Chapter 2 Conductor & Dielectric

2.1 The Conductor in Electrostatic Field

2.2 Capacitance and Capacitor

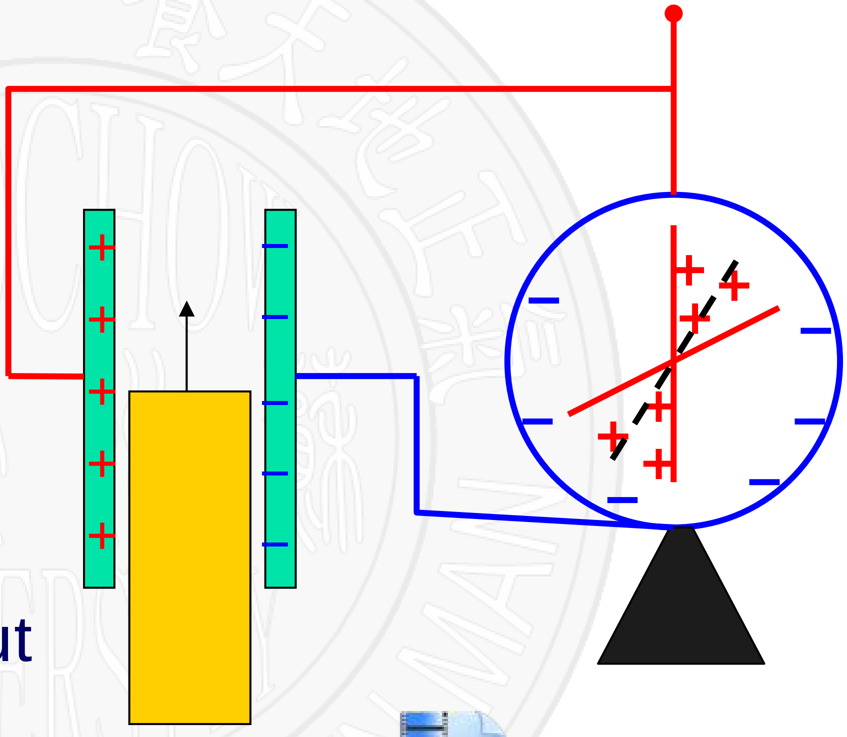
2.3 Dielectrics in Electric Field

2.4 The Energy Storage in Electric Field

2.3 Dielectrics in Electric Field

Faraday's experiment

When dielectric slab inserted into the capacitor, the angle of the needle of electroscope gets smaller.



What do you think about the phenomena ?

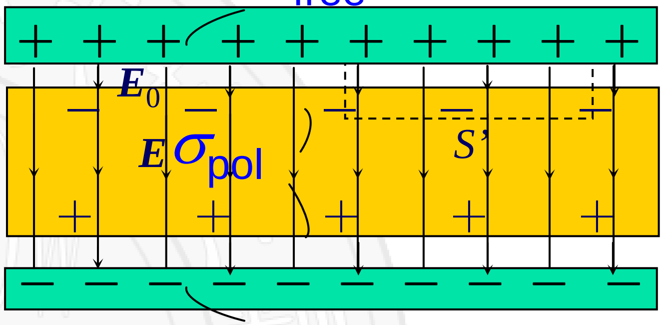
Faraday's demo

2.3 Dielectrics in Electric Field

Introduction

- Parallel plates capacitor:

$$C = \frac{\varepsilon_0 S}{d}$$ (No Dielectrics)



- Add a dielectric between the plates:

C increased
- Given Q, V decreased
- Given V,Q is increased

2.3 Dielectrics in Electric Field

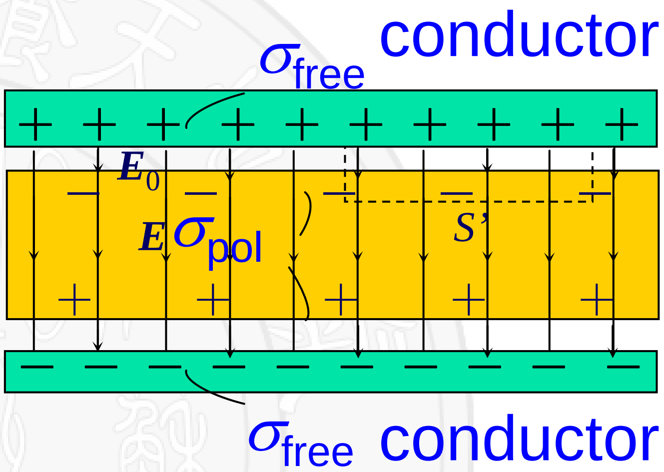
Introduction

- If Q is cont., V must be decreased

Why?

It means E decreased ?

$$V_P = \int_P^R \vec{E} \cdot d\vec{l}$$



Choose a Gaussian surface, broken lines

$$\oiint_{(S)} \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \sum_{(S内)} q_i$$

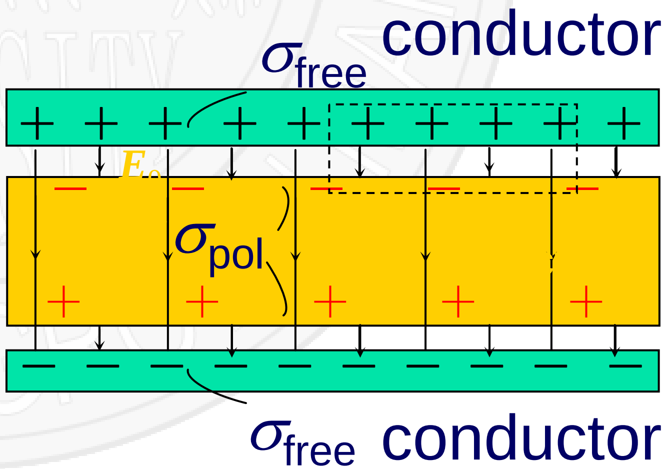
$\sum$q decreased. But free charge not changed on plate

2.3 Dielectrics in Electric Field

Introduction

So, negative charges appear inside Gaussian surface

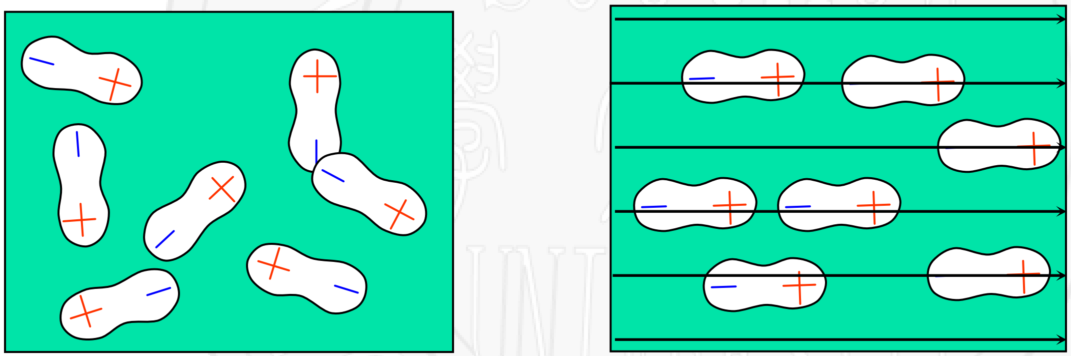
They generally appear at the surface of the insulator, called polarized (or induced) charged and produce field in opposite direction of original field inside a dielectrics.



2.3 Dielectrics in Electric Field

◈ The Molecular Theory of Induced Charges

Polar Molecules: Center of +nuclei and –electron not coincide.



Show polarization

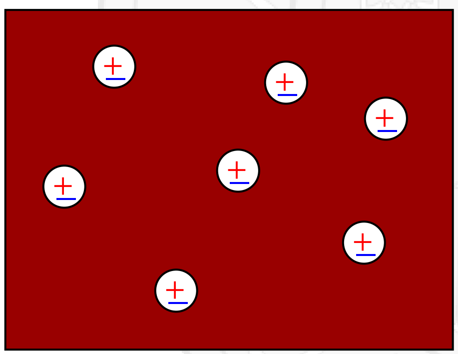
Orientation polarization

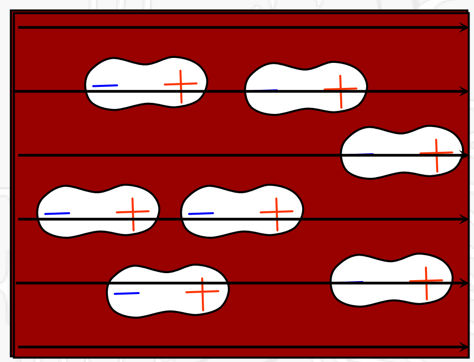
2.3 Dielectrics in Electric Field

The Molecular Theory of Induced Charges

Material made up of Molecules

Non-polar Molecules: Center of +nuclei and –electron coincide.





Show polarization

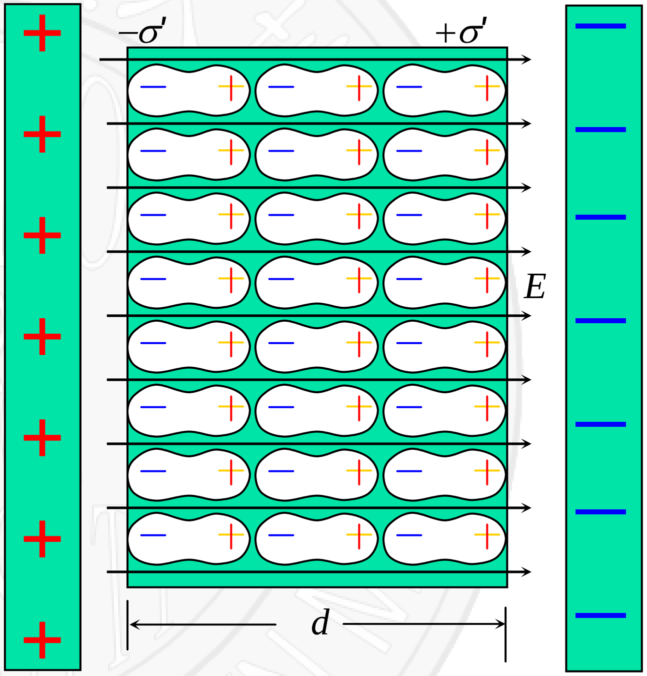
Show polarization

Displacement polarization

2.3 Dielectrics in Electric Field

◈ The Molecular Theory of Induced Charges

Whether Displacement or Orientation polarization, polarization charges will appear on surface of dielectrics



Within the remainder of the dielectric the net charge per unit volume remains zero.

Breakdown for Dielectrics

2.3 Dielectrics in Electric Field

The Molecular Theory of Induced Charges

The extent of polarization is described by a vector called polarization ***P***.

Definition of ***P***

$$\vec{\boldsymbol{P}} = \frac{\sum_{(\Delta V)} \vec{\boldsymbol{p}}_i}{\Delta \boldsymbol{V}}$$

If there are *n* molecules per unit volume, the average dipole moment is p_i ,

$$\overrightarrow{\boldsymbol{P}} = n\,\overrightarrow{\boldsymbol{p}}_i$$

Polarization is dipole moment per unit volume

2.3 Dielectrics in Electric Field

◈ Polarization Vector P & Bound Charges

In general, ***P*** will vary from point to point, vector function. ***P*** = ***P*** (x,y,z)

P's unit: C/m^2,P's direction is in the same direction of ***E***

The relationship between P and E:

The molecular charge separation ***l*** ∝ E， called linear or isotropic(各向同性)

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

Anisotropic Material

χ_e susceptibility of dielectrics , dimensionless

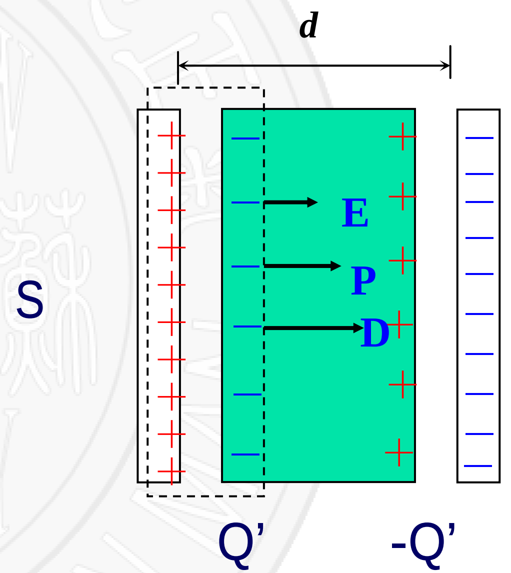
2.3 Dielectrics in Electric Field

◈ Polarization Vector P & Bound Charges

Polarized dielectric like a big dipole.

$$\sum_{(\Delta V)} \boldsymbol{p}_i = Q'd$$

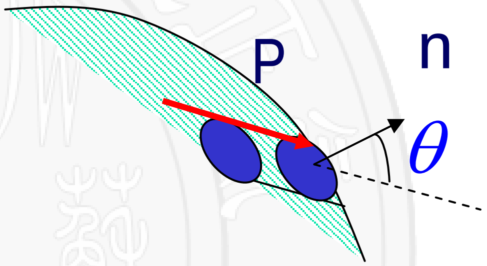
$$\boldsymbol{P} = \frac{\sum_{(\Delta V)} p_i}{\Delta V} = \frac{Q'd}{sd} = \sigma'$$



2.3 Dielectrics in Electric Field

Polarization Vector P & Bound Charges

In general



$$\vec{P}\cdot d\vec{S} = PdS\cos\theta$$

$$= nqldS\cos\theta = dq'$$

$$\frac{dq'}{dS} = nql\cos\theta = P\cos\theta = \vec{P}\cdot\vec{n}$$

$$\sigma' = \vec{P}\cdot\vec{n} = P\cos\theta$$

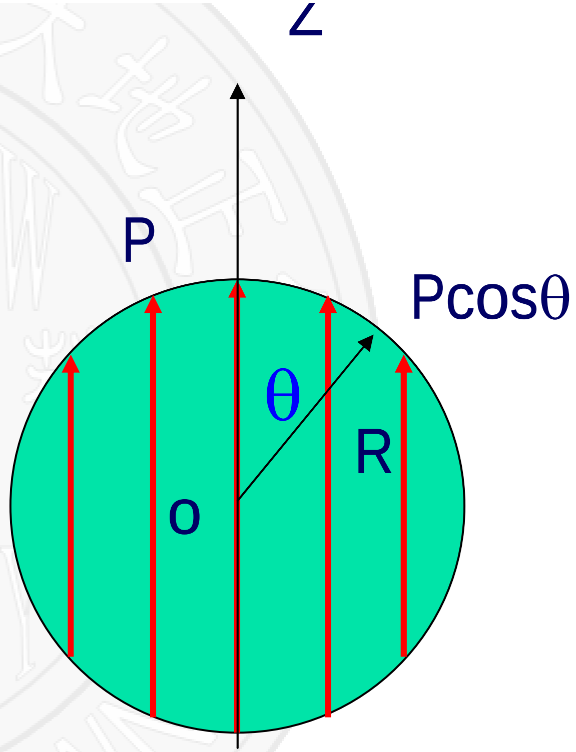
◈ Polarization Vector P & Bound Charges

$$\sigma' = \vec{P} \cdot \vec{n} = P\cos\theta$$

Find the electric field at origin

$ds = Rd\theta \cdot R\sin\theta \cdot d\phi$

$E' = P/3\varepsilon_0$



2.3 Dielectrics in Electric Field

◈ Polarization Vector P & Bound Charges

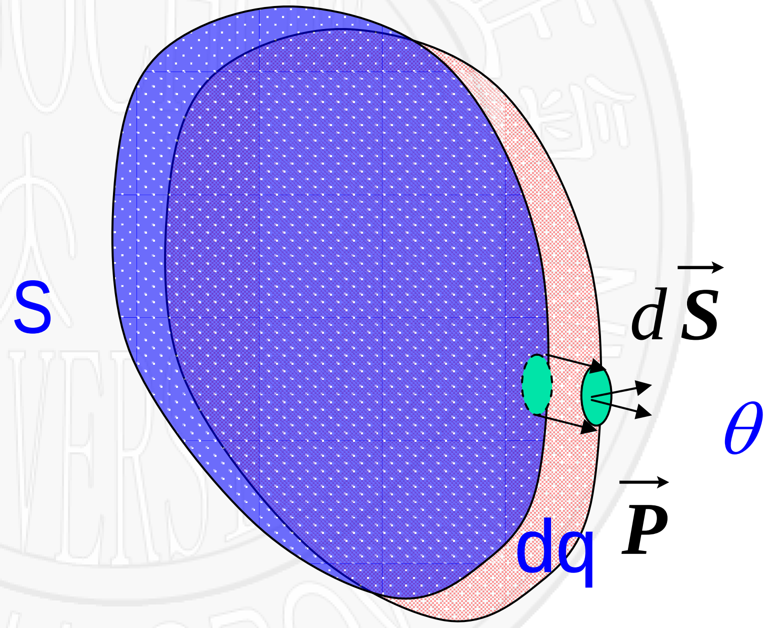
$$\vec{P}\cdot d\vec{S} = PdS\cos\theta = nq\cdot ldS\cos\theta$$

$$= nq\cdot \mathrm{dV}$$

$$= \mathrm{d}q$$

$$\oiint_{(S)} \vec{P}\cdot d\vec{S} = q$$

$$\oiint_{(S)} \vec{P}\cdot d\vec{S} = -q'$$



2.3 Dielectrics in Electric Field

Gauss's Law in dielectrics

$$\oiint_{(S)} \vec{E} \cdot \mathrm{d}\vec{S} = \frac{\sum\limits_{(S内)} q_i}{\varepsilon_0} = \frac{Q_0 + Q'}{\varepsilon_0}$$

$$\oiint_{(S)} \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0}\left(Q_0 - \oiint_{(S)} \boldsymbol{P} \cdot \mathrm{d}\boldsymbol{S}\right)$$

$$\oiint_{(S)} (\varepsilon_0 \vec{E} + \vec{P}) \cdot \mathrm{d}\vec{S} = Q_0$$

$$\oiint_{(S)} \vec{D} \cdot \mathrm{d}\vec{S} = Q_0$$

2.3 Dielectrics in Electric Field

◈ Gauss's Law in dielectrics

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Displacement Vector

The surface integral of ***D*** over any closed surface (the flux of ***D***) is equal to the free charge only within the surface.

For linear or isotropic dielectric

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

Anisotropic Material

$$\vec{D} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = (1 + \chi_e)\varepsilon_0 \vec{E}$$

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$$

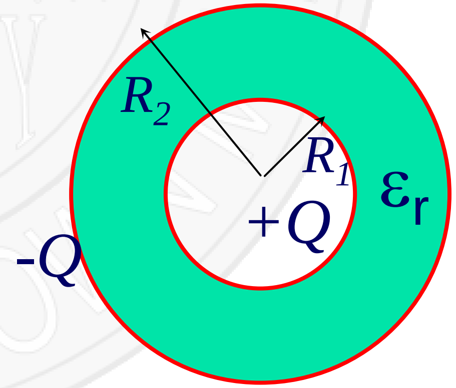
2.3 Dielectrics in Electric Field

◈ Gauss's Law in dielectrics

Example 2.6 A spherical capacitor consists of two concentric metal sphere shells of radii R_1 and $R_2(>R_1)$.The material between the sphere shells has a dielectric constant of ε_r. There is a charge $+Q$ on the inner sphere shell and a charge $-Q$ on the outer sphere shell. Find the capacitance and polarization charge.

Solution:

$$\oint\!\!\!\oint_{(S)} \boldsymbol{D}\cdot \mathrm{d}\boldsymbol{S} = Q_0$$



$$\oint\!\!\!\oint_{(S)} \boldsymbol{D}\cdot \mathrm{d}\boldsymbol{S} = D(4\pi r^2)$$

2.3 Dielectrics in Electric Field

Gauss's Law in dielectrics

$$4\pi r^2 D = Q$$

$$D = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{1}{4\pi\varepsilon_r\varepsilon_0}\frac{Q}{r^2}$$

$$V = \int_{R_1}^{R_2} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = \int_{R_1}^{R_2} E\cdot \mathrm{d}r = \int_{R_1}^{R_2}\frac{Q}{4\pi\varepsilon_r\varepsilon_0}\frac{\mathrm{d}r}{r^2}$$

$$= \frac{Q}{4\pi\varepsilon_r\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

2.3 Dielectrics in Electric Field

◈ Gauss's Law in dielectrics

$$P = \chi_e \varepsilon_0 E = \frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi r^2}$$

$$\sigma'_{inner} = -P_{R_1} = -\frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi R_1^2}$$

$$\sigma'_{outter} = P_{R_2} = \frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi R_2^2}$$

$$C = 4\pi \varepsilon_r \varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$