Tutorial for chapter 2

Summary

- Electrostatic Equilibrium

  - If there is no motion of charges (except thermal motion) in conductor, the state of the conductor is called electrostatic equilibrium.

- Static Conditions: The electric field in a conductor is zero.

- The properties of conductor under electrostatic equilibrium

  - A conductor carrying static charge is an equipotential volume and its surface an equipotential surface.

Tutorial for chapter 2

Summary

- The electric field from the surface is at right angles to it at all points.

- Any net static charge on a conductor must reside on its surface.

- Surface charge density *s* at any point will vary from point to point.

- The direction of ***E*** for points close to the surface is at right angles to the surface, pointing away from the surface if the charge is positive.

Tutorial for chapter 2

Summary

- Electric field near surface

$$E = \frac{\sigma}{\varepsilon_0}$$

- The distribution of charge for isolated conductor

As a general rule, the charge density tends to be high on isolated conducting surfaces whose radii of curvature are small, and conversely.

- Hollow Conductor
  - Inside a conductor , ***E*** is zero everywhere.

Tutorial for chapter 2

Summary

- The Capacitance of an Isolated Conductor

$$C = Q/V$$

- The Capacitance Between Two Conductors

$$C = Q/V$$

- The Combinations of Capacitors
  - Capacitors in series

$$l/C_{eq} = 1/C_1 + 1/C_2 + \cdots + 1/C_3$$

Tutorial for chapter 2

Summary

- *Capacitors in parallel*

$$C_{eq} = C_1 + C_2 + \cdots + C_n$$

- Dielectrics in Electric Field
  - Polarization Vector P

$$\boldsymbol{P} = \frac{\sum \boldsymbol{p}_i}{\Delta v} = n\boldsymbol{p}$$

  - The Relationship Between **P** and **E** For Homogenous Medium

$$\boldsymbol{P} = \chi_e \varepsilon_0 \boldsymbol{E}$$

Tutorial for chapter 2

Summary

- The Bound Charge
  - The Volume Density For Bound Charge

$$\rho' = 0$$

  - The Surface Density For Bound Charge

$$\sigma' = \boldsymbol{P}\cdot\boldsymbol{n} = P\cos\theta = P_n$$

- Gauss's law in Dielectrics

$$\oint\!\!\!\oint_{(S)} \boldsymbol{D}\cdot \mathrm{d}\boldsymbol{S} = \sum_{(S内)} Q_0$$

Tutorial for chapter 2

Summary

- The Energy Storage in Electric Field
  - The Energy Storage in a Capacitor

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

  - The Energy Density For Electric Field

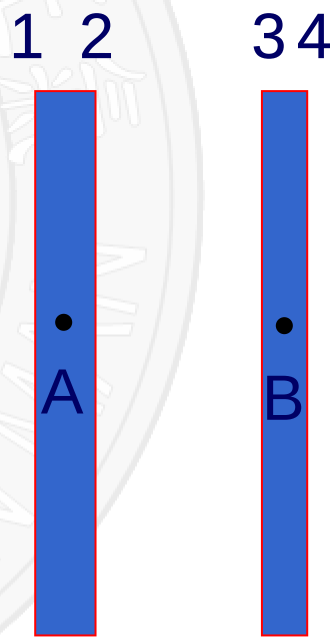
$$w = \frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E} = \frac{1}{2}\varepsilon E^2$$

  - The Energy Storage in Electric Field

$$W = \iiint_{(V)} w dV = \iiint_{(V)} \frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E} dV$$

Tutorial for chapter 2

2.1.4.Fig.2.14 shows two parallel conducting plates. Show (a) the facing surfaces carry equal but opposite charge density; (b) the backward surfaces carry equal and same charge density.i.e, $\sigma_2 = -\sigma_3$; $\sigma_1 = \sigma_4$.



$$E_A = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0}$$

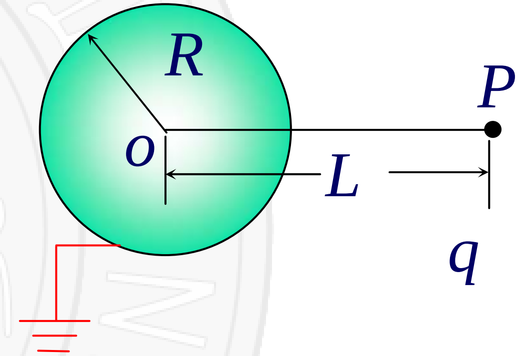
$$E_B = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0}$$

$$\sigma_2 = -\sigma_3$$

$$\sigma_1 = \sigma_4$$

Tutorial for chapter 2

2.1.5.Figure 2.15 shows a metallic sphere of radius *R* is grounded. A point charge *q* is placed at *P* a distance *L* from the center of the sphere. Find the induced charge on the sphere



Solution: Consider the center of the sphere, find the potential of the center

$$V_o=\frac{1}{4\pi\varepsilon_0}\frac{q}{L}+\oiint_{(S)}\frac{1}{4\pi\varepsilon_0}\frac{dq}{R}=\frac{1}{4\pi\varepsilon_0}\frac{q}{L}+\frac{1}{4\pi\varepsilon_0 R}\oiint_{(S)}dq$$

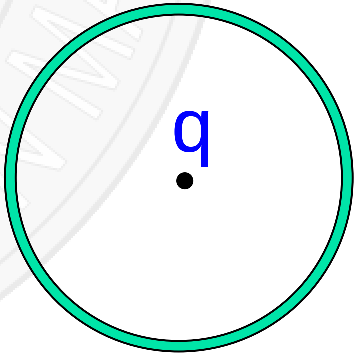
$$V_o=\frac{1}{4\pi\varepsilon_0}\frac{q}{L}+\frac{q'}{4\pi\varepsilon_0 R}=0$$

$$q'=-\frac{R}{L}q$$

If the sphere is not grounded, what's its potential?

Tutorial for chapter 2

2.1.9. An uncharged spherical, thin, metallic shell has a point charge q at its center. Give expressions for the electric potential (a) inside the shell and (b) outside the shell. (c) Has the shell any effect on the field due to q? (d) Has the presence of q any effect on the shell? (e) If a second point charge is held outside the shell, does this outside charge experience a force? (f) Does the inside charge experience a force? (g) Is there a contradiction with Newton's third law here?



Tutorial for chapter 2

2.3.3.A parallel-plate capacitor of area S and distance d apart is filled with two dielectrics as in the figure. If the total charge is $\pm Q$ on the plates,

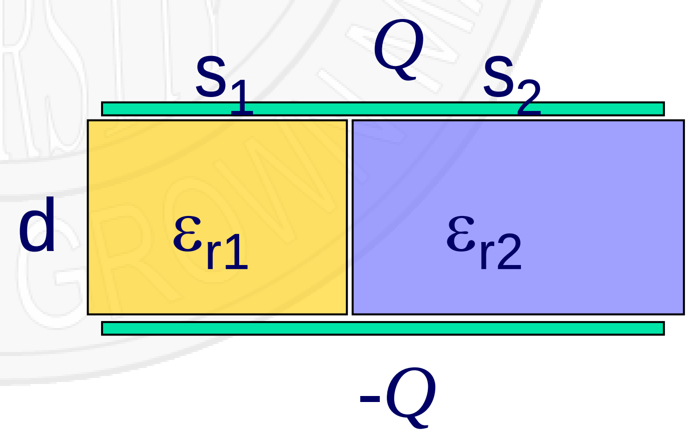
(a) How does the charge distribute on each part of the plate?

(b) Find the electric field in these dielectrics.

(c) Find the bound charge density on the surface of these dielectrics.

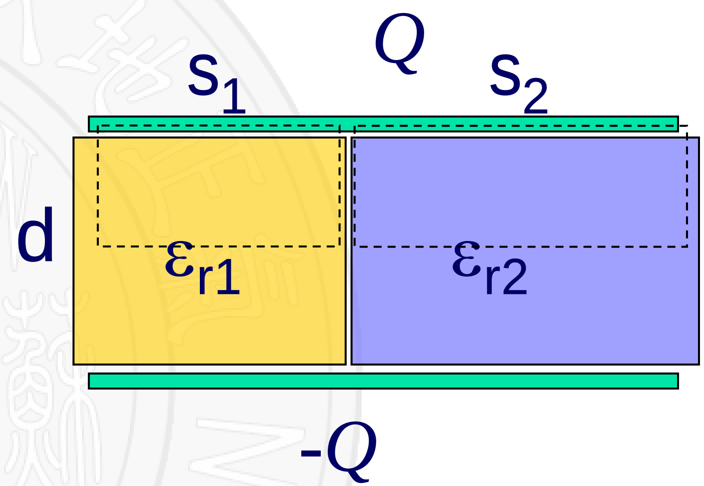
(d) Find that the capacitance

(e) Check this formula for all the limiting cases that you can think of



Tutorial for chapter 2

Solution: Applying Gauss's Law of Dielectrics in two part respectively, we got



Part I:

$$\oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = Q_0$$

$$\oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = D_1 S_1 = Q_1$$

$$D_1 = \frac{Q_1}{S_1}$$

Part II:

$$\oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = Q_0$$

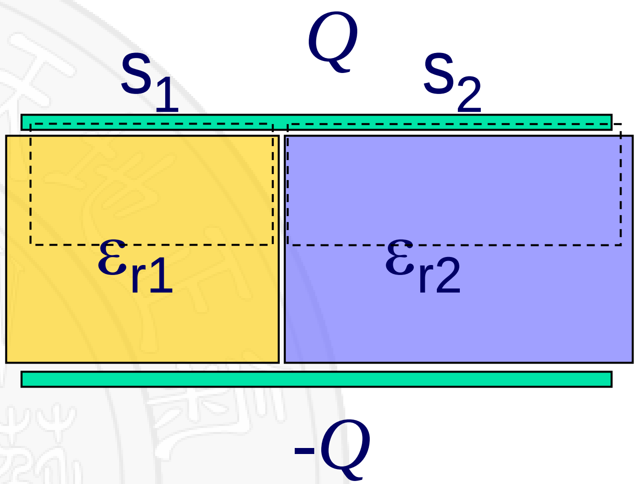
$$\oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = D_2 S_2 = Q_2$$

$$D_2 = \frac{Q_2}{S_2}$$

Tutorial for chapter 2

$$E_1 = \frac{D_1}{\varepsilon_{r1}\varepsilon_0} = \frac{Q_1}{\varepsilon_{r1}\varepsilon_0 S_1}$$

$E_1 = E_2$?



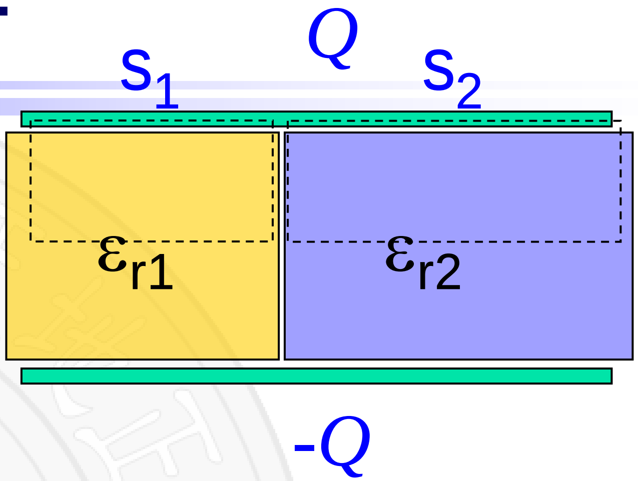
$$E_2 = \frac{D_2}{\varepsilon_{r2}\varepsilon_0} = \frac{Q_2}{\varepsilon_{r2}\varepsilon_0 S_2}$$

$$\begin{cases} \dfrac{Q_1}{\varepsilon_{r1} S_1} = \dfrac{Q_2}{\varepsilon_{r2} S_2} \\ Q = Q_1 + Q_2 \end{cases}$$

$$Q_1 = \frac{\varepsilon_{r1} S_1}{\varepsilon_{r1} S_1 + \varepsilon_{r2} S_2} Q$$

$$Q_2 = \frac{\varepsilon_{r2} S_2}{\varepsilon_{r1} S_1 + \varepsilon_{r2} S_2} Q$$

Tutorial for chapter 2



$$E_1 = \frac{Q_1}{\varepsilon_{r1}\varepsilon_0 S_1} = \frac{Q}{\varepsilon_0(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)} = E_2$$

$$\vec{\boldsymbol{P}} = \chi_e \varepsilon_0 \vec{\boldsymbol{E}}$$

$$\vec{\boldsymbol{P}}_1 = \chi_{e1}\varepsilon_0 E_1 = (\varepsilon_{r1} - 1)\varepsilon_0 \vec{E}_1$$

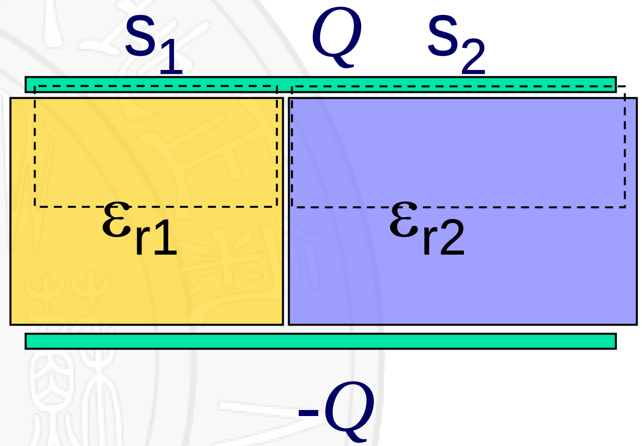
$$\sigma_{1up} = -(\varepsilon_{r1} - 1)\varepsilon_0 E_1 = (1 - \varepsilon_{r1})\frac{Q}{(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)}$$

$$\sigma_{1down} = -\sigma_{1up}$$

$$\sigma_{2up} = (1 - \varepsilon_{r2})\frac{Q}{(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)} \qquad \sigma_{2down} = -\sigma_{2up}$$

Tutorial for chapter 2

From the definition of capacitance and the following Eqs, We get



$$E_1 = \frac{Q}{\varepsilon_0(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)} = E_2$$

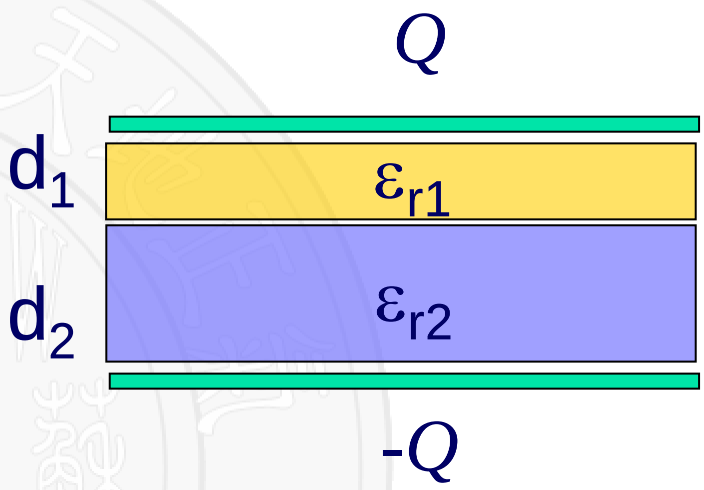
$$V = E_1 d = \frac{Qd}{\varepsilon_0(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)}$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0(\varepsilon_{r1}S_1 + \varepsilon_{r2}S_2)}{d}$$

Equivalent to capacitor in parallel

- If $S_1 = S_2 = S/2$ $$C = \frac{\varepsilon_0 S(\varepsilon_{r1} + \varepsilon_{r2})}{2d}$$
- If $S_1 = S$, $S_2 = 0$ $$C = \frac{\varepsilon_0 \varepsilon_{r1} S}{d}$$

Tutorial for chapter 2



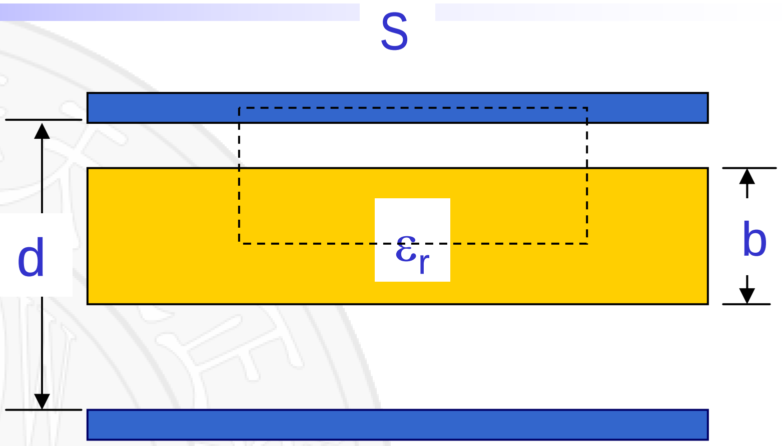
Tutorial for chapter 2

2.3.9.Figure 2.45shows a dielectric slab of thickness b and dielectric constant ε_r placed between the plates of a parallel-plate capacitor of plate area S and separation d. A potential difference V_0 is applied with no dielectric present. The battery is then disconnected and the dielectric slab inserted.

(a) Calculate the capacitance C_0 before the slab is inserted.
(b) Calculate the free charge Q_0.
(c) Calculate the electric field in the gap.
(d) Calculate the electric field in the dielectric.
(e) Calculate the potential difference between the plates.
(f) Calculate the capacitance with the slab in place.

Tutorial for chapter 2

Solution: The battery is disconnected after charging, the charges on plates don't change.



(a) $C_0=\varepsilon_0 S/d$

(b) $Q_0=C_0V_0=\varepsilon_0 SV_0/d$

$E_0=\sigma_0/\varepsilon_0=Q_0/S\varepsilon_0=V_0/d$

(c) The charges don't change, neither the surface charge density on the plates. So the electric field in the gap is also E_0

(d) From Gauss's law, we choose a Gaussian surface (broken lines), as shown in the figure

Tutorial for chapter 2

$$\oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = Q_0 \qquad \oiint_{(S)} \vec{\boldsymbol{D}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = D\Delta S = \sigma_0 \Delta S$$

$D = \sigma_0$ $\qquad$ $D=\varepsilon_r\varepsilon_0 E$

(d) $$E = \frac{D}{\varepsilon_r\varepsilon_0} = \frac{Q_0}{\varepsilon_r\varepsilon_0 S} = \frac{\varepsilon_0 S V_0}{\varepsilon_r\varepsilon_0 S d} = \frac{V_0}{\varepsilon_r d}$$

$E=E_0/\varepsilon_r$

(e) $$V = \int_{top}^{buttom} E dl = Eb + E_0(d-b) = \frac{V_0}{\varepsilon_r d} b + \frac{V_0}{d}(d-b)$$

Tutorial for chapter 2

(f)

$$C = \frac{Q}{V} = \frac{\varepsilon_0 S V_0}{[\frac{V_0 b}{\varepsilon_r d} + \frac{V_0 (d-b)}{d}]d} = \frac{\varepsilon_r \varepsilon_0 S}{b + \varepsilon_r (d-b)}$$

If b=d, then C=$\varepsilon_r\varepsilon_0$S/d,fulfiled with dielectrics

If b=0, then C=ε_0S/d, without dielectrics

Tutorial for chapter 2

2.3.11. Figure 2.45 shows that a spherical capacitor is filled with two dielectrics of dielectric constant ε_{r1} and ε_{r2}, respectively. If the inner and outer plates is charged $\pm Q$.

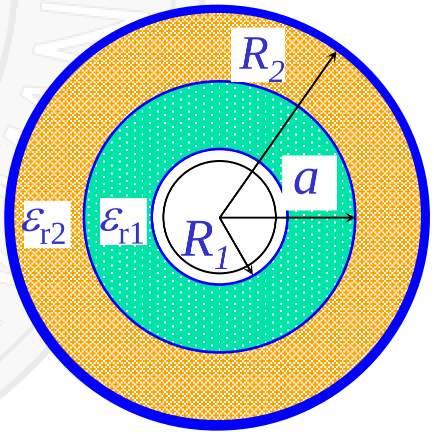
(a) Compute the electric field in these dielectrics.

(b) Compute the bound charge density on each surface of these dielectrics.

(c) Compute the potential difference V between the plates.

(d) Show the capacitance is given by

$$C = \frac{4\pi\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}aR_1R_2}{\varepsilon_{r2}R_2(a-R_1)+\varepsilon_{r1}R_1(R_2-a)}$$

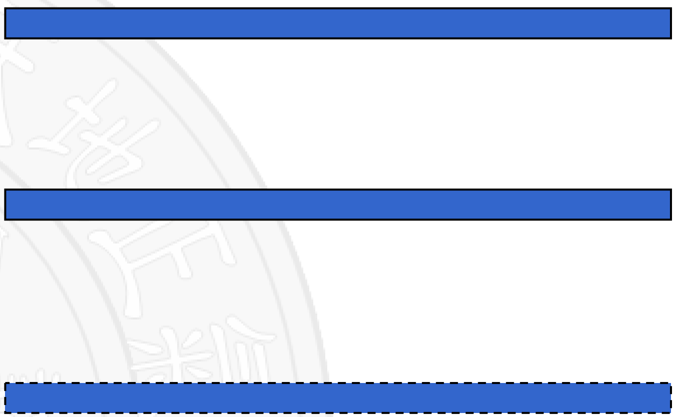


Tutorial for chapter 2

2.4.6.A parallel-plate capacitor has plates of area S and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. Derive expressions in terms of S, d, and V for (a) the new potential difference, (b) the initial and the final stored energy, and (c) the work required to separate the plates. (d) If the battery remains connecting to the capacitor, do these again.

Tutorial for chapter 2

Disconnected situation:



(a) The new potential difference:

V'=E×2d=2d×V/d=2V

(b) $$W_{initial} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 S}{d}V^2$$

$$W_{final} = \frac{1}{2}C'V'^2 = \frac{1}{2}\frac{\varepsilon_0 S}{2d}(2V)^2 = \frac{\varepsilon_0 S}{d}V^2$$

(c) $$A = W_{final} - W_{initial} = \frac{1}{2}\frac{\varepsilon_0 S}{d}V^2$$

Tutorial for chapter 2

$$A=\int_{d}^{2d}F\,dx=\int_{d}^{2d}QE\,dx=QE\int_{d}^{2d}dx=QEd$$

$$E=\frac{V}{d}\qquad Q=CV=\frac{\varepsilon_0 S}{d}V$$

$$A=QEd=\frac{\varepsilon_0 S V^2}{d}$$?

$$E=\frac{V}{2d}$$