Unconditionally Secure Revocable Storage: Tight Bounds, Optimal Construction, and Robustness

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Abstract

Data stored in cloud storage sometimes requires long-term security due to its sensitivity (e.g., genome data), and therefore, it also requires flexible access control for handling entities who can use the data. *Broadcast encryption* can partially provide such flexibility by specifying privileged receivers so that only they can decrypt a ciphertext. However, once privileged receivers are specified, they can be no longer dynamically added and/or removed. In this paper, we propose a new type of broadcast encryption which provides long-term security and appropriate access control, which we call unconditionally secure *revocable-storage broadcast encryption* (RS-BE). In RS-BE, privileged receivers of a ciphertext can be dynamically updated without revealing any information on the underlying plaintext. Specifically, we define a model and security of RS-BE, derive tight lower bounds on sizes of secret keys required for secure RS-BE, and propose a construction of RS-BE which meets all of these bounds. Our lower bounds can be applied to traditional broadcast encryption. Furthermore, to detect an improper update, we consider security against modification attacks to a ciphertext, and present a concrete construction secure against this type of attacks.

1 Introduction

1.1 Background

In recent years, the progress of cloud technologies has been remarkable, and cloud-based applications are becoming widespread. One area in which cloud technology has the potential to provide significant impact, is advanced medical treatment, and applications of cloud technology in this area is currently being investigated intensively [3, 41]. To provide such advanced medical services, it is required to store the data of individual patients using cloud storage. However, this data is generally very sensitive and should be protected carefully. Especially, when storing genome data using cloud storage, computationally secure encryption is considered to provide insufficient protection since genetic properties will be inherited by descendants of the genome owner, and thus, significantly long-term security is required [3, 4]. For example, even if we encrypt genome data using a 2048-bit RSA cryptosystem, which is considered sufficiently secure in most applications, security will only be guaranteed until 2030 [5], which is not sufficient for protecting genome privacy (which must take into account the privacy of our descendants).

A promising approach for obtaining sufficiently strong security for medical data is to utilize information-theoretically secure encryption, e.g. the one-time pad. However, the one-time pad is only a (standard) symmetric encryption scheme, and thus, not suitable for effective use in a cloud environment. Namely, in a cloud storage system, there are potentially many users who will be given permission to access the stored data, and these privileged users are furthermore dynamically determined. It is obvious that such a scenario cannot be easily handled by using only (standard) symmetric encryption. *Broadcast encryption* [19] which allows multiple receivers to decrypt a logically single ciphertext seems to partially yield the required functionality. However, when the sender encrypts a plaintext in broadcast encryption, he is forced to fix the set of privileged users and cannot dynamically add and/or remove receivers. For handling dynamic changes to the set of privileged receivers (in the context of attribute-based encryption [36]), Sahai, Seyalioglu, and Waters proposed *revocable-storage attribute-based encryption* [35] in which a ciphertext in a cloud storage system can be periodically updated according to a changing set of privileged users. However, their scheme is computationally secure and does not guarantee security against future powerful adversaries.

Therefore, it is important to investigate suitable cryptographic primitives which simultaneously provide a high level of security for sensitive data and sufficient flexibility to implement appropriate access control.

1.2 Our Contributions

In this paper, we propose the notion of unconditionally secure *revocable-storage broadcast encryption* (RS-BE) which yields information-theoretic security and the above required functionality for cloud storage. In a RS-BE scheme, similarly to broadcast encryption, the sender chooses a set of (initial) privileged users and encrypts a plaintext so that only these users can decrypt the ciphertext. Moreover, the *storage manager* can update the ciphertext to reflect changes in the set of privileged users. Here, the update procedure is carried out without revealing the plaintext, and thus, the storage manager cannot learn anything about the encrypted plaintext. We furthermore show tight lower bounds on the sizes of ciphertexts and secret keys in the unconditionally secure setting, and present an optimal construction which achieves these bounds as well as a robust construction which is resilient to a maliciously behaving storage manager.

More specifically, our contributions are as follows. Firstly, in Section 2, we give a formal model and security definitions of unconditionally secure RS-BE. Then, in Section 3, we clarify that it is possible to construct an unconditionally secure RS-BE scheme in which the ciphertext length is the same as the plaintext length. We note that this is an important and desired property since ciphertexts are stored in the cloud permanently or for a long time, and therefore, compactness of ciphertexts is one of the most important aspects to consider in the design of a RS-BE scheme. We then investigate lower bounds on the sizes of decryption keys, encryption keys, and the storage manager's keys under the condition that the ciphertext size is the same as the plaintext size. These bounds can also be seen as a generalization of the bounds for (traditional) broadcast encryption, and furthermore imply a tight bound on the size of encryption keys in broadcast encryption which, to the best of our knowledge, has not been clarified before our work. In Section 4, we show an unconditionally secure RS-BE scheme which meets all of these bounds with equalities. This means that these bounds are *tight* and the proposed construction is *optimal.* In Section 5, we furthermore consider a scenario in which a maliciously behaving storage manager can try to modify the encrypted plaintext. This is related to *non-malleability* in the context of ordinary encryption. In a RS-BE scheme, malleability may cause a serious problem since the ciphertext is periodically updated, but an improper update carried out by a malicious storage manager may not be immediately detectable by the users. Then, we present a concrete robust construction, which is provably secure against this type of attacks, based on an ordinary RS-BE scheme and *an algebraic manipulation detection code* (AMD-code for short) [16].

1.3 Related Work

Berkovits [6] first considered the concept of broadcast encryption, and Fiat and Naor [19] developed a formal and systematic approach to the construction of broadcast encryption schemes. Since then, broadcast encryption schemes have been improved both in the computationally secure setting [30, 18, 13, 21, 34] and in the unconditionally secure setting [8, 10, 25, 6, 19, 39, 20, 28, 32, 15, 33, 17], and used in various situations such as copyright protection in the real world. In particular, lower bounds on secret keys for unconditionally secure broadcast encryption (USBE for short) schemes have previously been investigated [8, 10, 25]. However, some problems nonetheless remain. Blundo and Cresti [8] derived lower bounds on USBE in the context of key predistribution schemes (KPS for short) [29, 7]. However, these bounds are specific to the application to KPS, and are not true lower bounds for USBE in general. Also, Blundo et al. [10] derived lower bounds for USBE, but these bounds are not tight. Furthermore, Kurosawa et al. [25] showed tight lower bounds on the size of decryption keys for USBE through equivalence between USBE and KPS, however, they did not mention lower bounds on encryption keys in their paper. In contrast, we derive tight lower bounds on both of the sizes of encryption keys and decryption keys for USBE without using such equivalence, and it turns out that the tight lower bound on the size of decryption keys in [25] is a special case of ours.

Recently, many researchers have investigated how we can *securely* use cloud data storage for various purposes [24, 35, 1, 22, 37, 38, 27, 26, 42]. Sahai, Seyalioglu, and Waters [35] first dealt with the concept of a revocable storage, and proposed revocable-storage attribute-based encryption (RS-ABE for short). They assume ciphertexts are stored in external storage, such as cloud data storage, and considered revocable attribute-based encryption [12, 2] with ciphertext updatable functionality (to be precise, [12] in the context of identity-based encryption). However, RS-ABE is only computationally secure, and hence cannot guarantee long-term security. In the unconditionally secure setting, proactive secret sharing schemes [23, 40, 31, 14] and fully dynamic secret sharing schemes [9] also provide functionality for updating shares. However, such updating functionality and its aim in these schemes are different from those in our RS-BE scheme. Hence, we cannot directly apply these techniques, and we need to define and to construct RS-BE schemes from scratch.

2 Revocable-Storage Broadcast Encryption

2.1 Model

In RS-BE, there are $n+2$ entities, a sender E, n users $U_1, \ldots, U_n$, and a storage manager SM. Let $\mathcal{U} := \{U_1, \ldots, U_n\}$ be a set of all users. First, E generates own encryption key ek, also generates n decryption keys $dk_1, \ldots, dk_n$ and a maintenance key mk behalf of $U_1, \ldots, U_n, SM$, and distributes them securely. E can specify a subset $\mathcal{S}$ (called a *privileged set*) of $\mathcal{U}$ such that $\mathcal{S} \neq \emptyset$, and encrypt a plaintext by using his encryption key ek so that only users in the privileged set can decrypt the resulting ciphertext. The ciphertext is stored and disclosed in an external storage such as cloud storage. A user U_i in the privileged set $\mathcal{S}$ takes the ciphertext from the storage himself, then he decrypts the ciphertext by using his decryption key dk_i. The storage manager SM can change *any* privileged set $\mathcal{S}$ of the ciphertext into *any* privileged set $\mathcal{S}'$ (even if *not* $\mathcal{S}' \subset \mathcal{S}$) by using his maintenance key mk without decryption (i.e., without revealing the underlying plaintext). At sender's request or by some kind of rule, the storage manager SM changes the privileged set of the ciphertext, and then SM replaces the old one with the new one.

Formally, RS-BE is executed as follows. Let $\mathcal{M}$ be a set of possible plaintexts. For any subset $\mathcal{J} := \{U_{i_1}, \ldots, U_{i_j}\} \subset \mathcal{U}$, let $\mathcal{C}_{\mathcal{J}}$ be a set of all possible ciphertexts for the privileged set $\mathcal{J}$, and let $\mathcal{C} := \bigcup_{\mathcal{J} \subset \mathcal{U}} \mathcal{C}_{\mathcal{J}}$. Let $\mathcal{EK}$ be a set of possible encryption keys, and let $\mathcal{MK}$ be a set of maintenance keys. Let $\mathcal{DK}_i$ be a set of possible decryption keys for U_i, and let $\mathcal{DK} := \bigcup_{i=1}^{n} \mathcal{DK}_i$.

Definition 1 (RS-BE)**.** *A revocable-storage broadcast encryption (*RS-BE *for short) scheme* Π *involves* $n+2$ *entities,* $E, U_1, U_2, \ldots, U_n$ *and* SM*, and consists of the following four-tuple of algorithms (Setup, Enc, Dec, Upd) with five spaces,* $\mathcal{M}, \mathcal{C}, \mathcal{EK}, \mathcal{DK}$*, and* $\mathcal{MK}$*, where all of the above algorithms except Setup are deterministic and all of the above spaces are finite.*

1. $(ek, mk, dk_1, \ldots, dk_n) \leftarrow Setup(n)$*: It takes the number of users* n *as input, and outputs an encryption key* $ek \in \mathcal{EK}$*,* n *decryption keys* $(dk_1, \ldots, dk_n) \in \prod_{i=1}^{n} \mathcal{DK}_i$*, and a maintenance key* $mk \in \mathcal{MK}$*.*

2. $c_{\mathcal{S}} \leftarrow Enc(ek, m, \mathcal{S})$*: It takes an encryption key* ek*, a plaintext* $m \in \mathcal{M}$*, and an initial privileged set* $\mathcal{S} \subset \mathcal{U}$ *as input, and outputs a ciphertext* $c_{\mathcal{S}}$*.*

3. m *or* $\perp \leftarrow Dec(dk_i, c_{\mathcal{S}}, \mathcal{S}, U_i)$*: It takes a decryption key* dk_i *of a user* U_i*, the ciphertext* $c_{\mathcal{S}}$*, the privileged set* $\mathcal{S}$*, and the identity* U_i *as input, and outputs* m *or* $\perp$*.*

4. $c_{\mathcal{S}'}$ *or* $\perp \leftarrow Upd(mk, c_{\mathcal{S}}, \mathcal{S}, \mathcal{S}')$*: It takes a maintenance key* mk*, the ciphertext* $c_{\mathcal{S}}$*, its privileged set* $\mathcal{S}$*, and a new privileged set* $\mathcal{S}'$ *as input, and outputs a ciphertext* $c_{\mathcal{S}'}$ *for* $\mathcal{S}'$ *or* $\perp$*.*

In RS-BE Π, we require the following correctness holds: (a) For all $n \in \mathbb{N}$, all $(ek, mk, dk_1, \ldots, dk_n) \leftarrow Setup(n)$, all $m \in \mathcal{M}$, all $\mathcal{S} \subset \mathcal{U}$, and all $U_i \in \mathcal{S}$, $m \leftarrow Dec(dk_i, Enc(ek, m, \mathcal{S}), \mathcal{S}, U_i)$. (b) For all $n \in \mathbb{N}$, all $(ek, mk, dk_1, \ldots, dk_n) \leftarrow Setup(n)$, all $m \in \mathcal{M}$, all $\mathcal{S}, \mathcal{S}' \subset \mathcal{U}$, $Upd(mk, Enc(ek, m, \mathcal{S}), \mathcal{S}') = Enc(ek, m, \mathcal{S}')$. (a) means the *decryption correctness* and (b) means the *updating correctness*.

In RS-BE, for simplicity we assume the one-time model where it is allowed for the sender to encrypt a plaintext and store a ciphertext only once. Note that it is unrestricted for the storage manager to execute the algorithm *Upd* (i.e. the ciphertext can be updated unboundedly).

2.2 Security Definition

We consider perfect secrecy against at most ω colluders and the storage manager. Here, we note that in principle, it is impossible to guarantee security against collusion of them since the storage manager can change any privileged set of a ciphertext into any privileged set. Therefore, we consider security in the case that at most ω colluders and the storage manager try to attack separately.[1] Namely, we consider the following two kinds of security notions: (1) At most ω colluders who are not included in the privileged set cannot get any information on the underlying plaintext from the ciphertext (a traditional security notion for broadcast encryption). (2) The storage manager cannot get any information on the underlying plaintext from the ciphertext. The reason why we consider the second one is that if the storage manager can obtain the underlying plaintext or some information on it, it is only necessary to encrypt the same plaintext with a new privileged set and replace an old ciphertext with the new one by a sender to change privileged sets. Hence, we require the storage manager can update the ciphertext without decryption (without leaking any information on the

[1] We also discuss a RS-BE scheme secure against collusion of at most ω colluders and the storage manager under a restricted transformation rule of the storage manager's key in Appendix A.

underlying plaintext). For any $\mathcal{J} := \{U_{i_1}, \ldots, U_{i_j}\} \subset \mathcal{U}$, let $\mathcal{DK}_{\mathcal{J}} := \mathcal{DK}_{i_1} \times \cdots \times \mathcal{DK}_{i_j}$ be a set of possible secret keys of $\mathcal{J}$. Let M, $C_{\mathcal{S}}$, EK, DK_i $(1 \le i \le n)$, $DK_{\mathcal{J}}$ $(\mathcal{J} \subset \mathcal{U})$, and MK be random variables which takes values on $\mathcal{M}$, $\mathcal{C}_{\mathcal{S}}$, $\mathcal{EK}$, $\mathcal{DK}_i$ $(1 \le i \le n)$, $\mathcal{DK}_{\mathcal{J}}$ $(\mathcal{J} \subset \mathcal{U})$, and $\mathcal{MK}$, respectively. Formally, security of RS-BE is defined as follows.

Definition 2 (Security of RS-BE). *Let Π be an RS-BE scheme. Π is said to be $(\le n, \le \omega)$-one-time secure if the following conditions are satisfied:*

(1) For any privileged set $\mathcal{S} \subset \mathcal{U}$, and any set of colluders $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \le \omega$, it holds that $H(M \mid C_{\mathcal{S}}, DK_{\mathcal{W}}) = H(M)$.

(2) For any privileged set $\mathcal{S} \subset \mathcal{U}$, it holds that $H(M \mid C_{\mathcal{S}}, MK) = H(M)$.

Remark 1. *In the model of RS-BE (Definition 1), if SM does not exist (i.e., mk is empty string and we do not consider the algorithm Upd), and we therefore do not consider the condition (2) in Definition 2, then Definitions 1 and 2 are the same as those of ($(\le n, \le \omega)$-one-time secure) traditional broadcast encryption schemes [19, 39, 8, 25]. Hence, we can say our scheme is natural extension of the broadcast encryption schemes.*

Remark 2. *The condition (1) in Definition 2 implies that the number of ciphertexts taken by $\mathcal{W}$ from the storage is at most one. However, it is natural to think that $\mathcal{W}$ can access the storage multiple time and take ciphertexts for various privileged sets. Namely, for more realistic definition, we should consider the following security condition (1') instead of (1):*

(1') For any privileged sets $\mathcal{S}_1, \ldots, \mathcal{S}_k \subset \mathcal{U}$ $(1 \le k \le 2^n)$, and any set of colluders $\mathcal{W} \subset \mathcal{U}$ such that $\left(\bigcup_{i=1}^{k} \mathcal{S}_i\right) \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \le \omega$, it holds that $H(M \mid C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_k}, DK_{\mathcal{W}}) = H(M)$.

For convenience, we call Π a strongly secure RS-BE scheme if it satisfies the conditions (1') and (2), and just call Π a secure RS-BE scheme if it satisfies Definition 2 (the conditions (1) and (2)). Actually, tight lower bounds on secret keys required for such a strongly secure RS-BE scheme are the same as those required for the secure RS-BE scheme (the bounds will appear in Theorem 2). Therefore, we can obtain the same optimal construction, in the sense that the construction meets equality in every lower bound, which will be proposed in Section 4. In addition to this, to deal with RS-BE as natural extension of traditional broadcast encryption, we consider the above weaker security definition (Definition 2).

3 Tight Lower Bounds on Sizes of Ciphertexts and Secret Keys

In this section, we show lower bounds on the sizes of ciphertexts and secret keys required for a $(\le n, \le \omega)$-one-time secure RS-BE scheme. As mentioned in [10, 28, 32, 33], in traditional broadcast encryption schemes, there is a trade-off between the ciphertext size and the secret key size. RS-BE schemes also have such a trade-off. Actually, if we ignore the size of a ciphertext, it is not difficult to construct an $(\le n, \le \omega)$-one-time secure RS-BE scheme which is fairly efficient in other aspects, and the concrete construction is as follows. A sender E has n secret keys $k_1, \ldots, k_n$ and a common key K shared among E and all users $U_1, \ldots, U_n$ as ek, each user U_i has k_i and K as dk_i, and a storage manager SM has $k_1, \ldots, k_n$ as mk. E encrypts a plaintext m by $ct_{i_j} := m + k_{i_j} + K$ for every $U_{i_j} \in \mathcal{S}$ $(1 \le j \le |\mathcal{S}|)$, and outputs $c_{\mathcal{S}} := (ct_{i_1}, \ldots, ct_{i_{|\mathcal{S}|}})$. For updating the ciphertext, SM computes $ct = ct_\ell - k_\ell = m + K$ for $U_\ell \in \mathcal{S}$ and $ct_{i_j} := ct + k_{i_j}$ for every $U_{i_j} \in \mathcal{S}'$ $(1 \le j \le |\mathcal{S}'|)$, and then SM outputs $c_{\mathcal{S}'} := (ct_{i_1}, \ldots, ct_{i_{|\mathcal{S}'|}})$. Then, we have $|c_{\mathcal{S}}| = |\mathcal{S}| \cdot |m|$ for every $\mathcal{S} \in \mathcal{U}$, $|ek| = (n+1)|m|$, $|dk_i| = 2|m|$ $(1 \le i \le n)$, and $|mk| = n|m|$. The sizes of secret keys of this scheme are significantly smaller than those of our construction which will be proposed in Section 4 though the ciphertext length is proportional to the cardinality of the privileged set; on the other hand, that of the proposed scheme is equal to the plaintext length for any privileged set.

However, when we consider applying RS-BE to a cloud storage, compactness of a ciphertext is one of the most important factors to be taken into account, since in such a scenario, a ciphertext is stored in cloud permanently or for a long-time, and thus, the ciphertext length should be as small as possible. For the above reason, we first investigate the *tight* lower bound on the size of ciphertexts, and then, derive lower bounds on sizes of secret keys under the condition that the ciphertext length is optimal.

Theorem 1. *Let Π be an $(\leq n, \leq \omega)$-one-time secure RS-BE scheme. Then, for any $\mathcal{S} \subset \mathcal{U}$, $H(C_{\mathcal{S}}) \geq H(M)$ and there exists a concrete construction which meets this bound with equality.*

Proof. For any $\mathcal{S} \subset \mathcal{U}$ and $U_i \in \mathcal{S}$, we have

$$\begin{aligned} H(C_{\mathcal{S}}) \geq & H(C_{\mathcal{S}} \mid DK_i) & (1) \\ \geq & H(C_{\mathcal{S}} \mid DK_i) - H(C_{\mathcal{S}} \mid DK_i, M) & (2) \\ = & I(C_{\mathcal{S}}; M \mid DK_i) = H(M \mid DK_i) - H(M \mid DK_i, C_{\mathcal{S}}) = H(M), \end{aligned}$$

where the last equality follows from independence of M and DK_i and the decryption correctness.

Then, we show a construction which meets this bound with equality. A secret key of the one-time pad is assigned for every possible $\mathcal{S} \subset \mathcal{U}$. Namely, $ek := (\{k_{\mathcal{S}} \mid \mathcal{S} \subset \mathcal{U}\})$, $dk_i := (k_{\emptyset}, \{k_{\mathcal{S}} \mid \mathcal{S} \subset \mathcal{U} \wedge U_i \in \mathcal{S}\})$ $(1 \leq i \leq n)$, and $mk := \{k_{\mathcal{S}} \mid \mathcal{S} \subset \mathcal{U} \wedge \mathcal{S} \neq \emptyset\}$, where each $k_{\mathcal{S}}$ is chosen from a finite field uniformly at random. In *Enc*, for any $\mathcal{S}$, it outputs $c_{\mathcal{S}} := m + k_{\emptyset} + k_{\mathcal{S}}$. In *Dec*, if $U_i \in \mathcal{S}$, it can output $m = c_{\mathcal{S}} - k_{\emptyset} - k_{\mathcal{S}}$. In *Upd*, for any $\mathcal{S}$ and $\mathcal{S}'$, it outputs $c_{\mathcal{S}'} := c_{\mathcal{S}} - k_{\mathcal{S}} + k_{\mathcal{S}'}$. This construction is $(\leq n, \leq \omega)$-one-time secure since any $\mathcal{W}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ does not have $k_{\mathcal{S}}$ and SM does not have $k_{\emptyset}$. □

Next, we derive lower bounds on sizes of secret keys when the ciphertext size is optimal (i.e. the ciphertext length is equal to the plaintext length).

Theorem 2. *Let Π be an $(\leq n, \leq \omega)$-one-time secure RS-BE scheme. Then, the following lower bounds hold under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$:*

(i) $H(EK) \geq \sum_{j=0}^{\omega} \binom{n}{j} H(M)$, *(ii)* $H(DK_i) \geq \sum_{j=0}^{\omega} \binom{n-1}{j} H(M)$ *for any* $i \in \{1, 2, \ldots, n\}$,

(iii) $H(MK) \geq \left(\sum_{j=0}^{\omega} \binom{n}{j} - 1 \right) H(M)$.

Proof. The proof follows from the following lemmata.

Lemma 1. *For any $\mathcal{S} \subset \mathcal{U}$ and any $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{W} \cap \mathcal{S} = \emptyset$ and $|\mathcal{W}| \leq \omega$, let Y_i $(1 \leq i \leq k)$ be a privileged set such that $Y_i \cap \mathcal{W} \neq \emptyset$. Then, we have $H(C_{\mathcal{S}} \mid M, C_{Y_1}, \ldots, C_{Y_k}, DK_{\mathcal{W}}) \geq H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. From (1) and (2) in Theorem 1, and the condition $H(C_{\mathcal{S}}) = H(M)$, we have $H(C_{\mathcal{S}} \mid DK_i) = H(C_{\mathcal{S}} \mid DK_i) - H(C_{\mathcal{S}} \mid DK_i, M)$ for any $\mathcal{S} \subset \mathcal{U}$ and $U_i \in \mathcal{S}$. Therefore, we have

$$H(C_{\mathcal{S}} \mid DK_i, M) = 0. \quad (3)$$

For $H(M, C_{\mathcal{S}}, C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}})$, we have

$$\begin{aligned} &H(M, C_{\mathcal{S}}, C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}}) \\ &\qquad = H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}) + H(M \mid DK_{\mathcal{W}}, C_{\mathcal{S}}) + H(C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}}, C_{\mathcal{S}}, M) \\ &\qquad = H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}) + H(M) + H(C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}}, C_{\mathcal{S}}, M) \qquad (4) \\ &\qquad = H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}) + H(M), \qquad (5) \end{aligned}$$

where (4) follows from the condition (1) of Definition 2, and (5) follows from (3) (i.e. $H(C_{Y_j} \mid DK_{\mathcal{W}}, M) = 0$) since $Y_j \cap \mathcal{W} \neq \emptyset$ for any Y_j $(1 \leq j \leq k)$.

On the other hand, for $H(M, C_{\mathcal{S}}, C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}})$, we have

$$\begin{aligned} &H(M, C_{\mathcal{S}}, C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}}) \\ &\qquad = H(M \mid DK_{\mathcal{W}}) + H(C_{Y_1}, \ldots, C_{Y_k} \mid DK_{\mathcal{W}}, M) + H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}, M, C_{Y_1}, \ldots, C_{Y_k}) \\ &\qquad = H(M) + H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}, M, C_{Y_1}, \ldots, C_{Y_k}), \qquad (6) \end{aligned}$$

where (6) follows from independence of M and $DK_{\mathcal{W}}$ and the same reason for (5).

Hence, from (5) and (6), we have

$$H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}, M, C_{Y_1}, \ldots, C_{Y_k}) = H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}). \qquad (7)$$

In the following, we show $H(C_{\mathcal{S}} \mid DK_{\mathcal{W}}) \geq H(M)$.

For $H(M, C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK)$, we have

$$\begin{aligned} H(M, C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) =& H(C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) + H(M \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK, C_{\mathcal{S}}) \\ =& H(C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK), \qquad (8) \end{aligned}$$

where (8) follows from the decryption correctness (i.e. $H(M \mid DK_{\mathcal{S}}, C_{\mathcal{S}}) = 0$).

On the other hand, for $H(M, C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK)$, we have

$$\begin{aligned} H(M, C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) =& H(M \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) + H(C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK, M) \\ =& H(M \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK), \qquad (9) \end{aligned}$$

where (9) follows from the algorithm *Enc* (i.e. $H(C_{\mathcal{S}} \mid EK, M) = 0$).

Hence, we have

$$\begin{aligned} H(C_{\mathcal{S}} | DK_{\mathcal{W}}) \geq& H(C_{\mathcal{S}} \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) \\ =& H(M \mid DK_{\mathcal{S}}, DK_{\mathcal{W}}, EK) \qquad (10) \\ =& H(M), \qquad (11) \end{aligned}$$

where (10) follows from (8) and (9), and (11) follows from independence of M and $(EK, DK_1, \ldots, DK_n)$.

From (7) and (11), we have $H(C_{\mathcal{S}} \mid M, C_{Y_1}, \ldots, C_{Y_k}, DK_{\mathcal{W}}) \geq H(M)$. □

Lemma 2. *We have* $H(EK) \geq \sum_{j=0}^{\omega} \binom{n}{j} H(M)$ *under the condition* $H(C_{\mathcal{S}}) = H(M)$ *for any* $\mathcal{S} \subset \mathcal{U}$.

Proof. Let $\mathscr{W} := \{\mathcal{W} \subset \mathcal{U} \mid |\mathcal{W}| \leq \omega\} = \{\mathcal{W}_1, \dots, \mathcal{W}_t\}$ be the family of all possible sets of colluders, where $t = \sum_{j=0}^{\omega} \binom{n}{j}$. Moreover, let $\mathscr{S}(\mathscr{W}) := \{\mathcal{S}_1, \dots, \mathcal{S}_t\}$, where $\mathcal{S}_i = \mathcal{U} \setminus \mathcal{W}_i$ such that $\mathcal{W}_i \in \mathscr{W}$ $(1 \leq i \leq t)$. Without loss of generality, $|\mathcal{S}_1| \geq \cdots \geq |\mathcal{S}_t|$. Then, we have

$$\begin{aligned}
H(EK) =& H(EK \mid M) && (12)\\
\geq& I(EK; C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_t} \mid M)\\
=& H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_t} \mid M) - H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_t} \mid M, EK)\\
=& H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_t} \mid M) && (13)\\
=& \sum_{j=1}^{t} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_{j-1}})\\
\geq& \sum_{j=1}^{t} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j})\\
\geq& \sum_{j=0}^{\omega} \binom{n}{j} H(M), && (14)
\end{aligned}$$

where (12) follows from independence of M and EK, (13) follows from the algorithm *Enc* (i.e. $H(C_{\mathcal{S}_i} \mid EK, M) = 0$ $(1 \leq i \leq t)$), and (14) follows from Lemma 1. □

Lemma 3. *For any $i \in \{1, \dots, n\}$, we have $H(DK_i) \geq \sum_{j=0}^{\omega} \binom{n-1}{j} H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. Let $\mathscr{W}^{(i)} := \{\mathcal{W} \subset \mathcal{U} \setminus \{U_i\} \mid |\mathcal{W}| \leq \omega\} = \{\mathcal{W}_1, \dots, \mathcal{W}_\ell\}$ be the family of all possible sets of colluders except for sets of colluders containing U_i, where $\ell = \sum_{j=0}^{\omega} \binom{n-1}{j}$. Moreover, let $\mathscr{S}(\mathscr{W}^{(i)}) := \{\mathcal{S}_1, \dots, \mathcal{S}_\ell\}$, where $\mathcal{S}_i = \mathcal{U} \setminus \mathcal{W}_i$ such that $\mathcal{W}_i \in \mathscr{W}^{(i)}$ $(1 \leq i \leq \ell)$. Without loss of generality, $|\mathcal{S}_1| \geq \cdots \geq |\mathcal{S}_\ell|$. We note $U_i \in \mathcal{S}$ for any $\mathcal{S} \in \mathscr{S}(\mathscr{W}^{(i)})$. Then, we have

$$\begin{aligned}
H(DK_i) =& H(DK_i \mid M) && (15)\\
\geq& I(DK_i; C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_\ell} \mid M)\\
=& H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_\ell} \mid M) - H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_\ell} \mid M, DK_i)\\
=& H(C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_\ell} \mid M) && (16)\\
=& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_{j-1}})\\
\geq& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \dots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j})\\
\geq& \sum_{j=0}^{\omega} \binom{n-1}{j} H(M), && (17)
\end{aligned}$$

where (15) follows from independence of M and DK_i, (16) follows from (3) in Lemma 1 (i.e. $H(C_{\mathcal{S}_j} \mid DK_i, M) = 0$ $(1 \leq j \leq \ell)$), and (17) follows from Lemma 1. □

Lemma 4. *We have $H(MK) \geq \left(\sum_{j=0}^{\omega} \binom{n}{j} - 1\right) H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. Let $\mathscr{W}$ and $\mathscr{S}(\mathscr{W})$ be the same as those in Lemma 2. Then, we have

$$\begin{aligned}
H(MK) \geq & H(MK \mid C_{\mathcal{S}_1}) \\
\geq & I(MK; C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_t} \mid C_{\mathcal{S}_1}) \\
= & H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_t} \mid C_{\mathcal{S}_1}) - H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_t} \mid C_{\mathcal{S}_1}, MK) \\
= & H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_t} \mid C_{\mathcal{S}_1}) \qquad (18) \\
= & \sum_{j=2}^{t} H(C_{\mathcal{S}_j} \mid C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}) \\
\geq & \sum_{j=2}^{t} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j}) \\
\geq & \left(\sum_{j=0}^{\omega} \binom{n}{j} - 1 \right) H(M), \qquad (19)
\end{aligned}$$

where (18) follows from the algorithm *Upd* (i.e. $H(C_{\mathcal{S}_i} \mid C_{\mathcal{S}_1}, MK) = 0$ $(2 \leq i \leq t)$), and (19) follows from Lemma 1. □

Now, the proof of Theorem 2 is completed. □

As we will see in the next section, the above lower bounds are tight since our construction will meet all the above bounds with equalities. Therefore, we define optimality of constructions of RS-BE as follows.

Definition 3 (Optimality). *A construction of an $(\leq n, \leq \omega)$-one-time secure RS-BE scheme is said to be* optimal *if it meets equality in every bound of (i)-(iii) in Theorem 2.*

In a similar way, we can also derive tight lower bounds on secret keys required for another class of RS-BE schemes, called $(t, \leq \omega)$-one-time secure RS-BE schemes [28, 32, 25, 15], in which the number of privileged users is constant in all time periods, and show an optimal construction under this condition (see Appendix B for details).

4 Optimal Construction

In this section, we propose an optimal construction of $(\leq n, \leq \omega)$-one-time secure RS-BE based on the Fiat–Naor KPS[2] [19], and then we fine-tune a construction of the Fiat–Naor KPS for constructing our RS-BE scheme since a session key is created by redundant operation in their scheme, though the sizes of secret keys in their scheme are optimal. We define the following families of sets: $\mathscr{W} := \{\mathcal{W} \subset \mathcal{U} \mid |\mathcal{W}| \leq \omega\}$, $\mathscr{W}^{(i)} := \{\mathcal{W} \subset \mathcal{U} \setminus \{U_i\} \mid |\mathcal{W}| \leq \omega\}$, and $\mathscr{W}(\mathcal{S}) := \{\mathcal{W} \in \mathscr{W} \mid (\mathcal{W} \cap \mathcal{S} = \emptyset \wedge |\mathcal{W}| = \min(\omega, n - |\mathcal{S}|)) \vee \mathcal{W} = \emptyset\}$. Our construction is as follows.

1. $(ek, mk, dk_1, \ldots, dk_n) \leftarrow$ *Setup*(n): Let q be a prime power such that $q > n$, and $\mathbb{F}_q$ be a finite field with q elements. For every $\mathcal{W} \in \mathscr{W}$, it chooses $r_{\mathcal{W}} \in \mathbb{F}_q$ uniformly at random. Then, it outputs $ek := \{r_{\mathcal{W}} \mid \mathcal{W} \in \mathscr{W}\}$, $dk_i := \{r_{\mathcal{W}} \mid \mathcal{W} \in \mathscr{W}^{(i)}\}$ $(1 \leq i \leq n)$, and $mk := \{r_{\mathcal{W}} \mid \mathcal{W} \in \mathscr{W} \setminus \{\emptyset\}\}$.

[2]If we define a construction which meets equality in every bound of (i) and (ii) in Theorem 2 as an optimal construction of $(\leq n, \leq \omega)$-one-time secure BE, then we can obtain such an optimal construction from the Fiat–Naor KPS scheme and the one-time pad.

2. $c_{\mathcal{S}} \leftarrow Enc(ek, m, \mathcal{S})$: For any privileged set $\mathcal{S}$, it computes a session key $k_{\mathcal{S}} := \sum_{\mathcal{W} \in \mathscr{W}(\mathcal{S})} r_{\mathcal{W}}$, and then outputs $c_{\mathcal{S}} := m + k_{\mathcal{S}}$.

3. m or $\perp \leftarrow Dec(dk_i, c_{\mathcal{S}}, \mathcal{S}, U_i)$: If $U_i \in \mathcal{S}$, then it computes $k_{\mathcal{S}}$ as in the algorithm *Enc* and outputs $m = c_{\mathcal{S}} - k_{\mathcal{S}}$. Otherwise, it outputs $\perp$.

4. $c_{\mathcal{S}'}$ or $\perp \leftarrow Upd(mk, c_{\mathcal{S}}, \mathcal{S}, \mathcal{S}')$: For any privileged sets $\mathcal{S}$ and $\mathcal{S}'$, it computes an updating key $uk_{\mathcal{S} \to \mathcal{S}'} := \sum_{\mathcal{W} \in \mathscr{W}(\mathcal{S}') \setminus \{\emptyset\}} r_{\mathcal{W}} - \sum_{\mathcal{W} \in \mathscr{W}(\mathcal{S}) \setminus \{\emptyset\}} r_{\mathcal{W}}$, and outputs $c_{\mathcal{S}'} := c_{\mathcal{S}} + uk_{\mathcal{S} \to \mathcal{S}'}$.

Theorem 3. *The resulting RS-BE scheme* Π *by the above construction is* $(\leq n, \leq \omega)$*-one-time secure and optimal.*

Proof. First, we show the above construction meets the condition (1) in Definition 2. Without loss of generality, we consider that $\mathcal{W} := \{U_1, \ldots, U_\omega\}$ is a set of colluders and $\mathcal{S} := \{U_{\omega+1} \ldots, U_n\}$ is a privileged set. Consider the case that the set of colluders $\mathcal{W}$ will guess $k_{\mathcal{S}}$ to obtain $m = c_{\mathcal{S}} - k_{\mathcal{S}}$ by using their decryption keys. However, $\mathcal{W}$ cannot compute $k_{\mathcal{S}}$ since they do not have $r_{\mathcal{W}}$. Therefore, the best strategy of $\mathcal{W}$ is to make a random guess at m as in the one-time pad. Thus, we have $H(M \mid C_{\mathcal{S}}, DK_{\mathcal{W}}) = H(M)$. Similarly, for any privileged set $\mathcal{S} \subset \mathcal{U}$, any set of colluders $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \leq \omega$ does not have $r_{\mathcal{W}}$, though $r_{\mathcal{W}}$ is used for computing $k_{\mathcal{S}}$. Hence, for any $\mathcal{S} \subset \mathcal{U}$, and any $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \leq \omega$, $H(M \mid C_{\mathcal{S}}, DK_{\mathcal{W}}) = H(M)$.

Next, we show the above construction meets the condition (2) in Definition 2. Since $1 \leq |\mathcal{S}| \leq n$, $r_\emptyset$ is always used for computing $k_{\mathcal{S}}$ for any $\mathcal{S} \subset \mathcal{U}$, whereas SM does not have $r_\emptyset$. Hence, he can only guess m randomly as in the one-time pad. Thus, for any $\mathcal{S} \subset \mathcal{U}$, $H(M \mid C_{\mathcal{S}}, MK) = H(M)$.

Moreover, it is straightforward to see that the above construction is optimal. $\square$

5 Robust Construction

We now consider a scenario in which a maliciously behaving storage manager can try to modify the encrypted plaintext. This is related to *non-malleability* in the context of ordinary encryption. In a RS-BE scheme, malleability may cause a serious problem since the ciphertext is periodically updated, but an improper update carried out by a malicious storage manager may not be immediately detectable by the users. More specifically, we consider security against a storage manager who tries to modify a ciphertext so that a user in the privileged set obtains a modified plaintext which differs from an original plaintext encrypted by the sender. In addition to this, since ciphertexts of RS-BE schemes are stored in external storage such as cloud storage (in other words, the ciphertexts are accessible at anytime), we should also consider security against such a modification attack by colluders. Formally, we consider two types of adversaries as in Definition 2, and define the robustness of RS-BE as follows.

Definition 4 (Robust RS-BE)**.** *Let* Π *be an* $(\leq n, \leq \omega)$*-one-time secure RS-BE scheme.* Π *is said to be* δ*-robust if* $\max\{P_1, P_2\} \leq 1 - \delta$*, where* P_1 *and* P_2 *are defined as follows:*

(3) For any $\mathcal{S}_1, \ldots, \mathcal{S}_k \subset \mathcal{U}$ $(1 \leq k \leq 2^n)$*, any* $U_i \in \mathcal{S}_k$*, and any* $\mathcal{W} \subset \mathcal{U}$ *such that* $\left(\bigcup_{i=1}^{k} \mathcal{S}_i\right) \cap \mathcal{W} = \emptyset$ *and* $|\mathcal{W}| \leq \omega$*, we define* $P_1(\mathcal{S}_1, \ldots, \mathcal{S}_k, U_i, \mathcal{W})$ *as:*

$$P_1(\mathcal{S}_1, \ldots, \mathcal{S}_k, U_i, \mathcal{W}) :=$$
$$\max_{c'_{\mathcal{S}_k}} \max_{c_{\mathcal{S}_1}, \ldots, c_{\mathcal{S}_k}} \max_{dk_{\mathcal{W}}} \Pr(m' \leftarrow Dec(dk_i, c'_{\mathcal{S}_k}, \mathcal{S}_k, U_i) \mid \{Enc(ek, m, \mathcal{S}_j)\}_{1 \leq j \leq k}, dk_{\mathcal{W}}),$$

where $m' \notin \{m, \perp\}$ *and* $c_{\mathcal{S}_j} = Enc(ek, m, \mathcal{S}_j)$ $(1 \le j \le k)$*. Note that* $Enc(ek, m, \mathcal{S}_{j+1}) = Upd(mk, Enc(ek, m, \mathcal{S}_j), \mathcal{S}_j, \mathcal{S}_{j+1})$ *for any* $\mathcal{S}_j, \mathcal{S}_{j+1}$ $(1 \le j \le k-1)$ *(the updating correctness). Then,* P_1 *is defined as* $P_1 := \max_{\mathcal{S}_1,\ldots,\mathcal{S}_k,U_i,\mathcal{W}} P_1(\mathcal{S}_1, \ldots, \mathcal{S}_k, U_i, \mathcal{W})$*.*

(4) For any $\mathcal{S}, \mathcal{S}' \subset \mathcal{U}$ *and any* $U_i \in \mathcal{S}'$*, we define* $P_2(\mathcal{S}, \mathcal{S}', U_i)$ *as:*

$$P_2(\mathcal{S}, \mathcal{S}', U_i) := \max_{c'_{\mathcal{S}'}} \max_{c_{\mathcal{S}}} \max_{mk} \Pr(m' \leftarrow Dec(dk_i, c'_{\mathcal{S}'}, \mathcal{S}', U_i) \mid Enc(ek, m, \mathcal{S}), mk),$$

where $m' \notin \{m, \perp\}$ *and* $c_{\mathcal{S}} = Enc(ek, m, \mathcal{S})$*. Then,* P_2 *is defined as* $P_2 := \max_{\mathcal{S},\mathcal{S}',U_i} P_2(\mathcal{S}, \mathcal{S}', U_i)$*.*

We can construct a robust scheme by using an *algebraic manipulation detection code* (AMD-code), which is defined as follows.

Definition 5 (AMD-code [16])**.** *Let* $\mathcal{M}_{\mathrm{AMD}}$ *be a set of messages such that* $|\mathcal{M}_{\mathrm{AMD}}| = \eta$*, and* $\mathbb{G}$ *be a commutative group of order* γ*. An algebraic manipulation detection code (AMD-code)* Φ *consists of the following two-tuple algorithms (*Encode*,* Decode*), where* Encode *is a probabilistic encoding map* $\mathsf{Encode} : \mathcal{M}_{\mathrm{AMD}} \to \mathbb{G}$ *and a deterministic decoding map* $\mathsf{Decode} : \mathbb{G} \to \mathcal{M}_{\mathrm{AMD}} \cup \{\perp\}$ *such that* $\mathsf{Decode}(\mathsf{Encode}(m)) = m$ *with probability one for every* $m \in \mathcal{M}_{\mathrm{AMD}}$*.* Φ *is an* $(\eta, \gamma, \varepsilon)$*-AMD-code if for every* $m \in \mathcal{M}_{\mathrm{AMD}}$ *and for every* $\delta \in \mathbb{G}$*, the probability that* $\mathsf{Decode}(\mathsf{Encode}(m) + \delta) \notin \{m, \perp\}$ *is at most* ε*.*

A robust RS-BE scheme is constructed by modifying the construction proposed in Section 4 as follows: Before encrypting a plaintext $m \in \mathbb{F}_q$, the *Enc* algorithm runs $\hat{m} \leftarrow \mathsf{Encode}(m)$; and after decrypting a ciphertext, then the *Dec* algorithm runs $m \leftarrow \mathsf{Decode}(\tilde{m})$, where $\tilde{m}$ is the decryption result.

We obtain the following theorem, and omit the proof since it is straightforward.

Theorem 4. *If* Φ *is a* (q, q, ε)*-AMD-code, then the resulting RS-BE scheme* Π *by the above construction is* $(\le n, \le \omega)$*-one-time secure and* ε*-robust.*

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A Collusion Resistant RS-BE Scheme

We consider security against collusion of at most ω colluders and a storage manager. Intuitively, if a storage manager can change any privileged set of a ciphertext into any privileged set by using his maintenance key mk, we cannot achieve RS-BE secure against collusion of a set of colluders and the storage manager. Therefore, here we simply set the following transformation rule for mk: For any $\mathcal{S}, \mathcal{S}' \subset \mathcal{U}$, *Upd*$(mk, c_{\mathcal{S}}, \mathcal{S}, \mathcal{S}')$ outputs an updated ciphertext $c_{\mathcal{S}'}$ if $\mathcal{S}' \subset \mathcal{S}$ holds, otherwise it outputs $\perp$.

We define collusion resistant security as follows.

Definition 6 (Collusion Resistant RS-BE)**.** *Let Π be an RS-BE scheme. Π is said to be collusion-resistantly $(\leq n, \leq \omega)$-one-time secure if the following conditions are satisfied: For any privileged set $\mathcal{S} \subset \mathcal{U}$, and any set of colluders $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \leq \omega$, it holds that*

$$H(M \mid C_{\mathcal{S}}, DK_{\mathcal{W}}, MK) = H(M).$$

A construction which satisfies Definition 6 is as follows.

1. $(ek, mk, dk_1, \ldots, dk_n) \leftarrow$ *Setup*(n): Let q be a prime power such that $q > n$, and $\mathbb{F}_q$ be a finite field with q elements. It chooses n polynomials $f^{(h)}(x) := \sum_{i=0}^{\omega} a_i x^i$ $(h = 1, \ldots, n)$ over $\mathbb{F}_q$ uniformly at random, and computes $n-1$ polynomials $g^{(\ell)}(x) := f^{(\ell)}(x) - f^{(\ell-1)}(x)$ $(2 \leq \ell \leq n)$. Then, it outputs $ek := f^{(1)}(x)$, $dk_i := (f^{(1)}(i), \ldots, f^{(n)}(i))$ $(1 \leq i \leq n)$, and $mk := (g^{(2)}(x), \ldots, g^{(n)}(x))$.

2. $c_{\mathcal{S}} \leftarrow$ *Enc*$(ek, m, \mathcal{S})$: Let $\mathcal{S} = \{U_{i_1}, \ldots, U_{i_k}\}$ $(1 \leq k \leq n)$ be a privileged set. For every U_{i_j}, it computes $c_{i_j}^{(1)} := m + f^{(1)}(i_j)$, and sets a counter $t := 1$. Finally, it outputs $c_{\mathcal{S}} := (t, c_{i_1}^{(t)}, \ldots, c_{i_k}^{(t)})$.

3. m or $\perp \leftarrow$ *Dec*$(dk_i, c_{\mathcal{S}}, \mathcal{S}, U_i)$: If $U_i \in \mathcal{S}$, it computes $m = c_i^{(t)} - f^{(t)}(i)$ and outputs it. Otherwise, it outputs $\perp$.

4. $c_{\mathcal{S}'}$ or $\perp \leftarrow$ *Upd*$(mk, c_{\mathcal{S}}, \mathcal{S}, \mathcal{S}')$: Let $\mathcal{S}' = \{U_{i_1}, \ldots, U_{i_k}\}$. If $\mathcal{S}' \subset \mathcal{S}$ does not hold, it outputs $\perp$. Otherwise, for every $U_{i_j} \in \mathcal{S}' \subset \mathcal{S}$, it computes $c_i^{(t+1)} := c_{i_j}^{(t)} + g^{(t+1)}(i_j)$ $(1 \leq j \leq k)$. Finally, it sets $t := t + 1$ and outputs $c_{\mathcal{S}'} := (t, c_{i_1}^{(t)}, \ldots, c_{i_k}^{(t)})$.

Theorem 5. *The resulting RS-BE scheme Π by the above construction is collusion-resistantly $(\leq n, \leq \omega)$-one-time secure.*

Proof. It is not so difficult to prove this theorem. Without loss of generality, we consider that $\mathcal{W} := \{U_1, \ldots, U_\omega\}$ is a set of colluders and $\mathcal{S} := \{U_{\omega+1} \ldots, U_n\}$ is a privileged set. Consider the case that the set of colluders $\mathcal{W}$ and the storage manager will guess $k_{\mathcal{S}}$ to obtain the plaintext m by the using their secret keys. Since each degree of x of $f^{(h)}(x)$ $(1 \leq h \leq n)$ is at most ω, at most ω colluders cannot obtain $f^{(h)}(x)$ from $f^{(h)}(1), \ldots, f^{(h)}(\omega)$ $(1 \leq h \leq n)$. Hence, they cannot obtain any information on $f^{(h)}(x)$ $(1 \leq h \leq n)$ even if they have $g^{(\ell)}(x)$ $(2 \leq \ell \leq n)$. Hence, for any $\mathcal{S} \subset \mathcal{U}$, and any $\mathcal{W} \subset \mathcal{U}$ such that $\mathcal{S} \cap \mathcal{W} = \emptyset$ and $|\mathcal{W}| \leq \omega$, $H(M \mid C_{\mathcal{S}}, DK_{\mathcal{W}}, MK) = H(M)$. □

B $(t, \leq \omega)$-one-time secure RS-BE

As in traditional broadcast encryption schemes [28, 32, 25, 15], we can also consider another class of RS-BE schemes, which is called $(t, \leq \omega)$-one-time secure RS-BE schemes, where $t + \omega \leq n$. A model and security of such a scheme are almost the same as that described in Section 2, and the only difference from those in Section 2 is that a sender can specify only a privileged set whose cardinality is exactly t (i.e., $|\mathcal{S}| = t$).

Then, we can derive lower bounds on secret keys in a similar way to Section 3, and these bounds can also be applied to traditional $(t, \leq \omega)$-one-time secure broadcast encryption schemes [28, 32, 25, 15].

Theorem 6. *Let Π be a $(t, \leq \omega)$-one-time secure RS-BE scheme. Then, for any $\mathcal{S} \subset \mathcal{U}$, the following lower bounds hold under the condition $H(C_{\mathcal{S}}) = H(M)$:*

$$\textit{(i) } H(EK) \geq \binom{t+\omega}{t} H(M), \quad \textit{(ii) } H(DK_i) \geq \binom{t+\omega-1}{t-1} H(M) \textit{ for any } i \in \{1, 2, \ldots, n\},$$

$$\textit{(iii) } H(MK) \geq \left(\binom{t+\omega}{t} - 1 \right) H(M).$$

Proof. The proof follows from the following lemmata.

Lemma 5. *We have $H(EK) \geq \binom{t+\omega}{t} H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. Without loss of generality, let $\mathcal{I} := \{U_1, \ldots, U_{t+\omega}\}$. Let $\mathscr{W} := \{\mathcal{W} \subset \mathcal{I} \mid |\mathcal{W}| = \omega\} = \{\mathcal{W}_1, \ldots, \mathcal{W}_\ell\}$ be the family of all possible set of colluders, where $\ell = \binom{t+\omega}{\omega} = \binom{t+\omega}{t}$. Moreover, let $\mathscr{S}(\mathscr{W}) := \{\mathcal{S}_1, \ldots, \mathcal{S}_\ell\}$, where $\mathcal{S}_i = \mathcal{I} \setminus \mathcal{W}_i$ such that $\mathcal{W}_i \in \mathscr{W}$ $(1 \leq i \leq \ell)$. Then, we have

$$\begin{aligned}
H(EK) =& H(EK \mid M) && (20)\\
\geq& I(EK; C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M)\\
=& H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M) - H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M, EK)\\
=& H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M) && (21)\\
=& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}})\\
\geq& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j})\\
\geq& \binom{t+\omega}{t} H(M), && (22)
\end{aligned}$$

where (20) follows from independence of M and EK, (21) follows from the algorithm *Enc* (i.e. $H(C_{\mathcal{S}_i} \mid EK, M) = 0$ $(1 \leq i \leq \ell)$), and (22) follows from Lemma 1. □

Lemma 6. *For any $i \in \{1, \ldots, n\}$, then we have $H(DK_i) \geq \sum_{j=0}^{\omega} \binom{n-1}{j} H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. Without loss of generality, let $\mathcal{I} := \{U_1, \ldots, U_i, \ldots, U_{t+\omega}\}$. Let $\mathscr{W}^{(i)} := \{\mathcal{W} \subset \mathcal{I} \setminus \{U_i\} \mid |\mathcal{W}| = \omega\} = \{\mathcal{W}_1, \ldots, \mathcal{W}_\ell\}$ be the family of all possible set of colluders except for sets of colluders containing U_i, where $\ell = \binom{t+\omega-1}{\omega} = \binom{t+\omega-1}{t-1}$. Let $\mathscr{S}(\mathscr{W}^{(i)}) := \{\mathcal{S}_1, \ldots, \mathcal{S}_\ell\}$, where $\mathcal{S}_i = \mathcal{I} \setminus \mathcal{W}_i$ such that $\mathcal{W}_i \in \mathscr{W}^{(i)}$ $(1 \leq i \leq \ell)$ We note $U_i \in \mathcal{S}$ for any $\mathcal{S} \in \mathscr{S}(\mathscr{W}^{(i)})$. Then, we have

$$\begin{aligned}
H(DK_i) =& H(DK_i \mid M) && (23)\\
\geq& I(DK_i; C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M)\\
=& H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M) - H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M, DK_i)\\
=& H(C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_\ell} \mid M) && (24)\\
=& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}})\\
\geq& \sum_{j=1}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j})
\end{aligned}$$

$$\geq \binom{t+\omega-1}{t-1} H(M), \tag{25}$$

where (23) follows from independence of M and DK_i, (24) follows from (3) in Lemma 1 (i.e. $H(C_{\mathcal{S}_j} \mid DK_i, M) = 0$ $(1 \leq j \leq \ell)$), and (25) follows from Lemma 1. □

Lemma 7. *We have $H(MK) \geq \left(\binom{t+\omega}{t} - 1\right) H(M)$ under the condition $H(C_{\mathcal{S}}) = H(M)$ for any $\mathcal{S} \subset \mathcal{U}$.*

Proof. Let $\mathcal{I}$, $\mathscr{W}$ and $\mathscr{S}(\mathscr{W})$ be the same as those in Lemma 5. Then, we have

$$\begin{aligned}
H(MK) \geq & H(MK \mid C_{\mathcal{S}_1}) \\
\geq & I(MK; C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_\ell} \mid C_{\mathcal{S}_1}) \\
= & H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_\ell} \mid C_{\mathcal{S}_1}) - H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_\ell} \mid C_{\mathcal{S}_1}, MK) \\
= & H(C_{\mathcal{S}_2}, \ldots, C_{\mathcal{S}_\ell} \mid C_{\mathcal{S}_1}) & (26) \\
= & \sum_{j=2}^{\ell} H(C_{\mathcal{S}_j} \mid C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}) \\
\geq & \sum_{j=2}^{\ell} H(C_{\mathcal{S}_j} \mid M, C_{\mathcal{S}_1}, \ldots, C_{\mathcal{S}_{j-1}}, DK_{\mathcal{W}_j}) \\
\geq & \left(\binom{t+\omega}{t} - 1 \right) H(M), & (27)
\end{aligned}$$

where (26) follows from the algorithm *Upd* (i.e. $H(C_{\mathcal{S}_i} \mid C_{\mathcal{S}_1}, MK) = 0$ $(2 \leq i \leq \ell)$), and (27) follows from Lemma 1. □

Now, the proof of Theorem 6 is completed. □

We can construct a $(t, \leq \omega)$-one-time secure RS-BE scheme based on the idea of our construction described in Section 4 and an ω-secure non-interactive t-conference KPS [11] as follows. We omit the security proof since it is easy to prove in a similar manner as the proof of Theorem 3. Also, We can consider a robust scheme in the same manner as the proposed robust scheme in Section 5.

1. $(ek, mk, dk_1, \ldots, dk_n) \leftarrow$ *Setup*(n): Let $\mathbb{F}_q$ be a finite field with q $(> n)$ elements, where q is a prime power. It chooses a symmetric polynomial $f(x_1, \ldots, x_t) := \sum_{i_1=0}^{\omega} \cdots \sum_{i_t=0}^{\omega} a_{i_1 i_2 \cdots i_t} x_1^{i_1} \cdots x_t^{i_t}$ over $\mathbb{F}_q$, where $a_{i_1 i_2 \cdots i_t} = a_{\sigma(i_1)\sigma(i_2)\cdots\sigma(i_t)}$ for all permutations $\sigma = (\sigma(i_1), \sigma(i_2), \ldots, \sigma(i_t))$. Also, it conputes $g(x_1, x_2, \ldots, x_t) := f(x_1, x_2, \ldots, x_t) - a_{00\cdots 0}$. Then, it outputs $ek := f(x_1, x_2, \ldots, x_t)$, $dk_i := f(i, x_2, \ldots, x_t)$ $(1 \leq i \leq n)$, and $mk := g(x_1, x_2, \ldots, x_t)$.

2. $c_{\mathcal{S}} \leftarrow$ *Enc*$(ek, m, \mathcal{S})$: For any privileged set $\mathcal{S} := \{U_{i_1}, \ldots, U_{i_t}\}$, it computes a session key $k_{\mathcal{S}} := f(i_1, \ldots, i_t)$, and then outputs $c_{\mathcal{S}} := m + k_{\mathcal{S}}$.

3. m or $\perp \leftarrow$ *Dec*$(dk_i, c_{\mathcal{S}}, \mathcal{S}, U_i)$: If $U_i \in \mathcal{S}$, then it computes $k_{\mathcal{S}}$ as in the algorithm *Enc* and outputs $m = c_{\mathcal{S}} - k_{\mathcal{S}}$. Otherwise, it outputs $\perp$.

4. $c_{\mathcal{S}'}$ or $\perp \leftarrow$ *Upd*$(mk, c_{\mathcal{S}}, \mathcal{S}, \mathcal{S}')$: For any pair of privileged sets $\mathcal{S} := \{U_{i_1}, \ldots, U_{i_t}\}$ and $\mathcal{S}' := \{U_{j_1}, \ldots, U_{j_t}\}$, it computes and outputs $c_{\mathcal{S}'} := c_{\mathcal{S}} + g(j_1, \ldots, j_t) - g(i_1, \ldots, i_t)$.

Theorem 7. *The resulting RS-BE scheme Π by the above construction is $(t, \leq \omega)$-one-time secure and meets equality in every bound of (i)–(iii) in Theorem 6.*