Chapter 3 Steady Electric Current

3.1 Steady-State Condition

3.2 Resistance and the Ohm’s Law

3.3 Electromotive Force and Kirchhoff’s law

3.2 Resistance and Ohm's Law

◈ Resistance Resistivity and Conductivity

✷ Consider that a potential difference is applied on the same shapes and sizes but different materials(wood and copper). The currents will be very different.

- The reason same shapes and sizes, different resistance

- Resistance is defined as

$$R = \frac{V}{I}$$

- Unit: 1Ω=1V/A

3.2 Resistance and Ohm's Law

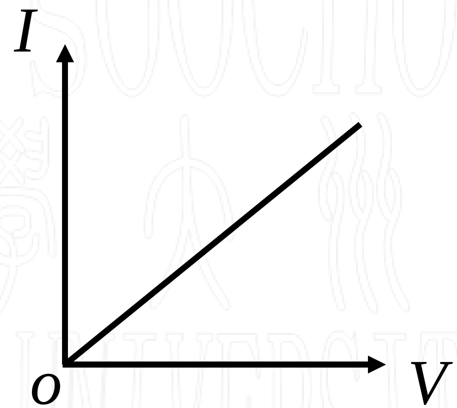
◈ Ohm's Law

✷ The current passing through a conductor is proportional to the potential difference.

$$I = \frac{V}{R}$$

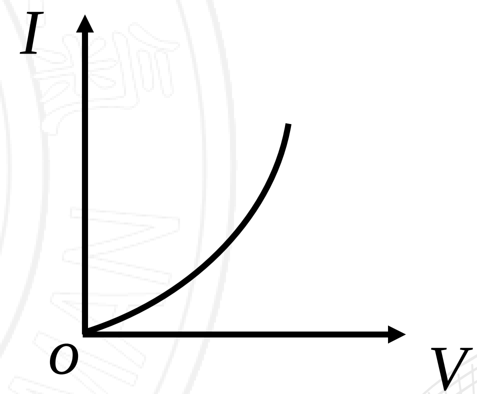
$$G = \frac{1}{R}$$

Linear resistance



This conductor obeys Ohm's law.

nonlinear resistance



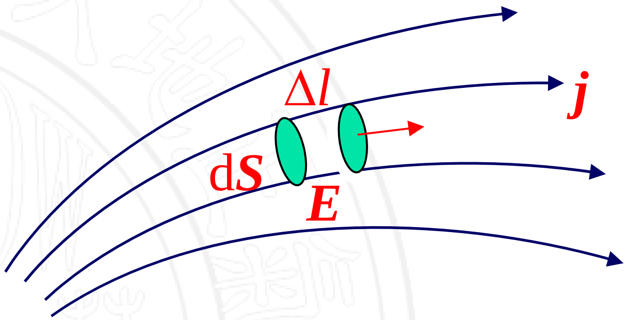
This conductor doesn't obey Ohm's law.

G: conductance, unit: siemens

3.2 Resistance and Ohm's Law

Differential Form Ohm's Law

$$\Delta V = E\Delta l \qquad \Delta R = \rho\frac{\Delta l}{\Delta S}$$



$$\Delta I = \frac{\Delta V}{\Delta R} = \frac{E\Delta l}{\rho\frac{\Delta l}{\Delta S}} = \frac{E\Delta S}{\rho}$$

$$j = \frac{\Delta I}{\Delta S} = \frac{1}{\rho}E = \sigma E$$

ρ: resistivity. σ:conductivity

$$\vec{J} = \sigma\vec{\mathbf{E}}$$

Electric fields cause charges to move

It reflects the proportionality between E and J in each point

3.2 Resistance and Ohm's Law

- Resistance

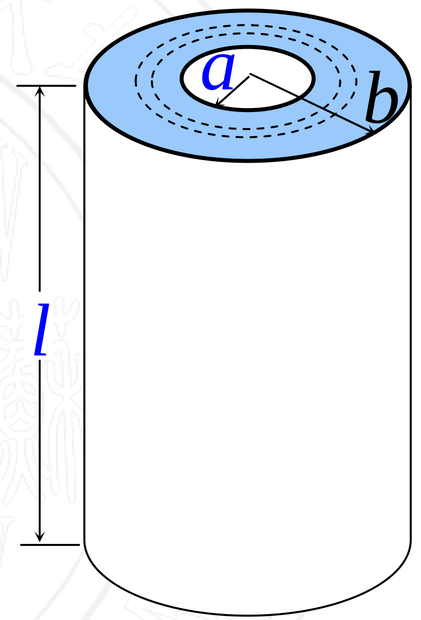
ρ resistivity

$$\rho_t = \rho_0[1+\alpha(t-t_0)]$$

$$R_t = R_0[1+\alpha(t-t_0)]$$

$$R = \rho\frac{L}{S}$$

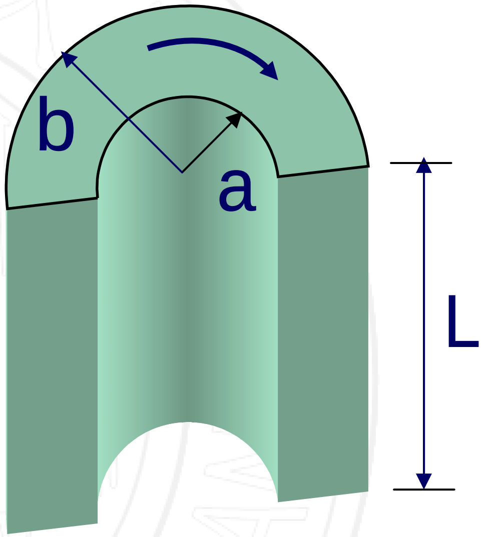
$$R = \int \rho \frac{dl}{S}$$



3.2 Resistance and Ohm's Law

- Resistance

What is the Resistance as in the figure shown ?



3.2 Resistance and Ohm's Law

Last time ...

Electric current I:

$$I = \frac{dq}{dt}$$

Electric current density j

$$j = \frac{\mathrm{d}I}{\mathrm{d}S_{\perp}}$$

Continuity equation:

$$\oint\!\!\oint_{(S)} \vec{j} \cdot \mathrm{d}\vec{S} = -\frac{dq}{dt}$$

3.2 Resistance and Ohm's Law

Last time...

Steady-state condition:

$$\oiint_{(S)} \vec{j} \cdot \mathrm{d}\vec{S} = 0$$

Ohm's law:

Macroscopic: $I = \dfrac{V}{R}$

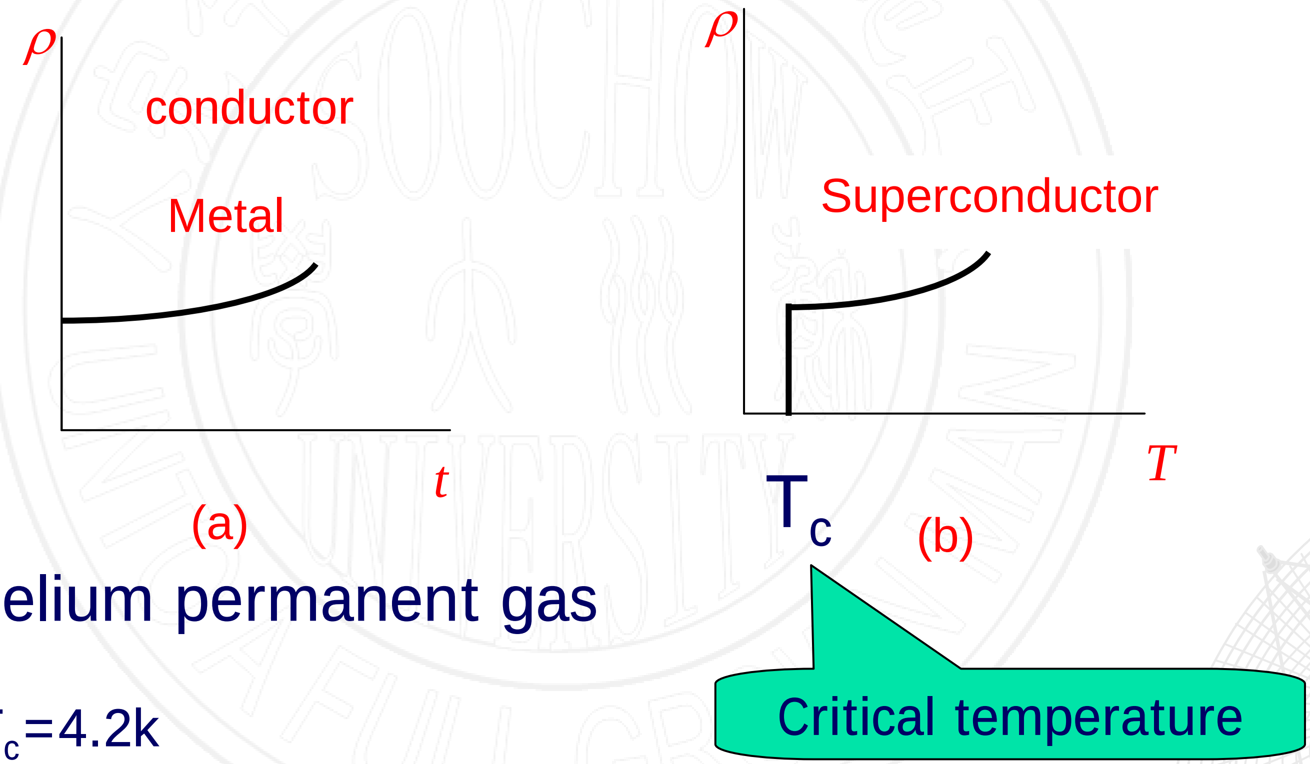
Microscopic: $\vec{J} = \sigma \vec{E}$

R=resistance $R = \int \rho \dfrac{\mathrm{d}l}{S}$

3.2 Resistance and Ohm's Law

- Superconductivity

$$\rho_t = \rho_0 [1 + \alpha (t - t_0)]$$



Onnes helium permanent gas

Mercury T_c=4.2k

3.2 Resistance and Ohm's Law

Ohm's Law in a Microscopic View

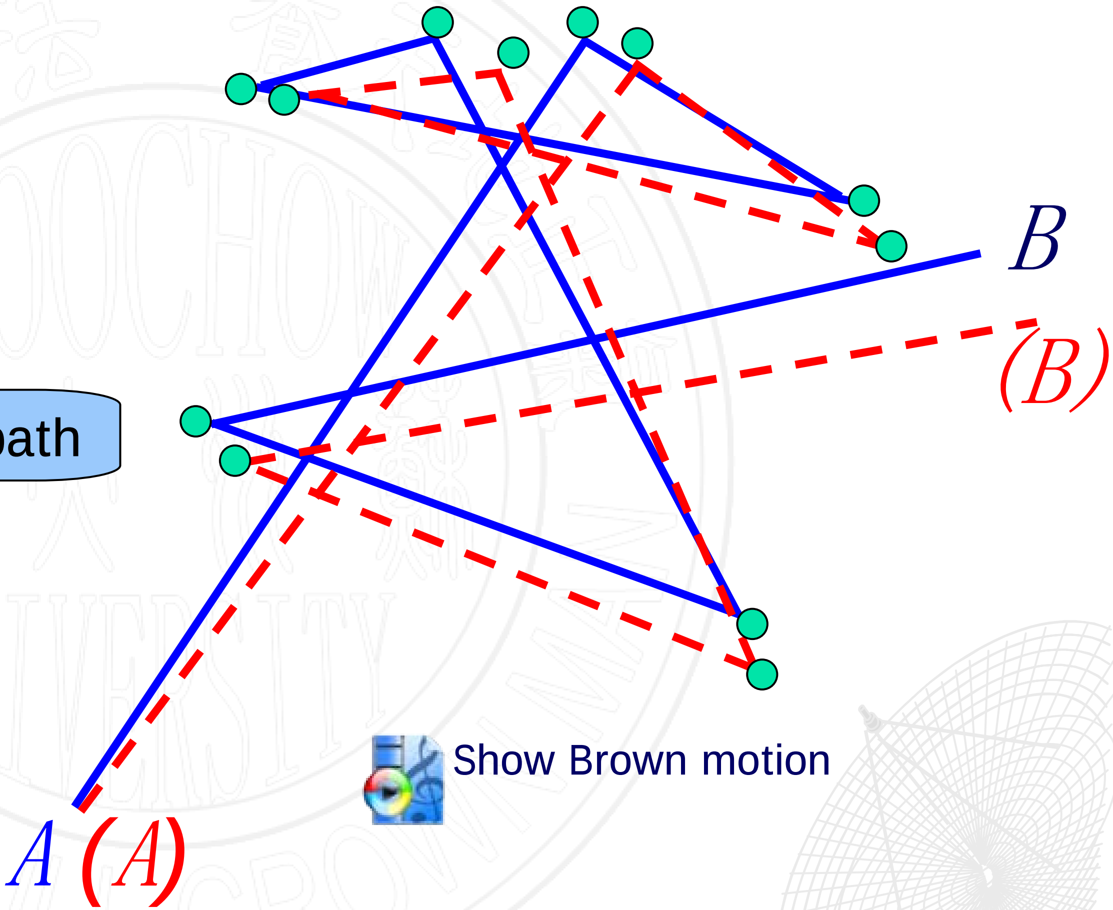
$$\rho_t = \rho_0[1+\alpha(t-t_0)]$$

$$a = \frac{qE}{m}$$

Average free path

$$u_d = \frac{1}{2}a\left(\frac{\lambda}{\overline{v}}\right) = \frac{qE\lambda}{2m\overline{v}}$$

Average speed



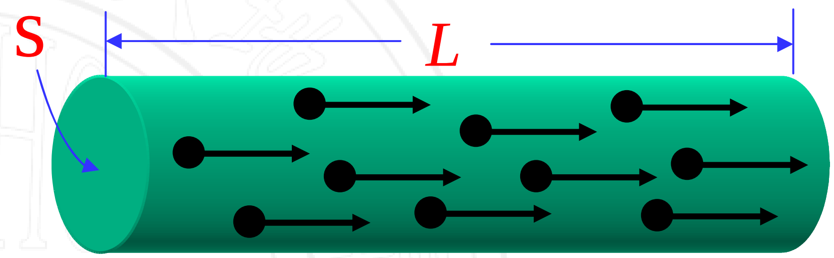
Show Brown motion

3.2 Resistance and Ohm's Law

◈ Ohm's Law in a Microscopic View

$$I=\frac{nSlq}{t}=\frac{nSlq}{l/u_d}=nqsu_d$$

Charge: q



Number density: n

$$j=\frac{\mathrm{d}I}{\mathrm{d}S_\perp}=nqu_d=nq\frac{qE}{2m}\left(\frac{\lambda}{\bar{v}}\right)$$

$$j=nq\frac{q}{2m}\left(\frac{\lambda}{\bar{v}}\right)E \qquad (\vec{J}=\sigma\vec{E})$$

$$\sigma=nq\frac{q}{2m}\left(\frac{\lambda}{\bar{v}}\right) \qquad \rho=\frac{1}{\sigma}=\frac{2m\bar{v}}{nq^2\lambda}$$

3.2 Resistance and Ohm's Law

The Combinations of Resistors

$$\oiint_{(S)} \vec{j} \cdot \mathrm{d}\vec{S} = 0$$

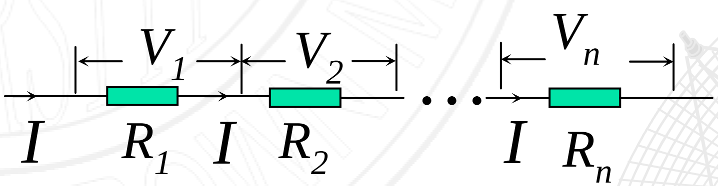
$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

Theory for steady field

- Resistors in series

$$V = V_1 + V_2 ... + V_n = IR_1 + IR_2 + ... + IR_n = I(R_1 + R_2 + ... + R_n)$$

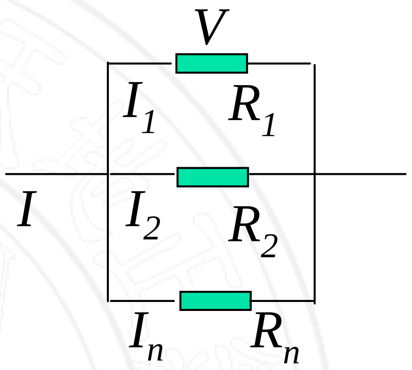
$$R = R_1 + R_2 + ... + R_3$$



3.2 Resistance and Ohm's Law

The Combinations of Resistors

- Resistors in parallel



$$I = I_1 + I_2 + \cdots + I_n$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_n} = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Reciprocal