Chapter 2 Conductor & Dielectric

2.1 The Conductor in Electrostatic Field

2.2 Capacitance and Capacitor

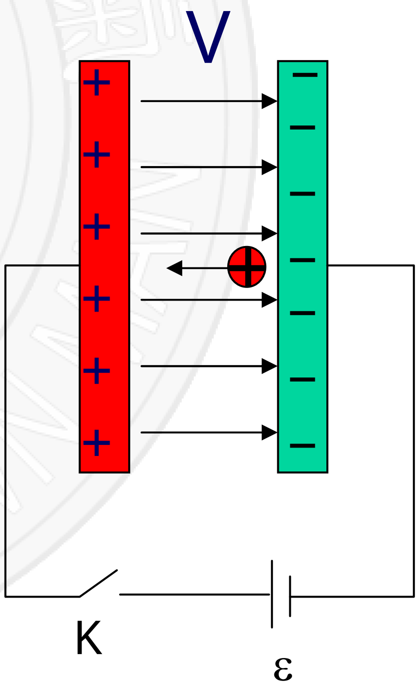
2.3 Dielectrics in Electric Field

2.4 The Energy Storage in Electric Field

2.4 Energy Storage in Electric Field

Energy stored in electric field

- Consider a capacitor with charge +/-q
- How much work is needed to bring a positive charge dq from the negative plate to the positive plate?
- Note: we are charging the capacitor!



$$dW = V(q)dq = \frac{q}{C}dq$$

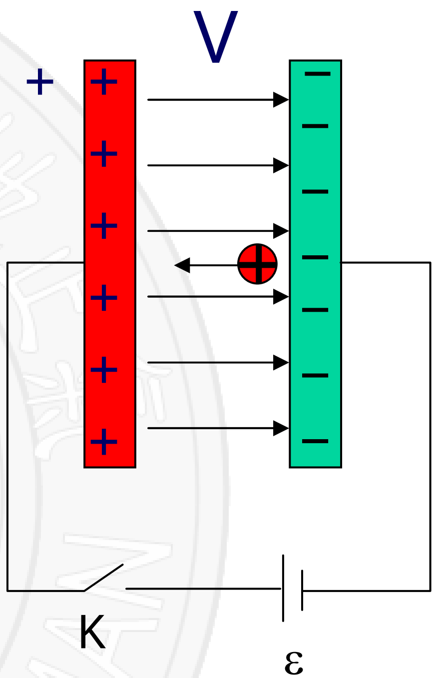
- How much work is needed to charge the capacitor from scratch?

2.4 Energy Storage in Electric Field

Energy stored in electric field

- The work is needed to charge the capacitor from scratch

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$



- Energy stored in the capacitor:

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Show Energy storage in Cap.

From Walter Lewin's Lecture

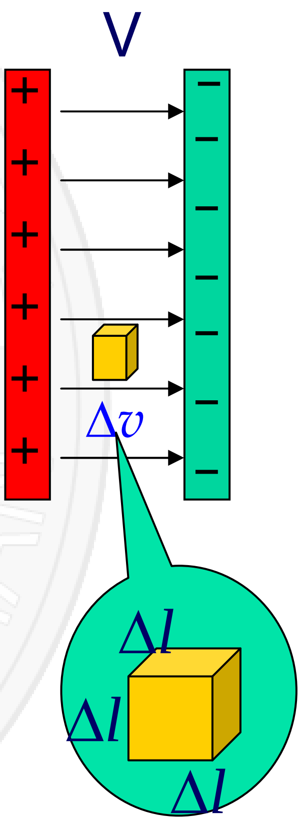
2.4 Energy Storage in Electric Field

Energy stored in electric field

- Where does the energy store?

Consider a small capacitor

Field



$$\Delta C = \frac{\varepsilon_0 (\Delta l)^2}{\Delta l}$$

$$\Delta W = \frac{1}{2} \Delta C V^2$$

$$V = E \Delta l$$

$$\Delta W = \frac{1}{2} \Delta C V^2 = \frac{1}{2} \frac{\varepsilon_0 (\Delta l)^2}{\Delta l} (E \Delta l)^2$$

$$\Delta W = \frac{1}{2} \varepsilon_0 E^2 (\Delta l)^3 = = \frac{1}{2} \varepsilon_0 E^2 \Delta v$$

2.4 Energy Storage in Electric Field

Energy stored in electric field

- Energy density w_e: energy per unit volume

$$w = \lim_{\Delta v \to 0} \frac{\Delta W}{\Delta v} = \frac{1}{2}\varepsilon_0 E^2$$

- If there are dielectrics in electric field

$$w = \frac{1}{2}\varepsilon_r \varepsilon_0 E^2 = \frac{1}{2}\vec{\boldsymbol{D}} \cdot \vec{\boldsymbol{E}}$$

- What is the essential part of energy?

2.4 Energy Storage in Electric Field

Energy stored in electric field

- What is the essential part of energy?

$$w = \frac{1}{2}\vec{\boldsymbol{D}} \cdot \vec{\boldsymbol{E}} = \frac{1}{2}(\varepsilon_0 \vec{\boldsymbol{E}} + \vec{\boldsymbol{P}}) \cdot \vec{\boldsymbol{E}}$$

$$= \frac{1}{2}\varepsilon_0 \boldsymbol{E}^2 + \frac{1}{2}\vec{\boldsymbol{P}} \cdot \vec{\boldsymbol{E}}$$

In free space, P=0

$$w = \frac{1}{2}\varepsilon_0 \boldsymbol{E}^2$$

E is the essential part

2.4 Energy Storage in Electric Field

Energy stored in electric field

- The total energy stored in field

$$W = \iiint_V w\,dv = \frac{1}{2}\iiint_V \varepsilon E^2 dv$$

Example 2.7 An isolated conducting sphere of radius *R*, in a vacuum, carries a charge q,

(a) Compute the total energy stored in the surrounding space.

(b) What is the radius R_0 of a spherical surface such that half the stored energy lies with it?

2.4 Energy Storage in Electric Field

Energy stored in electric field

Solution:

$$E=\begin{cases} 0 & (r<R) \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} \end{cases}$$

The energy d*W* that lies in a spherical shell betweens the radii *r* and *r* + d*r* is

$$\mathrm{d}W = \varepsilon E^2 4\pi r^2 dr = \frac{q^2}{8\pi\varepsilon_0}\frac{dr}{r^2}$$

$$W = \int \mathrm{d}W = \frac{q^2}{8\pi\varepsilon_0}\int_R^{\infty}\frac{\mathrm{d}r}{r^2} = \frac{q^2}{8\pi\varepsilon_0 R} = \frac{1}{2}\frac{q^2}{C}?$$

2.4 Energy Storage in Electric Field

Energy stored in electric field

(b) Find R_0 which makes the energy stored from radius R to R_0 equal to the total.

$$\frac{1}{2}W = \frac{q^2}{8\pi\varepsilon_0}\int_R^{R_0}\frac{\mathrm{d}r}{r^2}$$

$$\frac{q^2}{16\pi\varepsilon_0 R} = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{R_0}\right)$$

$R_0=2R$