Chapter 1 Electrostatic Field

1.1 Charge and Matter

1.2 Electric Field and Intensity

1.3 The Gauss's Law For **E**

1.4 Electric Potential

1.3 Gauss's Law For Electric Field

Review…

◈ Coulomb's Law

$$\vec{\boldsymbol{F}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\hat{\boldsymbol{r}}$$

- Repulsion if they have same signs
- Attraction if they have opposite signs
- Superposition Principle

$$\vec{\boldsymbol{F}}_1 = \vec{\boldsymbol{F}}_{21} + \vec{\boldsymbol{F}}_{31} \cdots \vec{\boldsymbol{F}}_{i1}$$

1.3 Gauss's Law For Electric Field

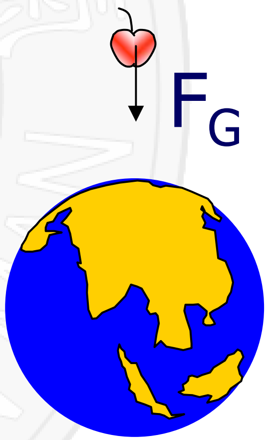
Review…

◇ Electric Field

- How do the forces act on the charges?

Doesn't need time and media

  - Action-at-a-distance?
  - Something else?



- Electric Field like Gravity field
  - Mass creates gravity field
  - Charge creates electric field
  - Any charge will experience electric force.

1.3 Gauss's Law For Electric Field

Review…

- The electric field of a point charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

- Superposition principle

  A group of point charge

$$\vec{E} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r^2}\hat{\boldsymbol{r}} \qquad \text{Vector Sum}$$

  A continuous distribution of charges

$$\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\boldsymbol{r}} \qquad \text{Vector Integration}$$

1.3 Gauss's Law For Electric Field

Review…

- Electric field of a dipole

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

- The electric field of a ring of charge

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

- Infinite charged line

$$E=\frac{\lambda}{2\pi\varepsilon_0 x}$$

1.3 Gauss's Law For Electric Field

- General speaking, we can compute electric field due to any charged system by SP.

? Q: Why to talk about Gauss's Law

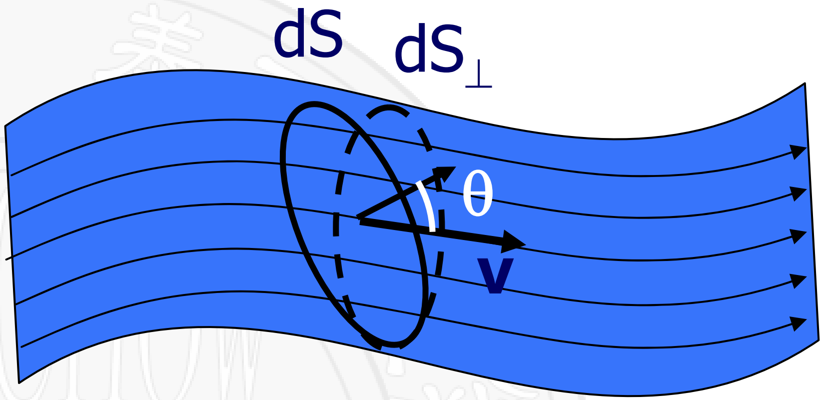
A: Another way to compute electric field and another view of electric field.

SP works to find electric field,
but it is very difficult (laborious).

1.3 Gauss's Law For Electric Field

The concept of flux

- Water flow



  - River ~ streamlines
  - The tangent to streamline gives v's direction
  - Velocity field, Vectorial field

$$\vec{v}(x,y,z) = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

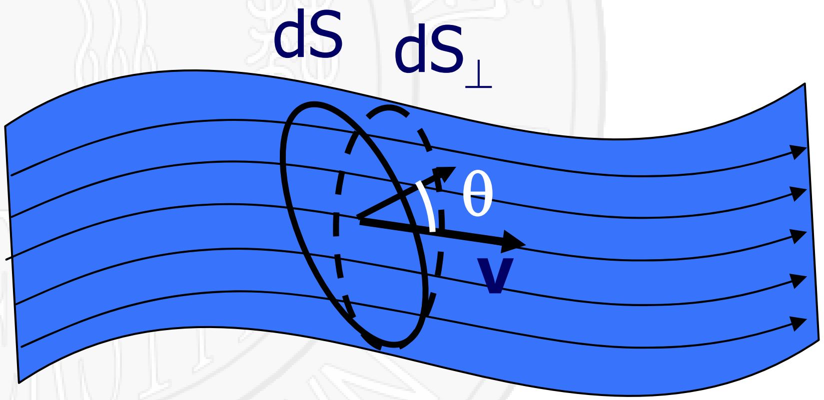
1.3 Gauss's Law For Electric Field

The concept of flux

- Flux=Flow Quantity, volume flow rate

$$dQ = v \cdot dS_{\perp} = v \cdot dS\cos\theta = \vec{v} \cdot d\vec{S}$$

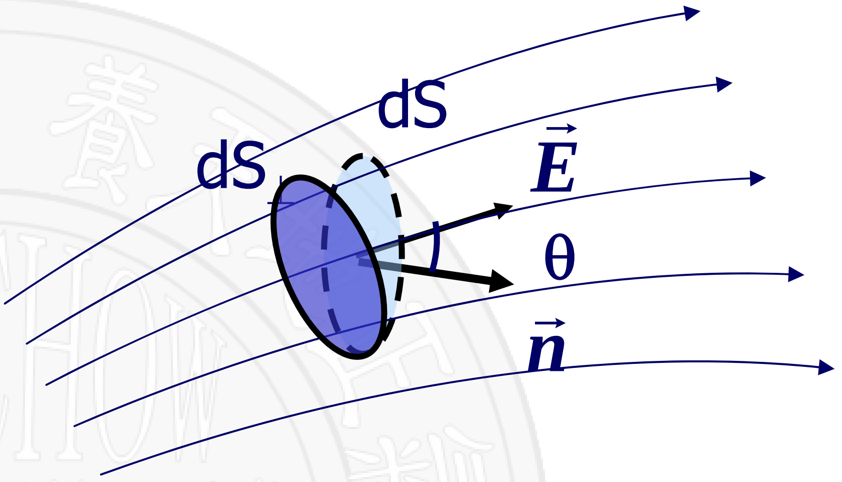
$$Q = \iint_{(S)} \vec{v} \cdot d\vec{S}$$



1.3 Gauss's Law For Electric Field

The concept of flux

- Flux of Electric field

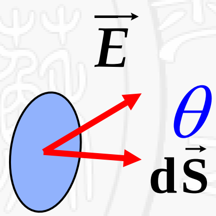


$$d\phi_{\vec{E}} = E \cdot dS_{\perp}$$

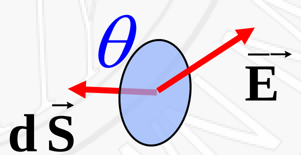
$$= E \cdot dS\cos\theta$$

$$= \vec{E} \cdot d\vec{S}$$

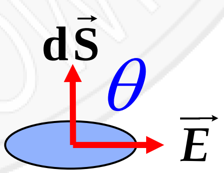
- $\theta < \dfrac{\pi}{2},\quad d\phi_E > 0$



- $\theta > \dfrac{\pi}{2},\quad d\phi_E < 0$



- $\theta = \dfrac{\pi}{2},\quad d\phi_E = 0$

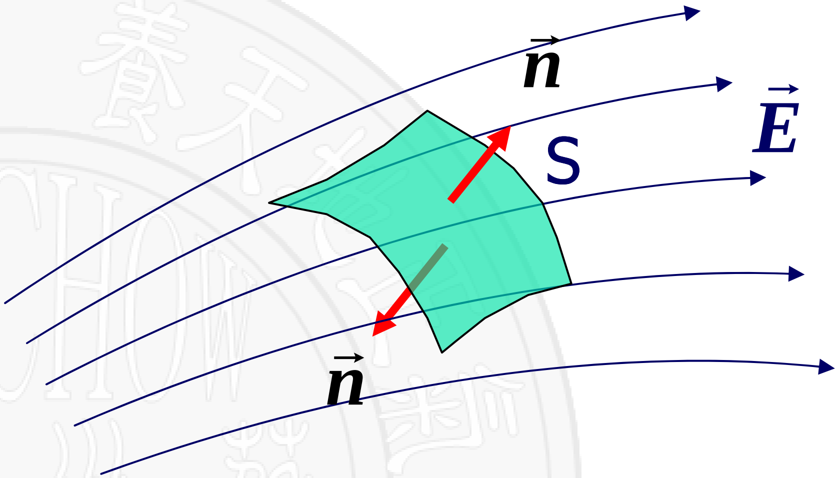


1.3 Gauss's Law For Electric Field

The concept of flux

- Flux of Electric field

$$\phi_{\vec{E}} = \iint_{(S)} \vec{E} \cdot d\vec{S}$$



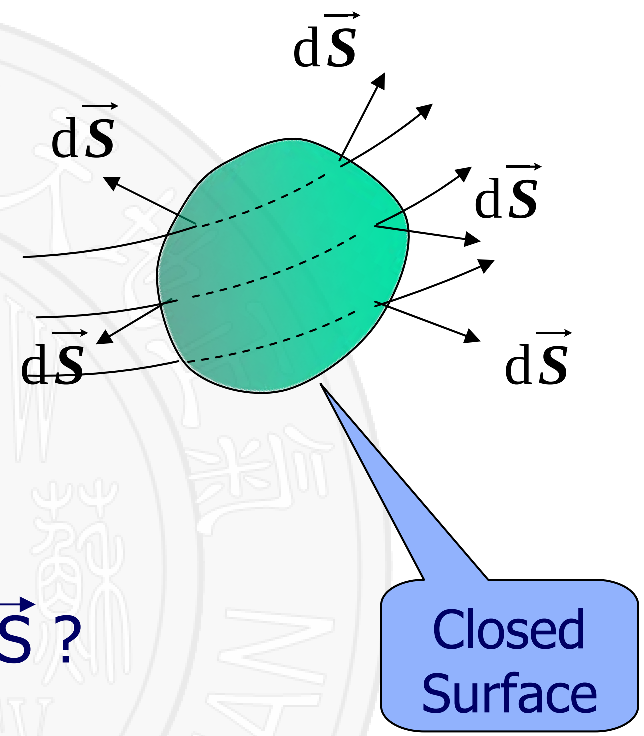
- $\phi_E > 0$, normal line pointing up↗.
- $\phi_E > 0$, normal line pointing down↙.
- ϕ_E=The numbers of field line through S

1.3 Gauss's Law For Electric Field

The concept of flux

- Flux for a Closed surface

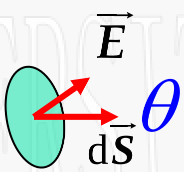
$$\phi_E = \oiint_{(S)} \vec{E} \cdot d\vec{S}$$



- What is the direction of $d\vec{S}$?

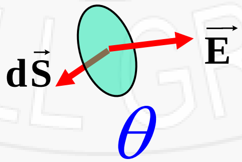
  Outward normal direction of $d\vec{S}$?

$$\theta < \frac{\pi}{2},\ d\phi_E > 0$$



Out

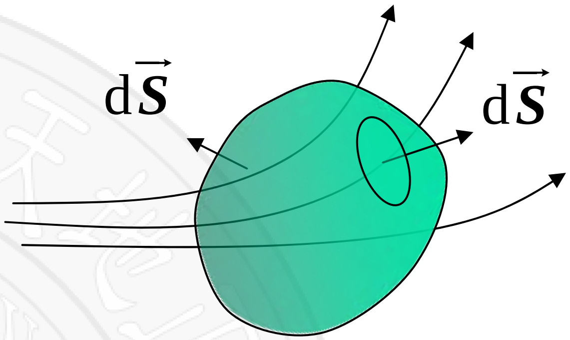
$$\theta > \frac{\pi}{2},\ d\phi_E < 0$$



In

1.3 Gauss's Law For Electric Field

The concept of flux



- Flux of Electric field

$$\phi_E = \oiint_{(S)} \vec{E} \cdot d\vec{S}$$

$\phi_E > 0$, Lines out more than in

Net Positive Charge inside

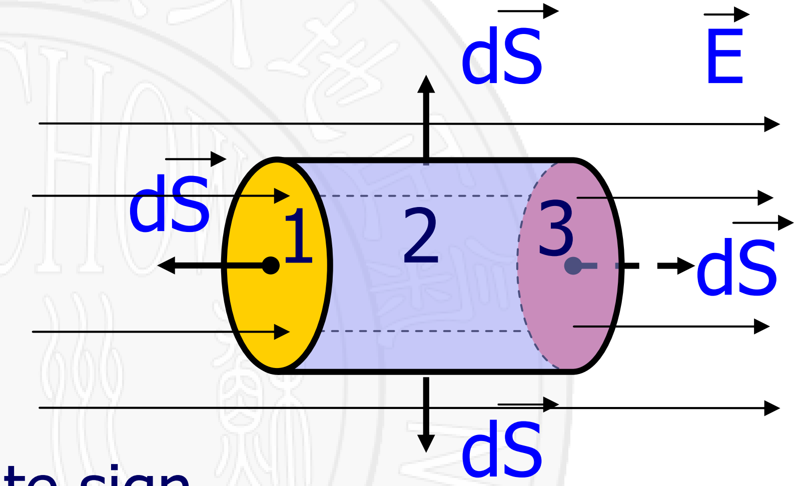
$\phi_E < 0$, Lines in more than out

Net Negative Charge inside

1.3 Gauss's Law For Electric Field

- Example for Flux of Electric field

A closed cylinder in an uniform electric field. Find the total flux.



$\phi_{tot} = \phi_1 + \phi_2 + \phi_3$

- $\Phi_2 = 0$, $\vec{E} \perp \vec{dS}$
- $|\Phi_1| = |\Phi_3|$, But opposite sign
- The total flux through the cylinder is zero!
- Lines Inward =Lines Outward

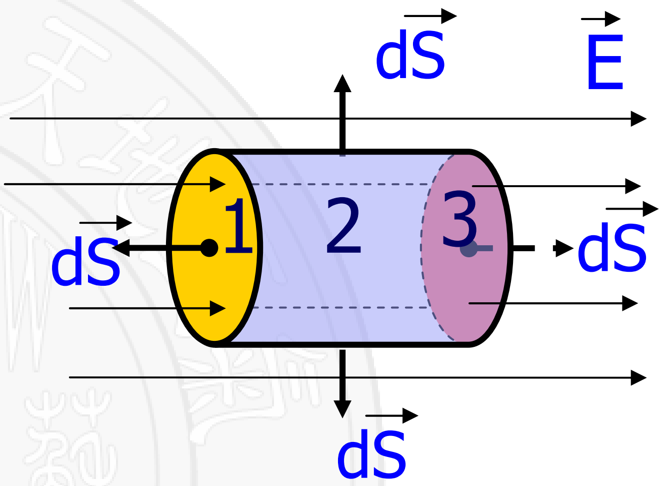
1.3 Gauss's Law For Electric Field

- Example for Flux of Electric field

Any charges in the cylinder?

No, all lines get in must get out!

Think about meaning of ϕ_E



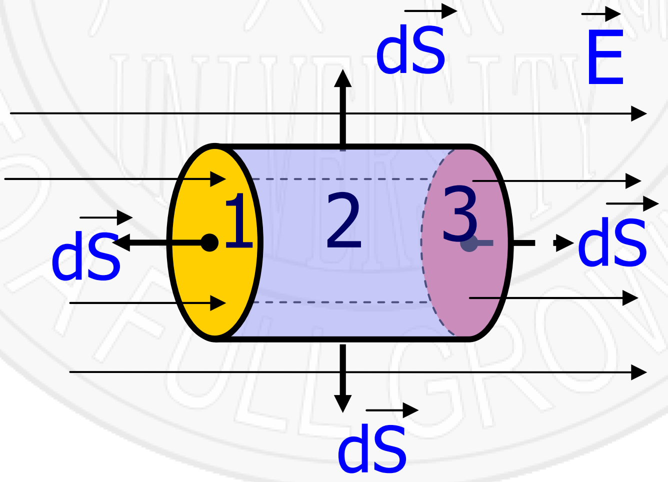
proportional to the field lines through the surface…

1.3 Gauss's Law For Electric Field

The concept of flux

Conclusion

- The electric flux through a closed surface that does not contain charges equals to zero.
- The electric flux through a closed surface that does contain a net charge doesn't equal to zero.



1.3 Gauss's Law For Electric Field

Gauss's Law

What's the exact relationship between charge and flux ?

- Gauss's law

The outward flux through any closed surface is equal to the algebraic sum of the enclosed charges divided by ε_0,i.e.,

$$\oiint_{(S)} \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \sum_{(S内)} q_i$$

通过任意闭合曲面的电通量等于该曲面所包围的电量的代数和除以ε_0

1.3 Gauss's Law For Electric Field

Gauss's Law

- Demonstration
  - A point charge at the center of a sphere

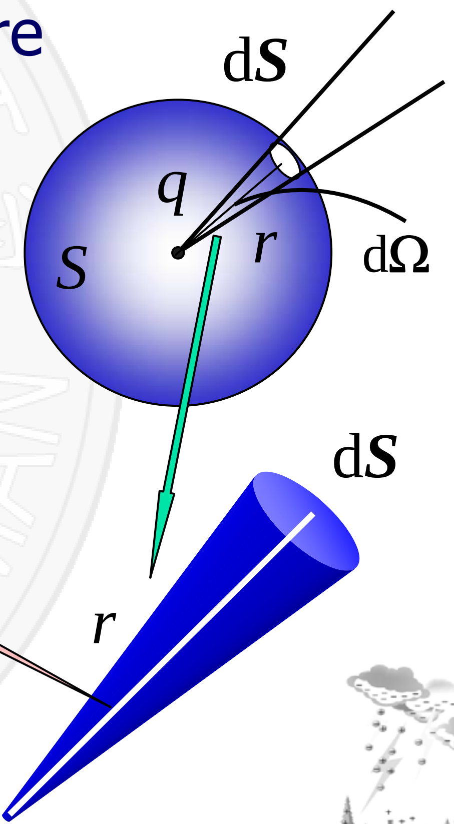
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

Solid angle

$$\mathrm{d}\phi_E = \vec{E}\cdot\mathrm{d}\vec{S} = \frac{q}{4\pi\varepsilon_0}\frac{\mathrm{d}S}{r^2} = \frac{q}{4\pi\varepsilon_0}\mathrm{d}\Omega$$

$$\phi_E = \oiint_{(S)}\vec{E}\cdot d\vec{S} = \oiint_{(S)}\frac{q}{4\pi\varepsilon_0}\frac{d\boldsymbol{S}}{r^2}$$

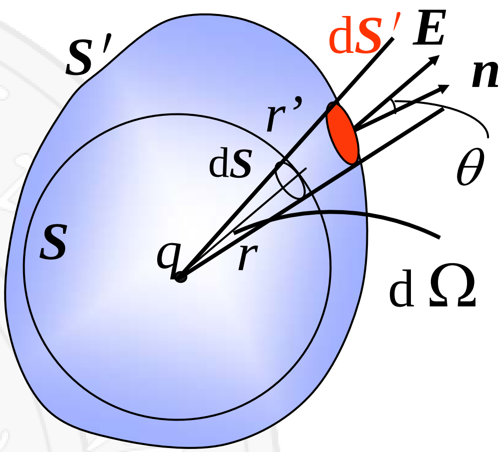
$$= \frac{q}{4\pi\varepsilon_0}\oiint_{(S)} d\Omega = \frac{q}{\varepsilon_0}$$



1.3 Gauss's Law For Electric Field

Gauss's Law

- A point charge in a closed surface



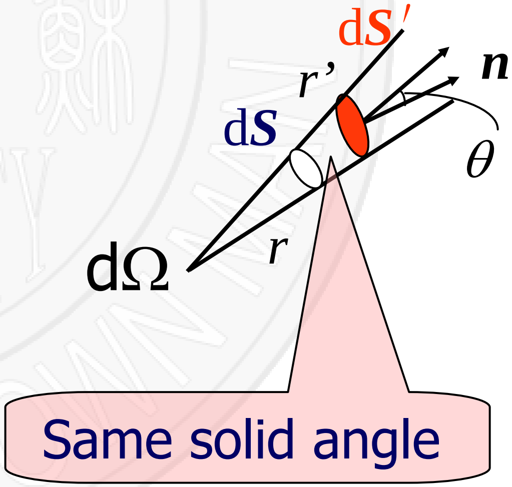
$$\mathrm{d}\phi_E = \vec{E}\cdot \mathrm{d}\vec{S}' = \frac{q}{4\pi\varepsilon_0}\frac{\mathrm{d}S'\cos\theta}{r'^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\mathrm{d}\Omega$$

$$\mathrm{d}\Omega = \frac{\mathrm{d}S'\cos\theta}{r'^2} = \frac{\mathrm{d}S\cos\theta}{r^2}$$

$$\phi_E = \oiint_{(S)} \vec{E}\cdot d\vec{S} = \oiint_{(S)} \frac{q}{4\pi\varepsilon_0}\frac{d\boldsymbol{S}}{r^2}$$

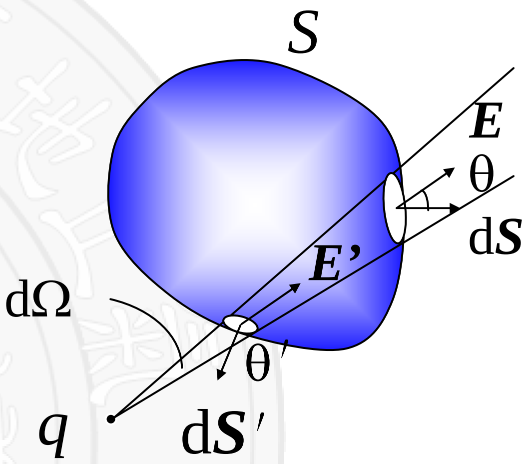
$$= \frac{q}{4\pi\varepsilon_0}\oiint_{(S)} d\Omega = \frac{q}{\varepsilon_0}$$



1.3 Gauss's Law For Electric Field

Gauss's Law

- A charge outside of a closed surface



$$d\phi_E = \vec{E}\cdot d\vec{S} + \vec{E}'\cdot d\vec{S}'$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{dS\cos\theta}{r^2} + \frac{q}{4\pi\varepsilon_0}\frac{dS'\cos\theta'}{r'^2}$$

$$= \frac{1}{4\pi\varepsilon_0}d\Omega + \frac{1}{4\pi\varepsilon_0}(-d\Omega) = 0$$

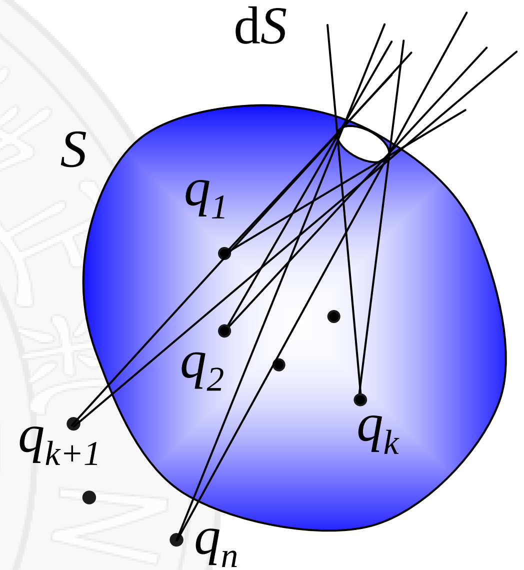
$$\phi_E = \oiint_{(S)} \vec{E}\cdot d\vec{S} = 0$$

- The electric flux through a closed surface that does not contain charges equals to zero.

1.3 Gauss's Law For Electric Field

Gauss's Law

- A group of charge, some inside, some outside.



$$\phi_E = \oiint_{(S)} (\boldsymbol{E}_1 + \boldsymbol{E}_2 + \cdots + \boldsymbol{E}_k + \boldsymbol{E}_{k+1} + \cdots + \boldsymbol{E}_n) \cdot d\boldsymbol{S}$$

$$= \oiint_{(S)} \boldsymbol{E}_1 \cdot d\boldsymbol{S} + \oiint_{(S)} \boldsymbol{E}_2 \cdot d\boldsymbol{S} + \cdots + \oiint_{(S)} \boldsymbol{E}_k \cdot d\boldsymbol{S}$$

$$+ \oiint_{(S)} \boldsymbol{E}_{k+1} \cdot d\boldsymbol{S} + \cdots + \oiint_{(S)} \boldsymbol{E}_n \cdot d\boldsymbol{S}$$

$$\phi_E = \oiint_{(S)} \boldsymbol{E} \cdot d\boldsymbol{S} = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \ldots + \frac{q_k}{\varepsilon_0} + 0 + \ldots + 0 = \frac{\sum_{i=1}^{k} q_i}{\varepsilon_0}$$

1.3 Gauss's Law For Electric Field

Gauss's Law

The electrostatic field is a divergent field.

Show divergent field

Show source of field

Show sink of field

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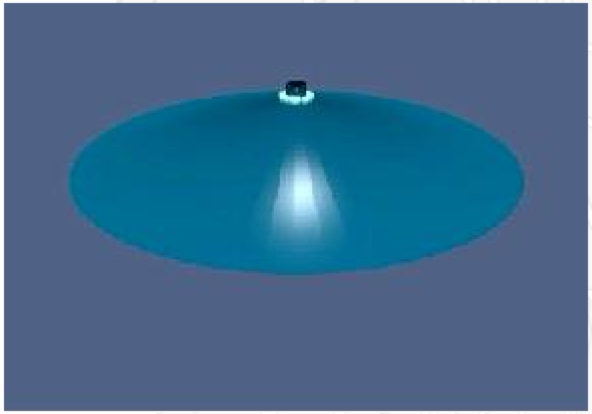
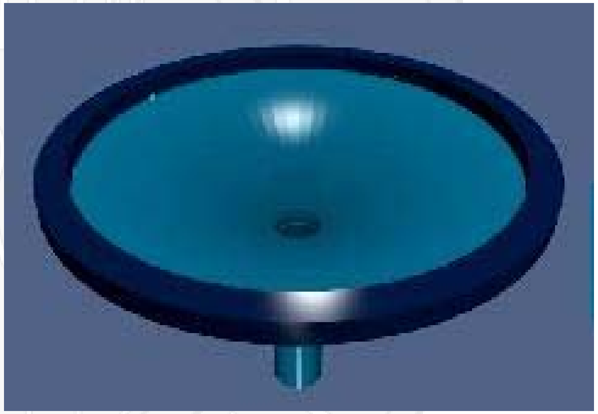
1.3 Gauss's Law For Electric Field

Gauss's Law

- The electrostatic field is a divergent field.
- The lines originate from +Q, end up –Q.





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source

sink

1.3 Gauss's Law For Electric Field

Gauss's Law

- Why is Gauss's law so important?
  - It relates the electric field E with its sources Q
  - Given Q distribution, find E (integral form)
  - Given E, find Q (differential form)
- Is Gauss's law always true?
  - Yes, no matter what E or what S

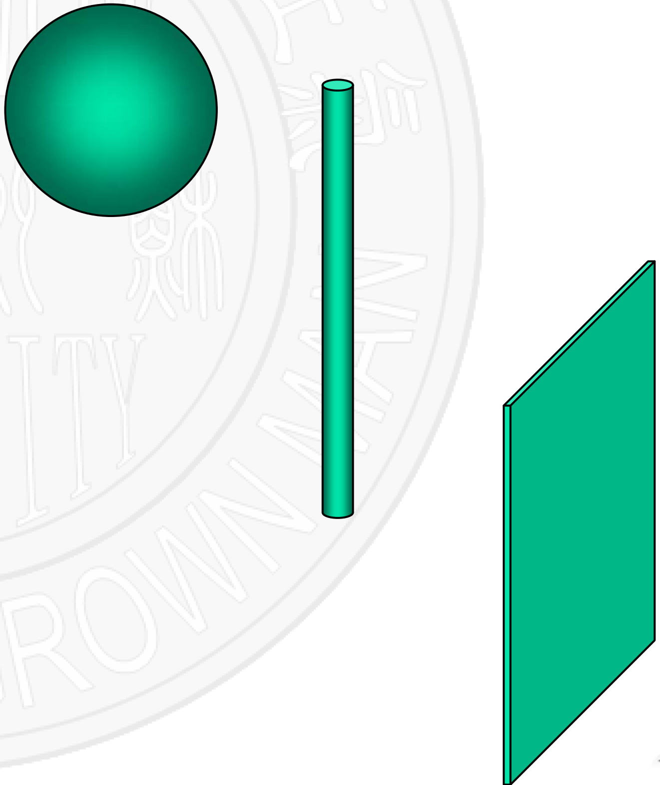
1.3 Gauss's Law For Electric Field

Gauss's Law

- Is Gauss's law always useful?

  Yes, very useful when to find E, if

  - *Spherical symmetry*
  - *cylindrical symmetry*
  - *Surface symmetry*



1.3 Gauss's Law For Electric Field

Applications of Gauss's Law

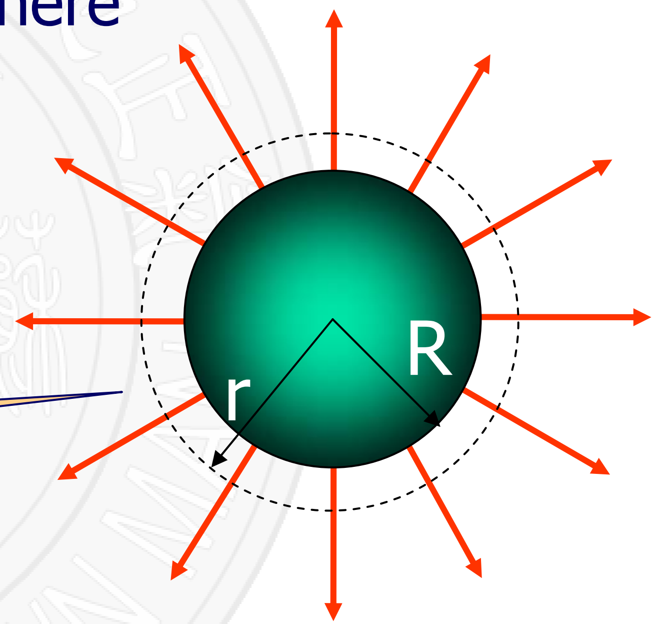
Spherically symmetric charge distribution

Charge q uni. distri. inside of a sphere R . Expressions of ***E*** for all points

Solution: Outside (r>R)

Chose concentric sphere as

Gaussian surface



Spherical symmetry, same E everywhere on sphere surface, the direction is radial.

1.3 Gauss's Law For Electric Field

Applications of Gauss's Law

- Spherically symmetric charge distribution

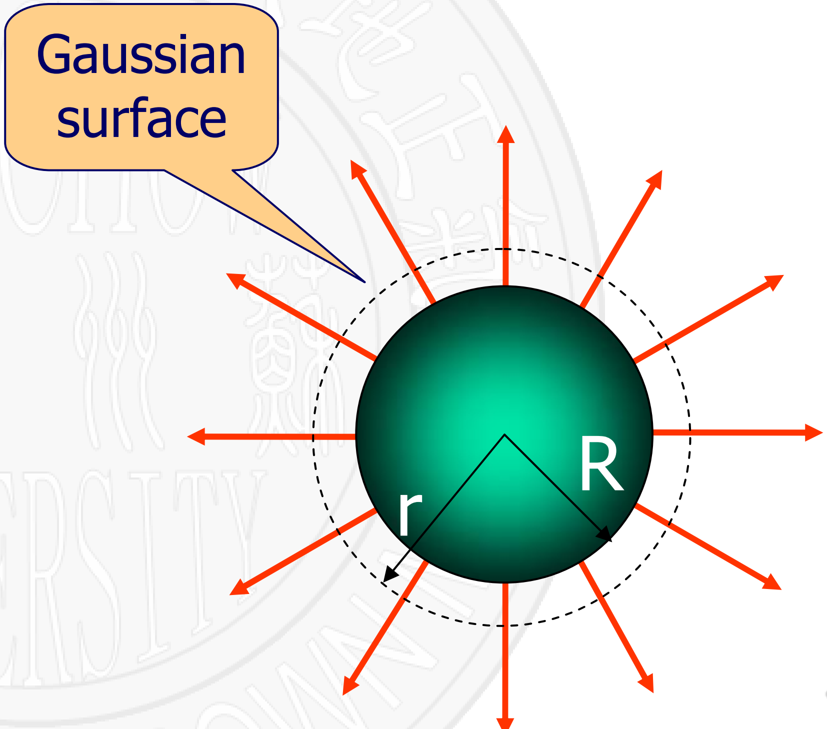
$$\oiint_{(S)} \vec{E} \cdot d\vec{S} = E \cdot 4\pi r^2$$

$$\oiint_{(S)} \vec{E} \cdot d\vec{S} = \frac{\sum q_i}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

the same as point charge.



1.3 Gauss's Law For Electric Field

- Applications of Gauss's Law
  - Spherically symmetric charge distribution

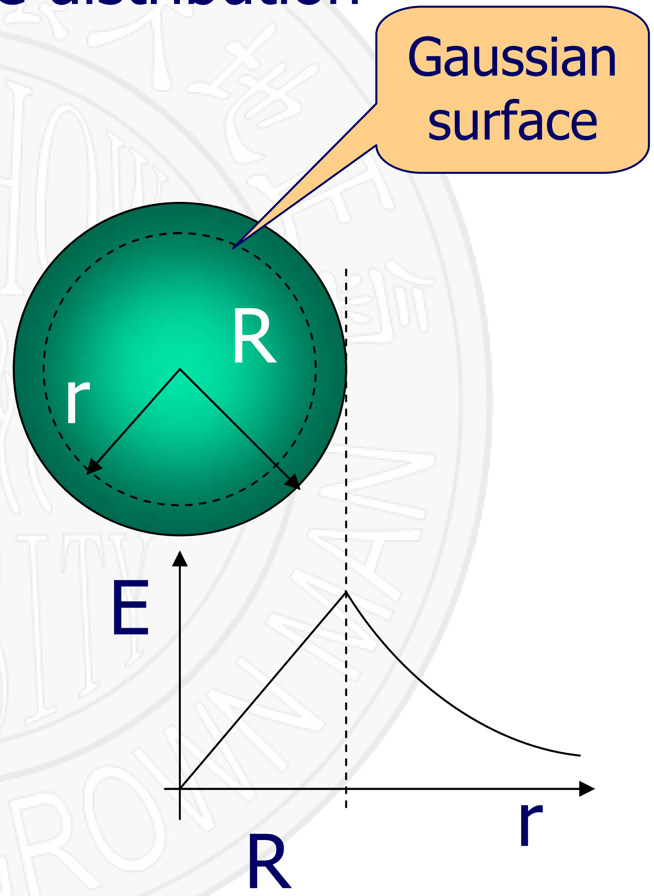
Inside (r<R)

$$\oiint_{(S)} \vec{E}\cdot d\vec{S} = E\cdot 4\pi r^2$$

$$E\cdot 4\pi r^2 = \frac{q'}{\varepsilon_0}$$

$$q' = \frac{q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = \frac{qr^3}{R^3} \Rightarrow$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$



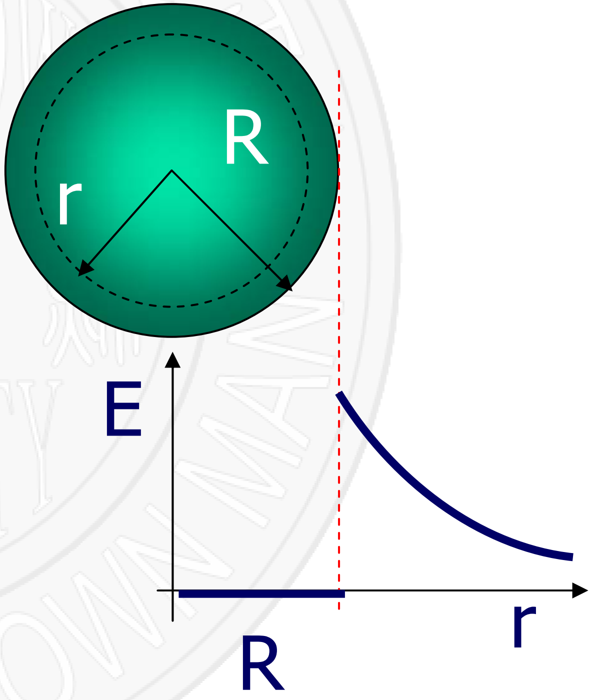
1.3 Gauss's Law For Electric Field

- Applications of Gauss's Law
  - Spherically symmetric charge distribution

If it is a spherical shell
or metal sphere, q′=0

E=0 (r<R)

E is equal to zero everywhere
inside of the sphere.



1.3 Gauss's Law For Electric Field

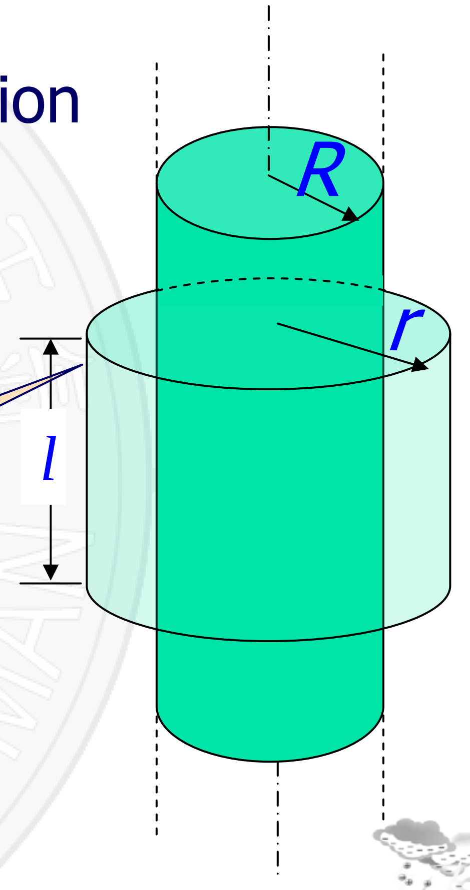
Applications of Gauss's Law

- Cylindrically symmetric charge distribution

Uni. charged infinite cylinder R, density ρ. E's expression for all points

Solution: Outside (r>R)

Chose coaxial cylinder as Gaussian surface

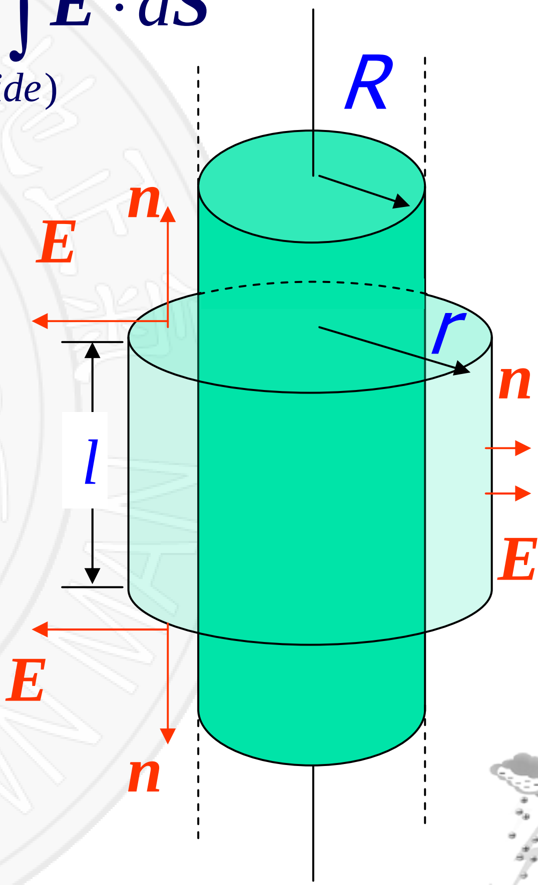


Cylindrical symmetry, E is the same everywhere at any coaxial cylinder, the direction is radial.

1.3 Gauss's Law For Electric Field

$$\oiint_{(tot)} \vec{E}\cdot d\vec{S} = \underbrace{\iint_{(top)} \vec{E}\cdot d\vec{S}}_{=0} + \underbrace{\iint_{(bottom)} \vec{E}\cdot d\vec{S}}_{=0} + \iint_{(side)} \vec{E}\cdot d\vec{S}$$

$$= \iint_{(side)} \vec{E}\cdot d\vec{S} = E2\pi rl$$



From Gauss's law

$$E2\pi rl = \frac{1}{\varepsilon_0}\sum_{(S)} q_i = \frac{1}{\varepsilon_0}\rho\pi R^2 l$$

$$E = \frac{1}{\varepsilon_0}\frac{\rho\pi R^2}{2\pi r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E \propto \frac{1}{r}$$

infinite charged line

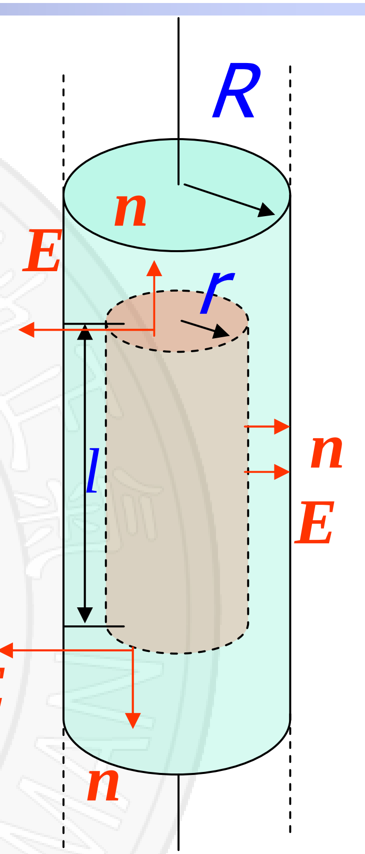
1.3 Gauss's Law For Electric Field

Inside (r<R)

=0   =0

$$\oiint_{(S)} \vec{E}\cdot d\vec{S} = \iint_{(S_{up})} \vec{E}\cdot d\vec{S} + \iint_{(S_{down})} \vec{E}\cdot d\vec{S} + \iint_{(S_{cylinder})} \vec{E}\cdot d\vec{S}$$

$$= \iint_{(S_{cylinder})} \vec{E}\cdot d\vec{S} = E2\pi rl$$

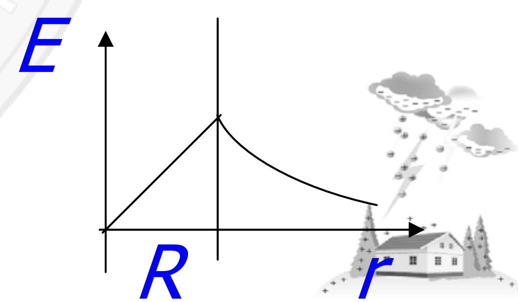


From Gauss's law

$$E2\pi rl = \frac{1}{\varepsilon_0}\sum_{(S)} q_i = \frac{1}{\varepsilon_0}\rho\pi r^2 l$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

$$E \propto r$$

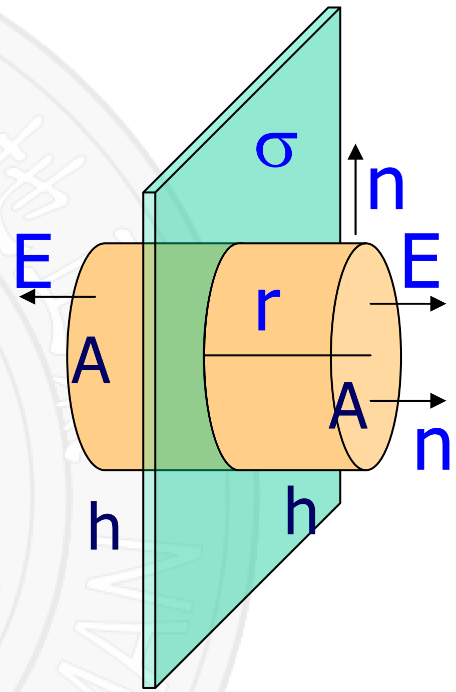


1.3 Gauss's Law For Electric Field

- Applications of Gauss's Law
  - Surface symmetric distribution

    Infinite sheet of charge, density σ

Solution: Choose cylinder as Gaussian surface!



Area of bottom A and height of 2h

Same E on each bottom

$$\oiint_{(tot)} \vec{E}\cdot d\vec{S} = \iint_{(left)} \vec{E}\cdot d\vec{S} + \iint_{(right)} \vec{E}\cdot d\vec{S} + \iint_{(side)} \vec{E}\cdot d\vec{S}$$

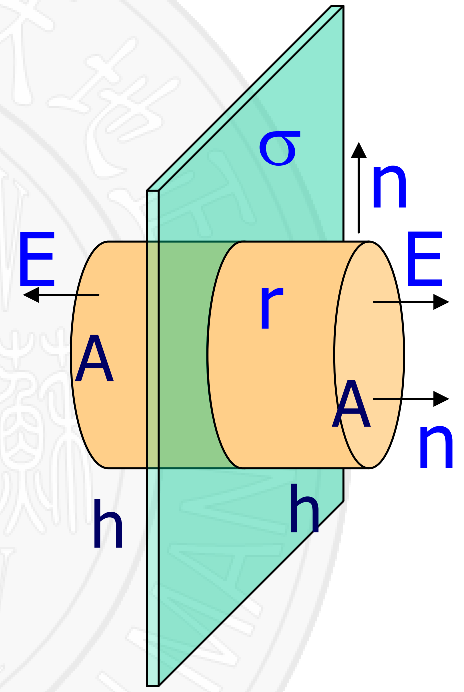
=0

1.3 Gauss's Law For Electric Field

- Applications of Gauss's Law
  - Surface symmetric distribution

$$\iint_{(left)} \vec{E} \cdot d\vec{S} = \iint_{(right)} \vec{E} \cdot d\vec{S} = 2EA$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



1.3 Gauss's Law For Electric Field

- Checklist for solving E&M problems
  - Read the problem (I am not joking!)
  - Look at the symmetries before choosing the best coordinate system
  - Look at the symmetries again and find out what cancels what and the direction of the vectors involved
  - Look for a way to avoid all complicated integration
  - Write down the complete solution (magnitudes and directions for all the different regions)