning bid of 750,000 for one 28GHz sub3 lot is substantially more than its bid for several other lots that include one 28GHz sub3 lot as part of a larger package. Some of the non-monotonicities are explained by constraints imposed by the activity rule, but others occur for packages unconstrained by the activity rule. All bids of size 6 or less were unconstrained.

Table 6. Non-monotonicity of Faultbasic's supplementary bids

				10GHz nat		28GHz nat		28GHz sub1		28GHz sub2		28GHz sub3		32GHz nat		40GHz nat	
Bidder	Round	Activity	Bid	price	q	price	q	price	q	price	q	price	q	price	q	price	q
Faultbasic	17	6	281,000	69,000		707,000		97,000		37,000		130,000	1	594,000		151,000	1
Faultbasic	SB	9	223,000	-		-	1	-		-		-	1	-		-	
Faultbasic	SB	9	389,000	-		-	1	-		-		-		-		-	1
Faultbasic	SB	6	535,000	-		-	1	-		-		-		-		-	
Faultbasic	SB	3	750,000	-		-		-		-		-	1	-		-	
Faultbasic	SB	6	350,000	-		-		-		-		-	1	-		-	1
Faultbasic	SB	9	380,000	-		-		-		-		-	1	-	1	-	
Faultbasic	SB	6	180,000	-		-		-	1	-	1	-	1	-		-	
Faultbasic	SB	4	160,000	-		-		-		-	1	-	1	-		-	
Faultbasic	SB	5	170,000	-		-		-	1	-		-	1	-		-	
Faultbasic	SB	8	230,000	-		-		-	1	-		-	1	-		-	1
Faultbasic	SB	9	264,000	-		-		-	1	-	1	-	1	-		-	1
Faultbasic	SB	7	218,000	-		-		-		-	1	-	1	-		-	1

Note: Winning bid in yellow.

BT's clock bids are shown in Table 7. BT switched its package whenever maintaining the same package would result in a bid in excess of 1,000,000. Until the last few rounds of the clock stage, BT focused entirely on the 32GHz and 40GHz lots. In round 15 and 16, BT shifted to include 28GHz lots. BT's bid in round 17 eliminated the remaining excess demand and shifted to categories with excess supply; it ended the clock stage.

Table 7. BT's clock bids

Bidder	Round	Activity	Bid	10GHz nat price	q	28GHz nat price	q	28GHz sub1 price	q	28GHz sub2 price	q	28GHz sub3 price	q	32GHz nat price	q	40GHz nat price	q
BT	1	42	420,000	10,000		60,000		20,000		10,000		30,000		60,000	6	30,000	2
BT	2	42	504,000	12,000		72,000		24,000		10,000		36,000		72,000	6	36,000	2
BT	3	42	596,000	14,000		86,000		29,000		11,000		43,000		86,000	6	40,000	2
BT	4	42	698,000	17,000		103,000		29,000		13,000		52,000		103,000	6	40,000	2
BT	5	42	832,000	20,000		124,000		29,000		16,000		62,000		124,000	6	44,000	2
BT	6	42	990,000	24,000		149,000		32,000		19,000		74,000		149,000	6	48,000	2
BT	7	36	991,000	29,000		179,000		38,000		23,000		89,000		179,000	5	48,000	2
BT	8	30	970,000	35,000		215,000		46,000		28,000		89,000		215,000	4	55,000	2
BT	9	30	912,000	42,000		258,000		46,000		34,000		89,000		258,000	2	66,000	6
BT	10	24	936,000	50,000		310,000		46,000		34,000		89,000		310,000	2	79,000	4
BT	11	21	967,000	60,000		372,000		46,000		34,000		89,000		341,000	2	95,000	3
BT	12	21	884,000	63,000		446,000		51,000		34,000		89,000		409,000	1	95,000	5
BT	13	18	911,000	66,000		535,000		61,000		34,000		89,000		491,000	1	105,000	4
BT	14	15	918,000	69,000		535,000		73,000		34,000		89,000		540,000	1	126,000	3
BT	15	15	935,000	69,000		535,000	1	88,000		34,000		98,000	1	594,000		151,000	2
BT	16	15	980,000	69,000		589,000	1	88,000	1	34,000	1	118,000	1	594,000		151,000	1
BT	17	12	896,000	69,000		707,000		97,000		37,000		130,000		594,000	1	151,000	2

In the supplementary round, BT submitted 545 bids, far more than any other bidder. Nearly all the bids that were not constrained by the activity rule were for approximately 1,000,000. The highest bid, including the winning bid, was for 1,001,000. The bids were non-monotonic. The winning bid was for a single 32GHz lot. There were hundreds of bids that included one or more 32GHz lots as well as other lots, yet were bid at 1,001,000 or less. Many of the non-monotonicities appear to be the result of the activity rule, as well as a tight constraint on bids of 1,001,000. BT had many supplementary bids that included 10GHz lots, despite the fact that BT never bid on the 10GHz lots in the clock stage.

TMobile consistently bid for 8 10GHz lots, 2 32GHz lots, and 1 40GHz lot throughout the clock stage. In the early rounds, some 28GHz lots were included as well. In the supplementary round, TMobile again focused on this core package, which it ultimately won with a bid of 8,500,000. Larger variations of this package were sharply constrained by the activity rule, as shown in Table 8. TMobile failed to bid on packages with more than 23 eligibility points during the clock stage; whereas according to its supplementary bids, its largest profitable package was for at least 26 eligibility points, and as a result all such packages larger than 23 were sharply constrained by the activity rule.

Table 8. TMobile's bids indicating activity rule constraint

Bidder	Round	Activity	Bid	10GHz nat price	q	28GHz nat price	q	28GHz sub1 price	q	28GHz sub2 price	q	28GHz sub3 price	q	32GHz nat price	q	40GHz nat price	q
TMobile	17	23	1,891,000	69,000	8	707,000		97,000		37,000		130,000		594,000	2	151,000	1
TMobile	SB	26	514,000	-	8	-		-		-		-	1	-	2	-	1
TMobile	SB	25	811,000	-	8	-		-	1	-		-		-	2	-	1
TMobile	SB	23	8,500,000	-	8	-		-		-		-		-	2	-	1
TMobile	SB	26	514,000	-	8	-	1	-		-		-	1	-	1	-	1
TMobile	SB	25	811,000	-	8	-	1	-	1	-		-		-	1	-	1
TMobile	SB	25	819,000	-	7	-		-		-		-	1	-	2	-	1
TMobile	SB	25	819,000	-	7	-	1	-		-		-	1	-	1	-	1
TMobile	SB	24	793,000	-	8	-		-		-	1	-		-	2	-	1
TMobile	SB	24	793,000	-	8	-	1	-		-	1	-		-	1	-	1
TMobile	SB	24	776,000	-	7	-		-	1	-		-		-	2	-	1
TMobile	SB	24	776,000	-	7	-	1	-	1	-		-		-	1	-	1
TMobile	SB	24	784,000	-	6	-		-		-		-	1	-	2	-	1
TMobile	SB	24	784,000	-	6	-	1	-		-		-	1	-	1	-	1
TMobile	SB	23	8,435,597	-	7	-		-		-	1	-		-	2	-	1

2.3 Optimization

The optimization at the end of the principal stage was performed using two independent optimizers. Both optimizers found the same solutions. There were two solutions with an optimal objective value of 15,792,999. The two solutions differed only in whether one 40GHz lot was assigned to BT or MLL. The randomly selected solution awarded MLL the 40GHz lot in question. With 10 bidders and 738 bids, the optimization took 0.16 seconds to solve the winner determination problem, 0.30 seconds to find the Vickrey prices (opportunity costs), and 0.11 seconds to determine the bidder-optimal core prices (base prices). Since the Vickrey prices did not violate any core constraints, these prices are the unique bidder-optimal core prices.

3 Assignment stage

The assignment stage was straightforward. Since the two 28GHz national lots were won by a single party, only three categories had multiple options to consider. Table 9 lists all the possibilities together with the bids for each bidder, and the opportunity cost and additional price for each winning bid.

Table 9. Assignment stage bids and outcome

10GHz national

Bidder	Option	Bid	Opp. cost	Price
Digiweb	Lots 1 and 2	0		
Digiweb	Lots 9 and 10	5,255	0	0
TMobile	Lots 1 to 8	0	0	0
TMobile	Lots 3 to 10	0		

32GHz national

Bidder	Option	Bid	Opp. cost	Price
BT	Lot 1	0		
BT	Lot 2	10,000		
BT	Lot 3	5,000		
BT	Lot 4	4,000		
BT	Lot 5	9,000	3,000	4,000
BT	Lot 6	0		
MLL	Lot 1	0		
MLL	Lot 2	3,000		
MLL	Lot 3	5,000		
MLL	Lot 4	5,000		
MLL	Lot 5	3,000		
MLL	Lot 6	0	0	0
Orange	Lots 1 and 2	105,000	11,130	11,130
Orange	Lots 2 and 3	15,000		
Orange	Lots 3 and 4	0		
Orange	Lots 4 and 5	15,000		
Orange	Lots 5 and 6	15,000		
TMobile	Lots 1 and 2	20,229		
TMobile	Lots 2 and 3	15,151		
TMobile	Lots 3 and 4	10,099	1,000	1,000
TMobile	Lots 4 and 5	5,048		
TMobile	Lots 5 and 6	0		

40GHz national

Bidder	Option	Bid	Opp. cost	Price
MLL	Lot 1	0		
MLL	Lot 2	5,000	0	0
MLL	Lot 5	5,000		
MLL	Lot 6	0		
TMobile	Lot 1	56,279	1,000	1,000
TMobile	Lot 2	54,279		
TMobile	Lot 5	2,219		
TMobile	Lot 6	0		
UKBB	Lots 1 to 4	1,000		
UKBB	Lots 2 to 5	0		
UKBB	Lots 3 to 6	0	0	0

Note: Winning bids in yellow.

The most interesting aspect of the assignment bids is how small the bids are relative to the winning principal stage bids, as shown in Table 10. The additional value that a bidder gets from its most preferred specific assignment was never more than 4% of the bidder's winning principal bid, and was only 1% across all bidders. This confirms that the approach of auctioning generic lots first was sound. Although bidders do care somewhat about specific assignments, the importance of the specific assignment is second-order—much less than a single bid increment. Moreover, as a result of the optimization, bidders were able to secure 92% of the total value potentially available, assuming each bidder got its first choice, as a result of desirable specific assignments. Overall, the additional payment was less than 10% of the additional value received.

Table 10. Comparison of principal and assignment stage bids

Bidder	Principal bid	Assignment bid (max)	Value achieved	Additional price	Assignment / principal	Value / assignment
Arqiva	1,599,000					
BT	1,001,000	10,000	9,000	4,000	1%	90%
Digiweb	142,000	5,255	5,255		4%	100%
Faultbasic	750,000					
MLL	250,000	10,000	5,000		4%	50%
Orange	2,999,999	105,000	105,000	11,130	4%	100%
RedM	34,000					
TMobile	8,500,000	76,508	66,378	2,000	1%	87%
Transfinite	97,000					
UKBB	420,000	1,000			0%	0%
Total	15,792,999	207,763	190,633	17,130	1%	92%

All the additional prices were Vickrey prices (opportunity cost), with the exception of BT's, which was 4,000 (1,000 more than the Vickrey price). Thus, as in the principal stage, there were strong incentives to bid full values in the assignment stage.

3.1 Optimization

Again two independent optimizers were used to find the optimal assignment. Both optimizers found the same unique solution. All assignments and prices were determined in a combined time of 0.05 seconds.

4 Conclusion

The 10-40 GHz auction successfully demonstrated the desirable features of the combinatorial clock auction. Bidding for generic lots greatly simplified the bidding. The bidders had strong incentives to submit truthful bids in the supplementary round and in the assignment stage. Assuming they did, then the outcome was fully efficient.

Competition in the auction was somewhat weak. This is the primary reason why all bidders were successful in winning spectrum and revenues were only 9% of values.

My only concern is with the activity rule in the principal stage. First, the eligibility point rule led to final clock prices that were higher than they may have been under a different rule, and the final clock prices were nearly five times as high as the base prices. Second, the eligibility point rule gives the bidders a high degree of flexibility in bidding on less desirable packages during the clock stage, which may undermine price discovery. In addition, if a bidder fails to bid on its largest profitable package in each round of the clock stage, the bidder's bids on large packages are sharply constrained in the supplementary round. Several bidders had supplementary bids constrained by the activity rule. These constraints likely reduced revenues, but I have little evidence to suggest that the constraints created any inefficiencies. One bidder, BT, seemed to use the flexibility of the eligibility point rule to submit a large number of supplementary bids, which were non-monotonic and for packages different from the preferences expressed in the primary rounds.

Overall, the identified anomalies likely did not compromise the efficiency of the auction. I view the outcome as a highly successful test of the combinatorial clock auction design.