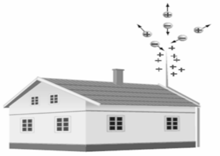
Chapter 2 Conductor & Dielectric

2.1 The Conductor in Electrostatic Field

2.2 Capacitance and Capacitor

2.3 Dielectrics in Electric Field

2.4 The Energy Storage in Electric Field

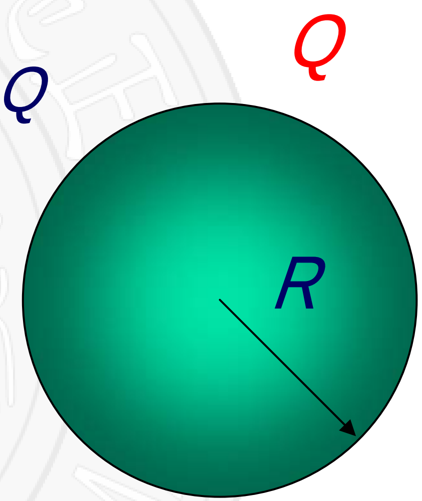


2.2 Capacitance and Capacitor

Capacitance for isolated conductor

- The capacitance for isolated conductor
  - The potential of the sphere *R*, *Q*

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$



  - Definition

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

Show overcharging

From Walter Lewin's Lecture

  - Units: 1C/1V=1F (very BIG unit)

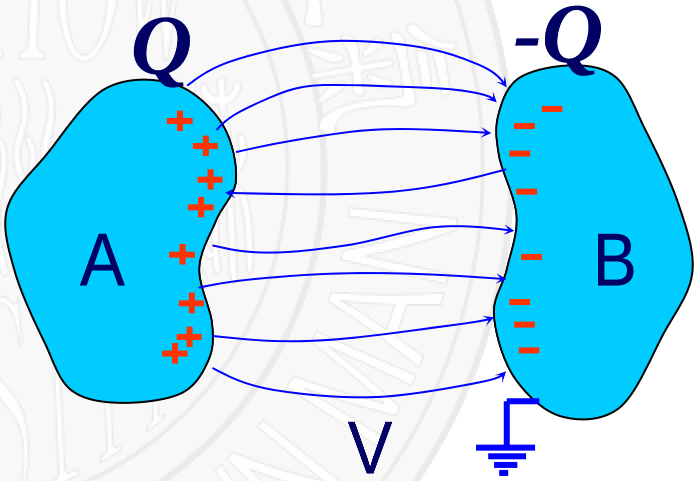
$$1F = 10^6\mu F = 10^{12} pF$$

Supercapacitor

2.2 Capacitance and Capacitor

Capacitance for Capacitor

- Capacitor: Combination of Conductors
  - Deposit charge +Q on one and -Q on the other
  - The ratio Q/V is cont.



- The Definition of capacitance

$$C = \frac{Q}{V}$$

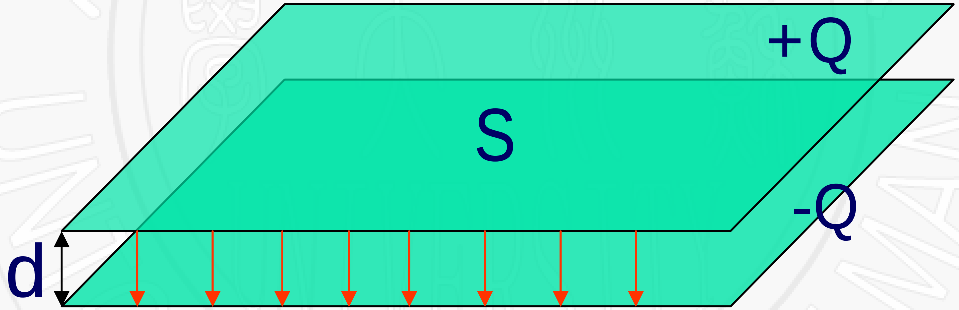
Potential difference A&B

2.2 Capacitance and Capacitor

Capacitance for Capacitor

- Calculation of Capacitance
  - Parallel-plate capacitor

    parallel plates, each of area S, at a distance d



$d^2 << S$ ~ infinite parallel planes

- Deposit +Q on top plate and –Q on bottom plate

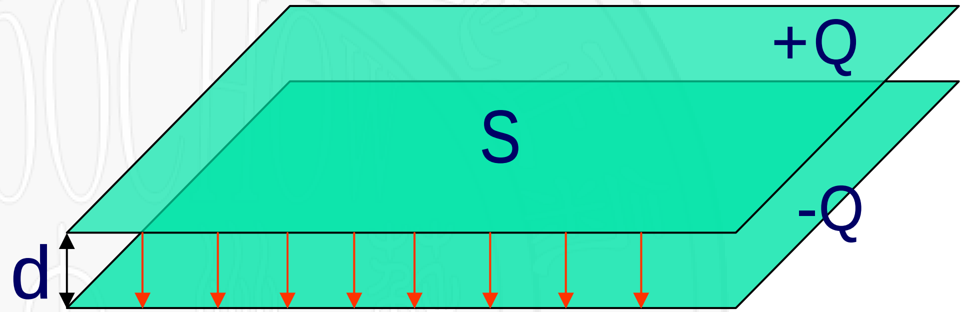
2.2 Capacitance and Capacitor

Capacitance for Capacitor

Calculation of Capacitance

- Find Potential V

$$V = Ed = \frac{Q}{S\varepsilon_0} d$$



- Find Capacitance:

$$C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d}$$

- C depends on *S* and *d*

2.2 Capacitance and Capacitor

Capacitance for Capacitor

Calculation of Capacitance

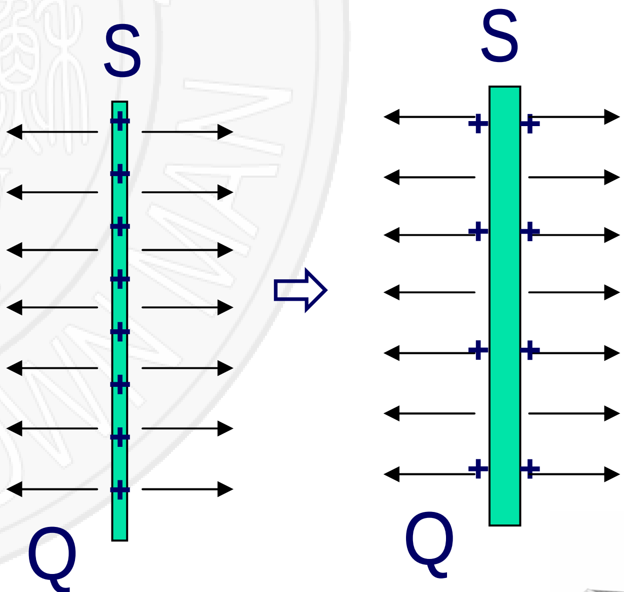
- Electric field on surface of conductor: σ/ε_0 or $\sigma/2\varepsilon_0$???
- Infinite plane of charges: $\sigma/2\varepsilon_0$
- Conductor surface: σ/ε_0

σ is different

$$\sigma = \frac{Q}{S} \Rightarrow E = \frac{Q}{2\varepsilon_0 S}$$

Consider a conductor

$$\sigma' = \frac{Q}{2S} \qquad E = \frac{Q}{2\varepsilon_0 S}$$



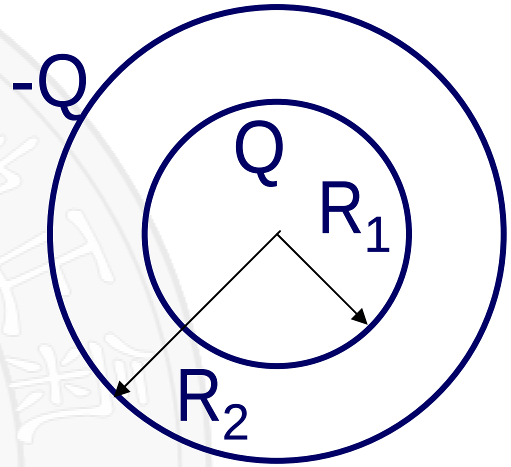
2.2 Capacitance and Capacitor

Capacitance for Capacitor

Calculation of Capacitance

- Spherical capacitor

Concentric spherical shells, R_1& R_2



- Deposit +Q on inner shell and –Q on outer shell

- Find Potential $V = \int_{R_1}^{R_2} E\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$

- Find Capacitance $C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$

2.2 Capacitance and Capacitor

Capacitance for Capacitor

Calculation of Capacitance

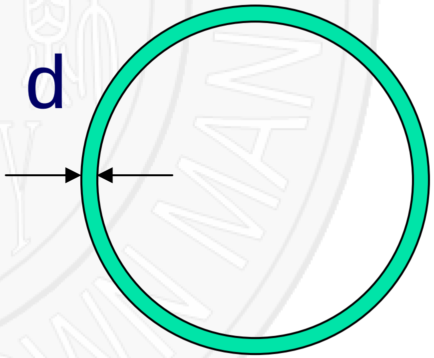
Spherical capacitor

- C depends only on the geometry of the arrangement

If $R_2 - R_1 = d << R_1 \rightarrow 0$

$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

$$= \frac{\varepsilon_0 4\pi R_1^2}{d} = \frac{\varepsilon_0 S}{d}$$

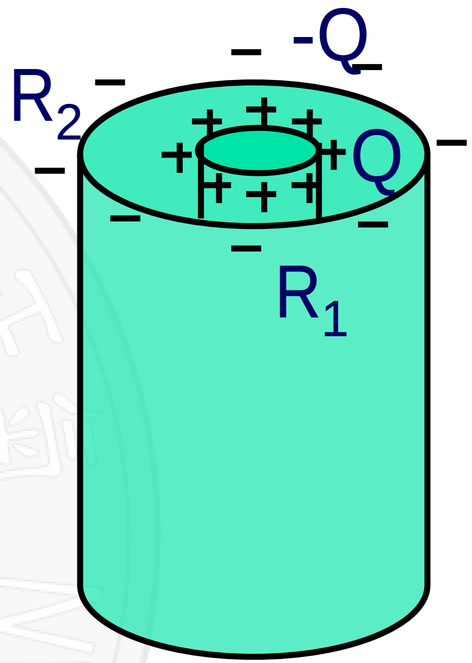


Same as parallel-plate capacitor!

2.2 Capacitance and Capacitor

◈ Capacitance for Capacitor

- Calculation of Capacitance
  - Cylindrical Capacitor



Coaxial cylindrical shells, R_1 & R_2,

- Deposit +Q on inner shell and –Q on outer shell

Find Potential $$V = \int_{R_1}^{R_2} E\mathrm{d}r = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{R_2}{R_1}$$

Find Capacitance $$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\frac{R_2}{R_1}}$$

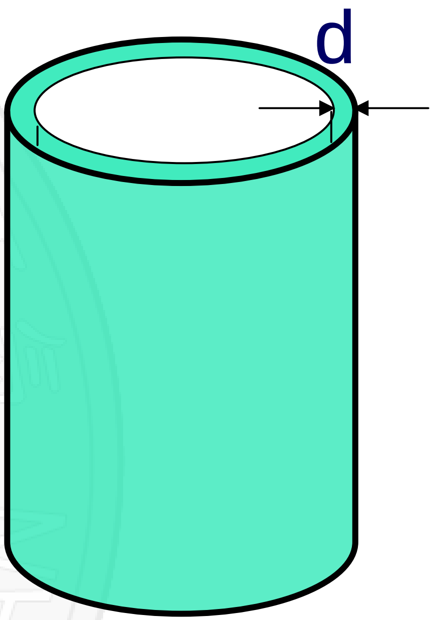
2.2 Capacitance and Capacitor

Capacitance for Capacitor

- Calculation of Capacitance
  - Cylindrical Capacitor

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\frac{R_2}{R_1}}$$

- C depends only on the geometry of the arrangement



If $R_2 - R_1 = d << R_1 \rightarrow 0$

$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{R_2}{R_1}} = \frac{2\pi\varepsilon_0 L}{\ln(1+\frac{d}{R_1})} \approx \frac{2\pi\varepsilon_0 L}{d/R_1} = \frac{\varepsilon_0 S}{d}$$

Supercapacitor

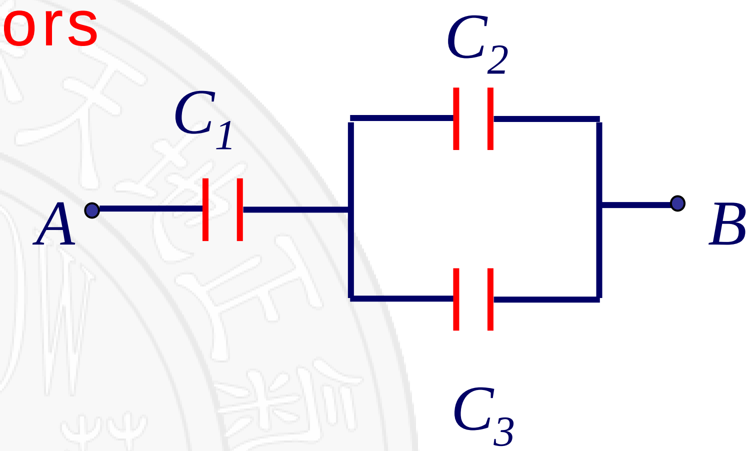
Same as parallel-plate capacitor!

2.2 Capacitance and Capacitor

The Combination of Capacitors

- Capacitors in series

In EE often treat the Combinations of Capacitors.



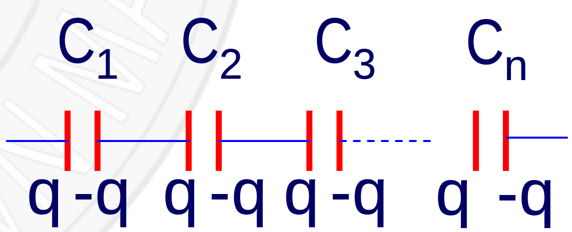
(A simple network)

Same charge q

$$V_1 = q/C_1,\ V_2 = q/C_2,\ \cdots\ V_n = q/C_n$$

$$V = (V_1 + V_2 + \cdots + V_n)$$

$$V = (q/C_1 + q/C_2 + \cdots + q/C_n)$$

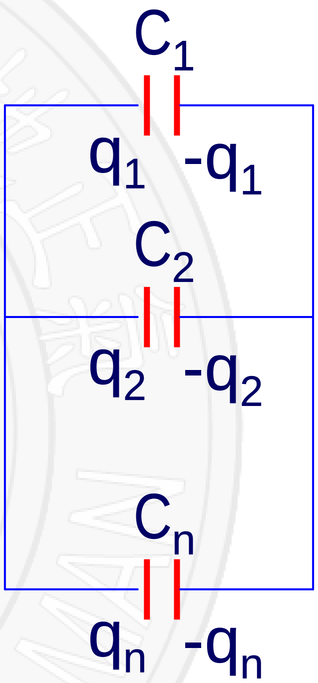


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

2.2 Capacitance and Capacitor

The Combination of Capacitors

- Capacitors in parallel



Same potential difference V

$q_1=C_1V$

$q_2=C_2V$

$\vdots$

$q_n=C_nV$

$q=q_1+q_2+\ldots+q_n=(C_1+C_2+\ldots C_n)V$

$C=q/V=(C_1+C_2+\ldots C_n)$

2.2 Capacitance and Capacitor

The Combination of Capacitors

Example 1.13 Two capacitors 200pF,300pF,capables of withstanding 500V, 900V without breakdown, in series.

(1)Find the equivalent C;

(2)Total voltage is 1000V, which one will breakdown first?

(3)what is the highest total voltage without breakdown?

Solution: According to the equation of equivalent C

(1): $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ ⇨ $C = \frac{200 \times 300}{200 + 300} = 120(pF)$

2.2 Capacitance and Capacitor

The Combination of Capacitors

(2): $V = V_1 + V_2 = 1000V$ $C_1V_1 = C_2V_2$

⇨ $V_1 = 600V$ $V_2 = 400V$

So,200pF,500V will breakdown first, and then...

(3): $Q_{1m} = C_1V_w = 200 \times 500 = 0.1\mu C$

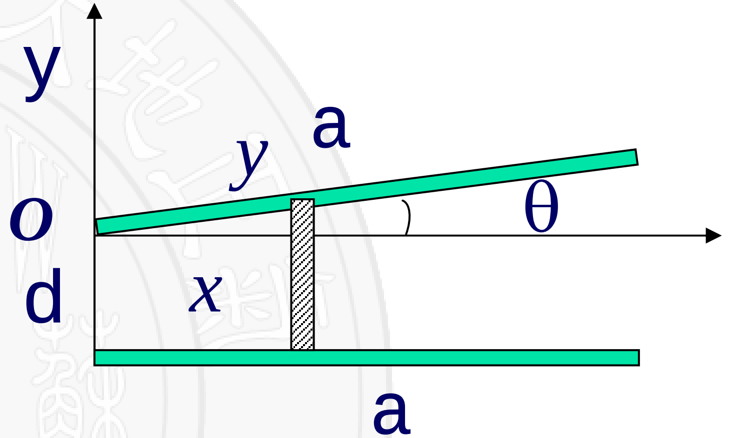
$Q_{2m} = C_1V_w = 300 \times 900 = 0.27\mu C$

$$V' = V_1 + V_2 = \frac{Q_{1m}}{C_1} + \frac{Q_{1m}}{C_2} = 500 + 333 = 833V$$

2.2 Capacitance and Capacitor

- Capacitance of not Parallel-plate capacitor

$$dC = \frac{\varepsilon_0 dS}{d} = \frac{\varepsilon_0 a dx}{d + x\tan\theta}$$



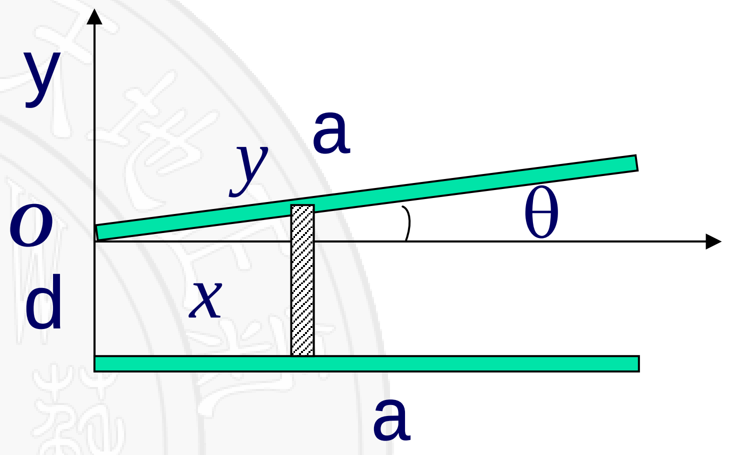
$$C = \int_0^a \frac{\varepsilon_0 a dx}{d + x\tan\theta} = \frac{\varepsilon_0 a}{\tan\theta}\ln(d + x\tan\theta)\Big|_0^a$$

$$C = \frac{\varepsilon_0 a}{\tan\theta}\ln\left(\frac{d + a\tan\theta}{d}\right) = \frac{\varepsilon_0 a}{\tan\theta}\ln\left(1 + \frac{a\tan\theta}{d}\right)$$

2.2 Capacitance and Capacitor

$$C = \frac{\varepsilon_0 a}{\tan\theta}\ln\left(\frac{\mathrm{d} + a\tan\theta}{\mathrm{d}}\right)$$

$$= \frac{\varepsilon_0 a}{\tan\theta}\ln\left(1 + \frac{a\tan\theta}{d}\right)$$



As θ is very small, tanθ≈ θ,and

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \cdots$$

$$C = \frac{\varepsilon_0 a}{\tan\theta}\frac{a\tan\theta}{d}\left(1 - \frac{1}{2}\frac{a\theta}{d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{a\theta}{2d}\right)$$